Exercise 3

Foundations of Agent Programming

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Exercise 3.1

Let Σ be the definitions of $Trans$ and $Final$ (Slides IIc/9-15) together with the additional axioms $Poss(A, s, s') \equiv s' = Do(A, s)$ and $Poss(B, s, s') \equiv s' = Do(B, s)$.

a) For every program δ and situation s give a recursive definition for a set $T_i(\delta, s)$ which contains all tuples (δ', s') such that δ' is the remaining program and s' the resulting situation after performing i transition steps according to the definition of $Trans$.

Show that for all $i \geq 1$: $T_i(A^*, S_0) = \{((nil; A^*), \underbrace{Do(A, \ldots, Do(A}_{i}, S_0)\ldots))\}$.

b) Give a definition for a set $Trace_k(\delta, s)$ which contains all final situations s' that are reachable from s via k transition steps of program δ.

Show that $\bigcup_k Trace_k(\delta_1, S_0) = \bigcup_k Trace_k(\delta_2, S_0)$ for $\delta_1 = A^*$ and $\delta_2 = A^{\|}$, assuming that for all $i \geq 1$ we have

$$T_i(A^{\|}, S_0) = \{((\underbrace{nil \parallel (\ldots \parallel (nil}_{i} \parallel A^{\|})\ldots)), \underbrace{Do(A, \ldots, Do(A}_{i}, S_0)\ldots))\}.$$

c) Show that $\bigcup_k Trace_k(\delta_1, S_0) \neq \bigcup_k Trace_k(\delta_2, S_0)$ when considering the programs $\delta_1 = A^* \parallel B^*$ and $\delta_2 = (A; B)^*$.

Exercise 3.2

Consider the *robot in a maze* from the lecture (Slides IIc/18ff) together with the following fluents:

$Facing(d)$	$\hat{=}$	the robot is facing direction $d \in \{0, 1, 2, 3\}$, where 0 corresponds to north, 1 to east, 2 to south and 3 to west
$At(x, y)$	$\hat{=}$	the robot is currently at position $(x, y) \in \mathbb{Z} \times \mathbb{Z}$, where $(x + 1, y + 1)$ marks the position diagonally southeast to (x, y)
$At(Exit)$	$\hat{=}$	the robot is currently at the exit
$Blocked$	$\hat{=}$	the position one step ahead of the robot is blocked

a) Introduce a sensing action $SenseExit$ that reports $true$ iff the robot is at the exit. Modify the program from Slide IIc/19 to account for this action.

b) Give an initial state axiom such that the robot starts at position $(0, 0)$ facing east and specify precondition axioms and effect axioms for the actions $TurnRight$, $TurnLeft$, $GoForward$, $SenseBlocked$ and $SenseExit$.

c) Determine the formula $Sensed[h, S_0]$ for the following history h:

$$h = (SenseBlocked, true), (TurnRight, true), (SenseBlocked, false), (GoForward, true).$$

d) Let Ax be the axiomatizations from a) and b) and h the history from c). Show that

$$Ax \wedge Sensed[h, S_0] \models Holds(At(0, 1), End[h, S_0]) \wedge Holds(Facing(2), End[h, S_0])$$