Chapter Overview

- From L_1 to L_2: SLD-derivations
- From L_2 to Prolog: Backtracking
- The cut in Prolog

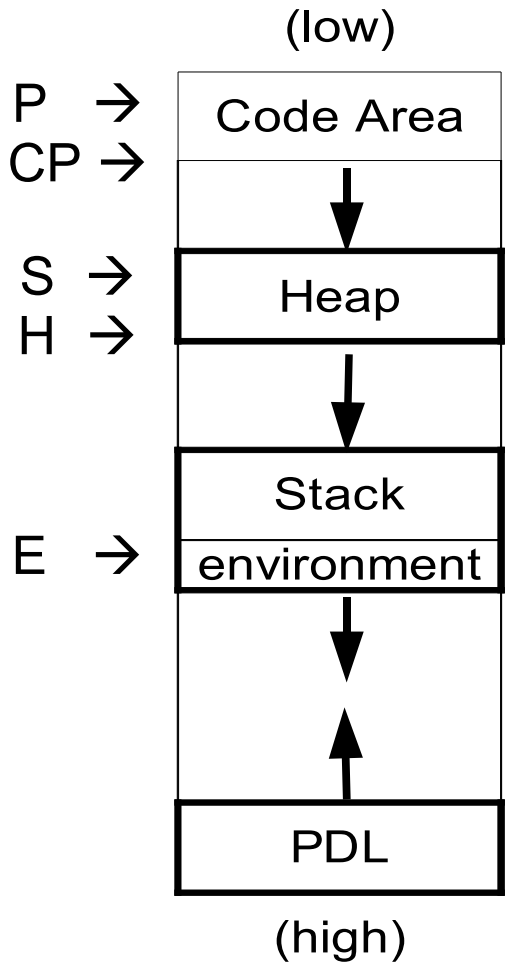
From L_1 to L_2

A program is a set of clauses - no two of which define the same predicate.

A query is a finite sequence of atoms.

WAM_2 Memory Layout

Registers



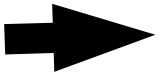
Argument and

term registers:

X_1, X_2, ...

The Basic Picture

```
first(X,[X|_]).
second(X,[_,X|_]).
sequence(Y,Z) :-
    first(X,Z),
    second(X,Y).
```



```
first/2:     program_code_for_first
             proceed
second/2:    program_code_for_second
             proceed
sequence/2:  allocate
             program_code_for_sequence
             query_code_for_first
             call first/2
             query_code_for_second
             call second/2
             deallocate
```

```
?- sequence([a,b],[b,c]),
   sequence([b,c],[c,d]).
```

```
             allocate
             query_code_for_sequence
             call sequence/2
             ...
```

Handling Facts

`call p/n` ≡ CP ← P + *instruction_size*(P);
P ← @(p/n);

`proceed` ≡ P ← CP;

instruction_size(P): number of memory units occupied by the instruction at address *P*

Handling Clauses

Y permanent variable in a clause H :- B_1, ..., B_m

$:\Leftrightarrow$

Y occurs in more than one of these m sets:

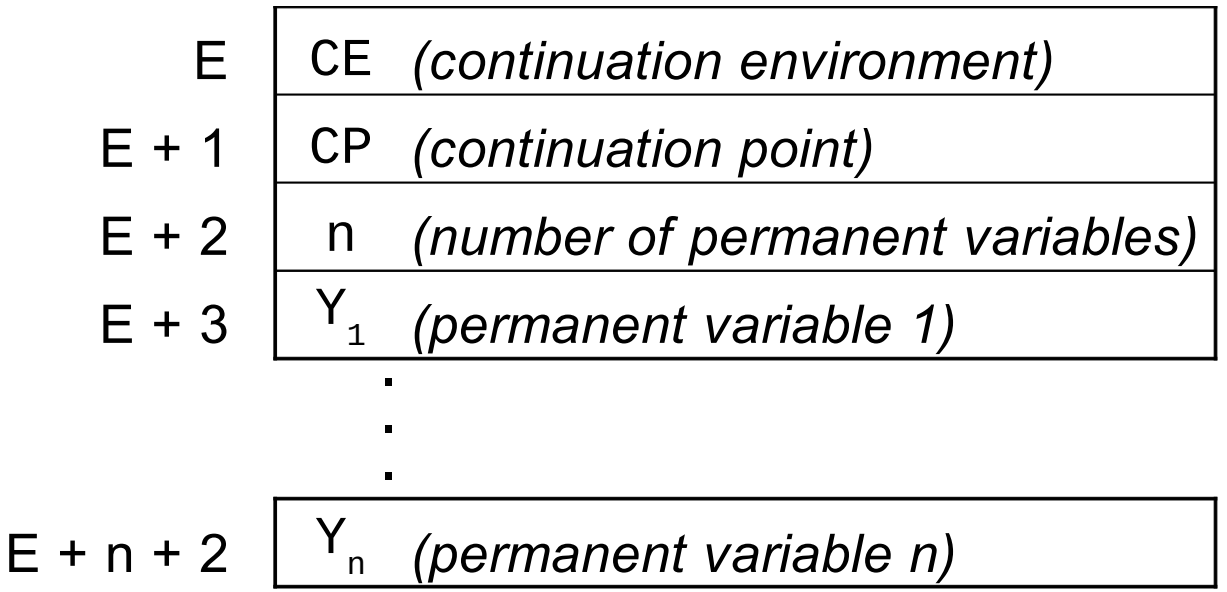
$Var(H) \cup Var(B_1), Var(B_2), ..., Var(B_m)$

To emphasize that a variable is permanent, we write Y_i.

Permanent variables are cells in the stack area.

Environmental Frame

The environment frame is used to save local data upon calling a procedure.



WAM_2 Machine Instructions

`allocate N` ≡ *newE* ← E + STACK[E+2] + 3;

STACK[*newE*] ← E;

STACK[*newE*+1] ← CP;

STACK[*newE*+2] ← N;

E ← *newE*;

P ← P + *instruction_size*(P);

`deallocate` ≡ P ← STACK[E+1];

E ← STACK[E];

Example

```
p/2:  allocate 2              %  p
      get_variable X3,A1      %     (X,
      get_variable Y1,A2      %       Y) :-
      put_value X3,A1         %          q(X,
      put_variable Y2,A2      %               Z
      call q/2                %               ),
      put_value Y2,A1         %          r(Z,
      put_value Y1,A2         %               Y
      call r/2                %               )
      deallocate              %                 .
```

WAM_2 Machine Code for `p(X,Y) :- q(X,Z),r(Z,Y).`

Handling the Query

The query is encoded like a clause but without the instructions for the missing head.

Prior to calling the code for the query, an initial environment is created:

STACK[E] ← E;

STACK[E+1] ← CP;

STACK[E+2] ← 0;

Following the (successful) call to the query, the register bindings are printed as the CAS.

From L_2 to Pure Prolog

A program is an arbitrary set of clauses.

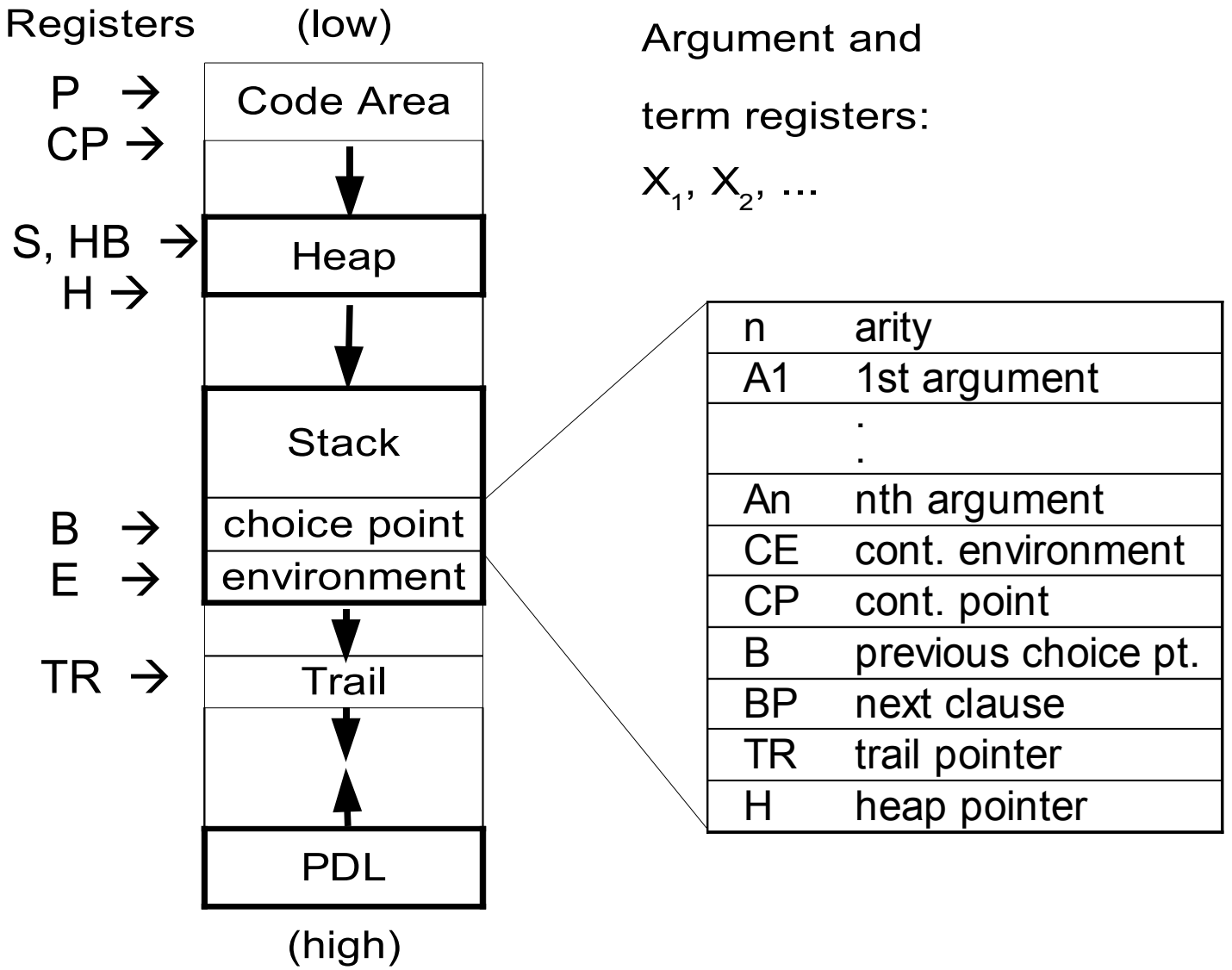
Choice Points vs. Environment Frames

Example:

```
a:-b(X),c(X).
b(X):-e(X).
c(1).
e(X):-f(X).
e(X):-g(X).
f(2).
g(1).
?-a.
```

This example shows that a choice point must protect each previously stored environment frame from deallocation in order that the environment is still available upon backtracking.

WAM Memory Layout



Trail

Backtracking requires to “undo” substitutions made after the choicepoint to which we jump back.

The trail contains the addresses of all variables which need to be unbound upon backtracking.

Register HB contains the value of H at the time of the latest choice point. Only variables below HB need to be reset.

Recording the Bindings

```
procedure bind(a1,a2: address);
        begin
            case STORE[a1] of
                ⟨REF,_⟩ :  STORE[a1] ← STORE[a2];
                           trail(a1);
                other   :  STORE[a2] ← STORE[a1];
                           trail(a2)
            endcase
        end bind
```

Trail Functions

- trail (*addr*: address)

pushes *addr* onto the trail, provided that

$$addr < HB$$

- unwind_trail ($addr_1$, $addr_2$: address)

resets all variables stored in $addr_1$, $addr_1 + 1$, ..., $addr_2 - 1$

The Trail and Unwind Operations

```
procedure trail(a: address);
    begin
        if (a < HB)
            then
                begin
                    TRAIL[TR] ← a;
                    TR ← TR+1
                end
    end trail

procedure unwind_trail(a1,a2: address);
    begin
        for i ← a1 to a2 – 1 do
            STORE[TRAIL[i]] ← ⟨REF,TRAIL[i] ⟩
    end unwind_trail
```

Choice Point Frame

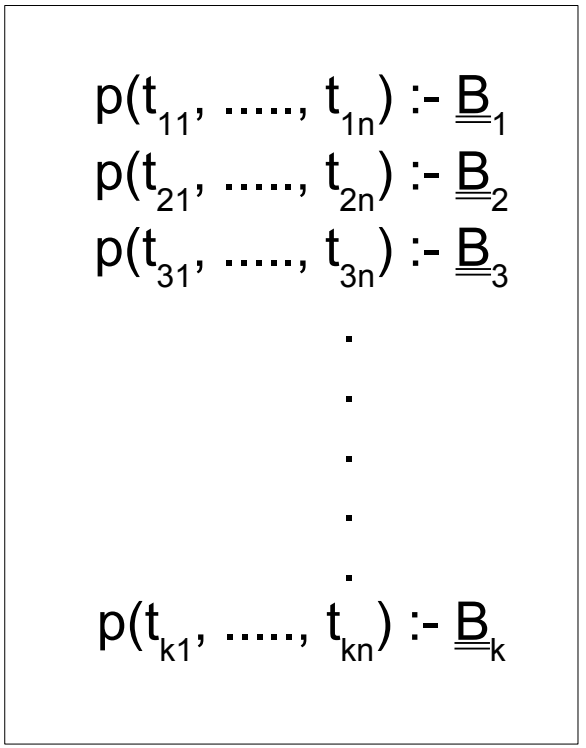
The choice point frame is used to save local data needed upon backtracking.

B	n	*(number of arguments)*
B+1	A1	*(argument register 1)*
	.	
	.	
	.	
B+n	An	*(argument register n)*
B+n+1	CE	*(continuation environment)*
B+n+2	CP	*(continuation point)*
B+n+3	B	*(previous choice point)*
B+n+4	BP	*(next clause)*
B+n+5	TR	*(trail pointer)*
B+n+6	H	*(heap pointer)*

WAM Machine Instructions (I)

```
allocate N ≡   if E > B
                   then newE ← E+STACK[E+2] + 3
                   else newE ← B+STACK[B] + 7;
               STACK[newE] ← E;
               STACK[newE+1] ← CP;
               STACK[newE+2] ← N;
               E ← newE;
               P ← P + instruction_size(P);
```

Coding Scheme for Backtracking



p/n : `try_me_else` L_1
code for first clause

L_1 : `retry_me_else` L_2
code for second clause

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L_{k-2} : `retry_me_else` L_{k-1}
code for third clause

L_{k-1} : `trust_me`
code for k-th clause

WAM Machine Instructions (II)

```
try_me_else L  ≡  if E > B
                      then newB ← E+STACK[E+2] + 3
                      else newB ← B+STACK[B] + 7;
                  STACK[newB] ← num_of_args;
                  n ← STACK[newB];
                  for i ← 1 to n do STACK[newB+i] ← A_i;
                  STACK[newB+n+1] ← E;
                  STACK[newB+n+2] ← CP;
                  STACK[newB+n+3] ← B;
                  STACK[newB+n+4] ← L;
                  STACK[newB+n+5] ← TR;
                  STACK[newB+n+6] ← H;
                  B ← newB;
                  HB ← H;
                  P ← P + instruction_size(P);
```

WAM Machine Instructions (III)

`retry_me_else L` ≡

n ← STACK[B];
for *i* ← 1 **to** *n* **do** A_i ← STACK[B+*i*];
E ← STACK[B+n+1];
CP ← STACK[B+n+2];
STACK[B+n+4] ← L;
unwind_trail(STACK[B+n+5],TR);
TR ← STACK[B+n+5];
H ← STACK[B+n+6];
HB ← H;
P ← P + *instruction_size*(P);

WAM Machine Instructions (IV)

`trust_me` ≡

$n \leftarrow$ STACK[B];
for $i \leftarrow 1$ **to** n **do** $A_i \leftarrow$ STACK[B+i];
E ← STACK[B+n+1];
CP ← STACK[B+n+2];
unwind_trail(STACK[B+n+5],TR);
TR ← STACK[B+n+5];
H ← STACK[B+n+6];
B ← STACK[B+n+3];
HB ← STACK[B+n+6];
P ← P + *instruction_size*(P);

WAM Machine Instructions (V)

`backtrack` ≡ P ← STACK[B + STACK[B]+4];
(whenever failure occurs)

`call p/n` ≡ CP ← P + *instruction_size*(P);
num_of_arg ← n;
P ← @ (p/n);

The Cut

The effect of the cut “!” is to forget any other alternative for the clause in which it appears and any other alternative arising from preceding body atoms in this clause.

This means to discard all choice points (CPs) created after the CP that was the current CP at the time the clause with the cut was called.

An additional cut register `B0` is used to store the appropriate CP where to return upon backtracking over a cut.

Call Revisited

`call p/n` ≡ CP ← P + *instruction_size*(P);
num_of_arg ← n;
B0 ← B;
P ← @ (p/n);

The value of B0 is saved in every choice point frame.

The Cut as First Body Atom

```
a/0: allocate 0          % a
     neck_cut            %    :- !,
     call b/0            %         b
     deallocate          %           .
```

WAM Machine Code for `a :- !, b.`

Machine Instruction

```
neck_cut   ≡   if B > B0 then begin
                    B ← B0;
                    HB ← STACK[B+STACK[B]+6];
                    tidy_trail
               end;
               P ← P + instruction_size(P);
```

The value of `B0` is saved in every choice point frame.

Auxiliary Function

```
procedure tidy_trail;
        begin
            i ← STACK[B+STACK[B]+5];
            while i < TR do
                if TRAIL[i] < HB then
                    i ← i+1
                else begin
                    TRAIL[i] ← TRAIL[TR-1];
                    TR ← TR-1;
                end
        end tidy_trail;
```

A Deep Cut

```
a/0:  allocate 1              %  a
      get_level Y1
      call b/0                %      :- b,
      cut Y1                  %          !,
      call c/0                %            c
      deallocate              %             .
```

WAM Machine Code for `a :- b, !, c.`

Machine Instructions

`get_level` Y_n ≡ STACK[E+2+*n*] ← B0;

P ← P + *instruction_size*(P);

`cut` Y_n ≡ **if** B > STACK[E+2+*n*] **then begin**

B ← STACK[E+2+*n*];

HB ← STACK[B+STACK[B]+6];

tidy_trail

end;

P ← P + *instruction_size*(P);

Objectives

- From L_1 to L_2: SLD-derivations
- From L_2 to Prolog: Backtracking
- The cut in Prolog