TECHNISCHE UNIVERSITÄT DRESDEN

FOUNDATIONS OF SEMANTIC WEB TECHNOLOGIES

Hypertableau I

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Dresden, 7 June 2013

DRESDEN concept Exzellenz aus Wissenschaft und Kultur

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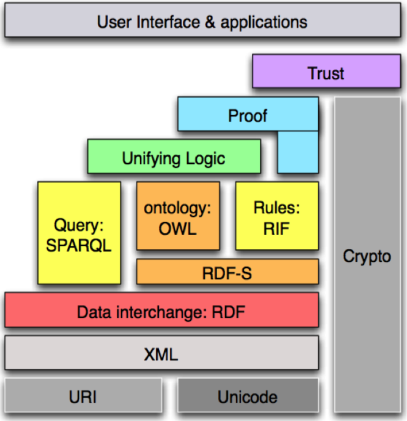
Content

Hypertableau



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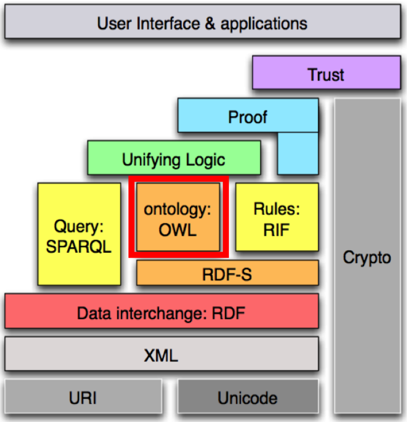

Agenda

- Motivation
- Recap: Translation into FOL
- Structural Transformation
- Translation into Clauses
- Summary

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Example Standard Tableau

Let the following TBox $\mathcal{T}$ and ABox $\mathcal{A}$ be given:

$$\mathcal{T} = \{\exists r.A \sqsubseteq A\} \qquad C_{\mathcal{T}} = \forall r.(\neg A) \sqcup A$$

$$\mathcal{A} = \{\neg A(a_0), r(a_0, b_1), r(b_1, a_1), \ldots, r(a_{n-1}, b_n), r(b_n, a_n), A(a_n)\}$$

$$a_0 \xrightarrow{r} b_1 \xrightarrow{r} a_1 \xrightarrow{r} b_2 \cdots\cdots\cdots \quad \cdots\cdots\cdots \rightarrow a_{n-1} \xrightarrow{r} b_n \xrightarrow{r} a_n$$

Assumption: we address the nodes in the tableau in alphabetic order, i.e., a's before b's

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L(a_0) &= \{\neg A, C_{\mathcal{T}}\} \\
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\end{aligned}$$

Now again decisions for a_n and $b_1, \ldots, b_n$

Is that Necessarily So?

- the algorithm constructs exponentially many branches, despite dependency directed backtracking
- translation of the formula into FOL:

$$
\begin{aligned}
& \forall r.(\neg A) \sqcup A \\
= & \forall x, y.\, [r(x,y) \wedge A(y) \rightarrow A(x)] \\
= & \forall x, y.\, [\neg r(x,y) \vee \neg A(y) \vee A(x)]
\end{aligned}
$$

- note: the formula does not have real non-determinism (Horn-clause)
- hypertableau exploits this

Idea Hypertableau

- translate KB axioms into FOL
  - rewrite axioms to obtain formulae of a certain structure
- axioms are translated such that non-determinism is avoided, if possible
- the formulae thus obtained become rules for constructing a model abstraction

Simple Hypertableau Example

1. Translation into Clauses

$$A \sqsubseteq A'$$
$$A \sqsubseteq \exists r.B$$
$$D \sqsubseteq E \sqcup F$$
$$F \sqsubseteq \bot$$
$$\exists r.\top \sqsubseteq C$$

$$A(a)$$
$$(D \sqcap \neg B)(d)$$

Simple Hypertableau Example

1. Translation into Clauses

$$A \sqsubseteq A' \qquad\qquad A(x) \to A'(x)$$

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- existential quantifiers treated as in the tableau

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$$F \sqsubseteq \bot \qquad F(x) \rightarrow \bot(x)$$
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D \sqsubseteq E \sqcup F & \quad & D(x) \rightarrow E(x) \vee F(x) \\
F \sqsubseteq \bot & \quad & F(x) \rightarrow \bot(x) \\
\exists r.\top \sqsubseteq C & \quad & r(x,y) \rightarrow C(x)
\end{array}
$$

$$
\begin{array}{r}
A(a) \\
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\begin{array}{rcl}
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A \sqsubseteq \exists r.B & \quad & A(x) \rightarrow \exists r.B(x) \\
D \sqsubseteq E \sqcup F & \quad & D(x) \rightarrow E(x) \vee F(x) \\
F \sqsubseteq \bot & \quad & F(x) \rightarrow \bot(x) \\
\exists r.\top \sqsubseteq C & \quad & r(x,y) \rightarrow C(x) \\
A(a) & \quad & \rightarrow A(a) \\
(D \sqcap \neg B)(d) & &
\end{array}
$$

- existential quantifiers treated as in the tableau

Simple Hypertableau Example

1. Translation into Clauses

$$A \sqsubseteq A' \qquad A(x) \rightarrow A'(x)$$
$$A \sqsubseteq \exists r.B \qquad A(x) \rightarrow \exists r.B(x)$$
$$D \sqsubseteq E \sqcup F \qquad D(x) \rightarrow E(x) \vee F(x)$$
$$F \sqsubseteq \bot \qquad F(x) \rightarrow \bot(x)$$
$$\exists r.\top \sqsubseteq C \qquad r(x,y) \rightarrow C(x)$$

$$A(a) \qquad \rightarrow A(a)$$
$$(D \sqcap \neg B)(d) \qquad \rightarrow D(d)$$
$$\rightarrow \neg B(d)$$

- existential quantifiers treated as in the tableau

Simple Hypertableau Example

a d

$L(a) = \{A\}$

$L(d) = \{D, \neg B\}$

$$
\begin{aligned}
A(x) &\rightarrow A'(x) \\
A(x) &\rightarrow \exists r.B(x) \\
D(x) &\rightarrow E(x) \vee F(x) \\
F(x) &\rightarrow \bot(x) \\
r(x,y) &\rightarrow C(x) \\
&\rightarrow A(a) \\
&\rightarrow D(d) \\
&\rightarrow \neg B(d)
\end{aligned}
$$

Simple Hypertableau Example

a d

$L(a) = \{A\}$

$L(d) = \{D, \neg B\}$

$$\begin{aligned} A(x) &\rightarrow A'(x) \\ A(x) &\rightarrow \exists r.B(x) \\ D(x) &\rightarrow E(x) \vee F(x) \\ F(x) &\rightarrow \bot(x) \\ r(x,y) &\rightarrow C(x) \\ &\rightarrow A(a) \\ &\rightarrow D(d) \\ &\rightarrow \neg B(d) \end{aligned}$$

Simple Hypertableau Example

a d

$L(a) = \{A\} \cup \{A'\}$

$L(d) = \{D, \neg B\}$

$$
\begin{aligned}
A(x) &\rightarrow A'(x) \\
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\end{aligned}
$$

$L(a) = \{A\} \cup \{A'\}$

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$$

$$
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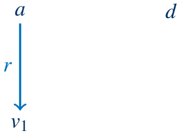
Simple Hypertableau Example

$$L(a) = \{A\} \cup \{A'\} \cup \{\exists r.B\}$$
$$L(d) = \{D, \neg B\} \cup \{E^1\}$$
$$L(v_1) = \{B\}$$

$$
\begin{aligned}
A(x) &\rightarrow A'(x) \\
A(x) &\rightarrow \exists r.B(x) \\
D(x) &\rightarrow E(x) \vee F(x) \\
F(x) &\rightarrow \bot(x) \\
r(x, y) &\rightarrow C(x) \\
&\rightarrow A(a) \\
&\rightarrow D(d) \\
&\rightarrow \neg B(d)
\end{aligned}
$$

Simple Hypertableau Example



$L(a) = \{A\} \cup \{A'\} \cup \{\exists r.B\}$

$L(d) = \{D, \neg B\} \cup \{E^1\}$

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$$
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$$

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$$L(a) = \{A\} \cup \{A'\} \cup \{\exists r.B\} \cup \{C\}$$
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$$L(a) = \{A\} \cup \{A'\} \cup \{\exists r.B\} \cup \{C\}$$

$$L(d) = \{D, \neg B\} \cup \{E^1\}$$

$$L(v_1) = \{B\}$$

- no more rules applicable $\rightsquigarrow$ satisfiability has been shown
- the only thing left from the tableau rules: an analogue of the $\exists$-rule

Agenda

- Motivation
- Recap: Translation into FOL
- Structural Transformation
- Translation into Clauses
- Summary

Translation into FOL

translation of TBox axioms into FOL via the mapping π with C, D complex classes, r a role and A an atomic class:

$$\pi(C \sqsubseteq D) = \forall x.(\pi_x(C) \rightarrow \pi_x(D)) \qquad \pi(C \equiv D) = \forall x.(\pi_x(C) \leftrightarrow \pi_x(D))$$

$$\begin{aligned} \pi_x(A) &= A(x) \\ \pi_x(\neg C) &= \neg\pi_x(C) \\ \pi_x(C \sqcap D) &= \pi_x(C) \wedge \pi_x(D) \\ \pi_x(C \sqcup D) &= \pi_x(C) \vee \pi_x(D) \\ \pi_x(\forall r.C) &= \forall y.(r(x,y) \rightarrow \pi_y(C)) \\ \pi_x(\exists r.C) &= \exists y.(r(x,y) \wedge \pi_y(C)) \end{aligned} \qquad \begin{aligned} \pi_y(A) &= A(y) \\ \pi_y(\neg C) &= \neg\pi_y(C) \\ \pi_y(C \sqcap D) &= \pi_y(C) \wedge \pi_y(D) \\ \pi_y(C \sqcup D) &= \pi_y(C) \vee \pi_y(D) \\ \pi_y(\forall r.C) &= \forall x.(r(y,x) \rightarrow \pi_x(C)) \\ \pi_y(\exists r.C) &= \exists x.(r(y,x) \wedge \pi_x(C)) \end{aligned}$$

TECHNISCHE UNIVERSITÄT DRESDEN

Motivation Normal Form

- the given translation creates rather complex formulae for complex axioms

$$\pi(C \sqsubseteq \exists r.(\forall s.(D \sqcup \exists r.D)))$$

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$$\pi(C \sqsubseteq \exists r.(\forall s.(D \sqcup \exists r.D)))$$
$$\forall x.\, [\pi_x(C) \rightarrow \pi_x(\exists r.(\forall s.(D \sqcup \exists r.D)))]$$

Motivation Normal Form

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$$\pi(C \sqsubseteq \exists r.(\forall s.(D \sqcup \exists r.D)))$$
$$\forall x. [\pi_x(C) \rightarrow \pi_x(\exists r.(\forall s.(D \sqcup \exists r.D)))]$$
$$\forall x. [C(x) \rightarrow \exists y.(r(x,y) \wedge \pi_y(\forall s.(D \sqcup \exists r.D)))]$$

TECHNISCHE UNIVERSITÄT DRESDEN

Motivation Normal Form

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$$\pi(C \sqsubseteq \exists r.(\forall s.(D \sqcup \exists r.D)))$$
$$\forall x.\,[\pi_x(C) \rightarrow \pi_x(\exists r.(\forall s.(D \sqcup \exists r.D)))]$$
$$\forall x.\,[C(x) \rightarrow \exists y.(r(x,y) \wedge \pi_y(\forall s.(D \sqcup \exists r.D)))]$$
$$\forall x.\,[C(x) \rightarrow \exists y.(r(x,y) \wedge \forall x.(s(y,x) \rightarrow \pi_x(D \sqcup \exists r.D)))]$$

Motivation Normal Form

- the given translation creates rather complex formulae for complex axioms

$$\pi(C \sqsubseteq \exists r.(\forall s.(D \sqcup \exists r.D)))$$
$$\forall x. [\pi_x(C) \to \pi_x(\exists r.(\forall s.(D \sqcup \exists r.D)))]$$
$$\forall x. [C(x) \to \exists y.(r(x,y) \wedge \pi_y(\forall s.(D \sqcup \exists r.D)))]$$
$$\forall x. [C(x) \to \exists y.(r(x,y) \wedge \forall x.(s(y,x) \to \pi_x(D \sqcup \exists r.D)))]$$
$$\forall x. [C(x) \to \exists y.(r(x,y) \wedge \forall x.(s(y,x) \to \pi_x(D) \vee \pi_x(\exists r.D)))]$$

TECHNISCHE UNIVERSITÄT DRESDEN

Motivation Normal Form

- the given translation creates rather complex formulae for complex axioms

$$
\begin{aligned}
&\pi(C \sqsubseteq \exists r.(\forall s.(D \sqcup \exists r.D))) \\
&\forall x.\,[\pi_x(C) \rightarrow \pi_x(\exists r.(\forall s.(D \sqcup \exists r.D)))] \\
&\forall x.\,[C(x) \rightarrow \exists y.(r(x,y) \wedge \pi_y(\forall s.(D \sqcup \exists r.D)))] \\
&\forall x.\,[C(x) \rightarrow \exists y.(r(x,y) \wedge \forall x.(s(y,x) \rightarrow \pi_x(D \sqcup \exists r.D)))] \\
&\forall x.\,[C(x) \rightarrow \exists y.(r(x,y) \wedge \forall x.(s(y,x) \rightarrow \pi_x(D) \vee \pi_x(\exists r.D)))] \\
&\forall x.\,[C(x) \rightarrow \exists y.(r(x,y) \wedge \forall x.(s(y,x) \rightarrow D(x) \vee \exists y.(r(x,y) \wedge \pi_y(D))))]
\end{aligned}
$$

Motivation Normal Form

- the given translation creates rather complex formulae for complex axioms

$$\pi(C \sqsubseteq \exists r.(\forall s.(D \sqcup \exists r.D)))$$
$$\forall x.\, [\pi_x(C) \rightarrow \pi_x(\exists r.(\forall s.(D \sqcup \exists r.D)))]$$
$$\forall x.\, [C(x) \rightarrow \exists y.(r(x,y) \wedge \pi_y(\forall s.(D \sqcup \exists r.D)))]$$
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Structural Transformation

- Structural Transformation introduces new concepts for complex subconcepts

$$C \sqsubseteq \exists r.(\forall s.(D \sqcup \exists r.D))$$

Structural Transformation

- Structural Transformation introduces new concepts for complex subconcepts

$$
\begin{aligned}
& C \sqsubseteq \exists r.(\forall s.(D \sqcup \exists r.D)) \\
\rightsquigarrow\ & C \sqsubseteq \exists r.Q_1 \\
& Q_1 \equiv \forall s.(D \sqcup \exists r.D)
\end{aligned}
$$

Structural Transformation

- Structural Transformation introduces new concepts for complex subconcepts

$$
\begin{aligned}
& C \sqsubseteq \exists r.(\forall s.(D \sqcup \exists r.D)) \\
\rightsquigarrow\ & C \sqsubseteq \exists r.Q_1 \\
& Q_1 \equiv \forall s.(D \sqcup \exists r.D) \\
\rightsquigarrow\ & Q_1 \equiv \forall s.Q_2 \\
& Q_2 \equiv D \sqcup \exists r.D
\end{aligned}
$$

Structural Transformation

- Structural Transformation introduces new concepts for complex subconcepts

$$
\begin{aligned}
& C \sqsubseteq \exists r.(\forall s.(D \sqcup \exists r.D)) \\
\rightsquigarrow\ & C \sqsubseteq \exists r.Q_1 \\
& Q_1 \equiv \forall s.(D \sqcup \exists r.D) \\
\rightsquigarrow\ & Q_1 \equiv \forall s.Q_2 \\
& Q_2 \equiv D \sqcup \exists r.D \\
\rightsquigarrow\ & Q_2 \equiv D \sqcup Q_3 \\
\rightsquigarrow\ & Q_3 \equiv \exists r.D
\end{aligned}
$$

Optimization of Structural Transformation

- do we have to introduce equivalence axioms?

$$A \sqsubseteq \forall r.(\forall r.B)$$

Optimization of Structural Transformation

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$$A \sqsubseteq \forall r.(\forall r.B)$$

$$\leadsto A \sqsubseteq \forall r.Q$$

$$\leadsto Q \sqsubseteq \forall r.B$$

Optimization of Structural Transformation

- do we have to introduce equivalence axioms?

$$A \sqsubseteq \forall r.(\forall r.B)$$

$$\rightsquigarrow \; A \sqsubseteq \forall r.Q$$

$$\rightsquigarrow \; Q \sqsubseteq \forall r.B \qquad \checkmark$$

Optimization of Structural Transformation

- do we have to introduce equivalence axioms?

$$A \sqsubseteq \forall r.(\forall r.B) \qquad\qquad \exists r.(A \sqcap B) \sqsubseteq C$$

$$\rightsquigarrow \; A \sqsubseteq \forall r.Q$$

$$\rightsquigarrow \; Q \sqsubseteq \forall r.B \qquad \checkmark$$

Optimization of Structural Transformation

- do we have to introduce equivalence axioms?

$$A \sqsubseteq \forall r.(\forall r.B) \qquad\qquad \exists r.(A \sqcap B) \sqsubseteq C$$

$$\rightsquigarrow\ A \sqsubseteq \forall r.Q \qquad\qquad \rightsquigarrow\ \exists r.Q \sqsubseteq C$$

$$\rightsquigarrow\ Q \sqsubseteq \forall r.B \quad \checkmark \qquad\qquad Q \sqsubseteq A \sqcap B$$

Optimization of Structural Transformation

- do we have to introduce equivalence axioms?

$$A \sqsubseteq \forall r.(\forall r.B)$$
$$\rightsquigarrow A \sqsubseteq \forall r.Q$$
$$\rightsquigarrow Q \sqsubseteq \forall r.B \quad \checkmark$$

$$\exists r.(A \sqcap B) \sqsubseteq C$$
$$\rightsquigarrow \exists r.Q \sqsubseteq C$$
$$Q \sqsubseteq A \sqcap B$$
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TECHNISCHE UNIVERSITÄT DRESDEN

Optimization of Structural Transformation

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$$\exists r.(A \sqcap B) \sqsubseteq C$$

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$$\rightsquigarrow Q \sqsubseteq A$$

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Well. Is that correct? Let $\mathcal{A} = \{r(a,b), A(b), B(b), \neg C(a)\}$

Optimization of Structural Transformation

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Well. Is that correct? Let $\mathcal{A} = \{r(a,b), A(b), B(b), \neg C(a)\}$

- In the context of a TBox with the original axiom, the ABox was contradictory.

Optimization of Structural Transformation

- do we have to introduce equivalence axioms?

$A \sqsubseteq \forall r.(\forall r.B)$	$\exists r.(A \sqcap B) \sqsubseteq C$
$\rightsquigarrow A \sqsubseteq \forall r.Q$	$\rightsquigarrow \exists r.Q \sqsubseteq C$
$\rightsquigarrow Q \sqsubseteq \forall r.B$ ✓	$Q \sqsubseteq A \sqcap B$
	$\rightsquigarrow Q \sqsubseteq A$
	$\rightsquigarrow Q \sqsubseteq B$

Well. Is that correct? Let $\mathcal{A} = \{r(a,b), A(b), B(b), \neg C(a)\}$

- In the context of a TBox with the original axiom, the ABox was contradictory.
- In the context of a TBox with the rewritten axiom, the ABox is satisfiable

Polarity for Optimized Transformation

- we have to take care if subexpressions occur “positively” or “negatively”
- $A \sqsubseteq B$ is just $\neg A \sqcup B$
- thus, A occurs negatively in the axiom and B positively
- when replacing a negatively occurring concept, we have to use $\sqsupseteq$

Polarity for Optimized Transformation

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$$\exists r.(A \sqcap B) \sqsubseteq C$$

$$\rightsquigarrow \exists r.Q \sqsubseteq C$$

$$Q \sqsupseteq A \sqcap B$$

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$$\rightsquigarrow A \sqcap B \sqsubseteq Q$$

$$C_{\mathcal{T}} = (\forall r.(\neg Q) \sqcup C) \sqcap (\neg A \sqcup \neg B \sqcup Q)$$

Tableau for Unsatisfiability

$$C_{\mathcal{T}} = (\forall r.(\neg Q) \sqcup C) \sqcap (\neg A \sqcup \neg B \sqcup Q)$$
$$\mathcal{A} = \{r(a,b), A(b), B(b), \neg C(a)\}$$

$$L(a) = \{\neg C\}$$
$$L(b) = \{A, B\}$$

Tableau for Unsatisfiability

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$$L(a) = \{\neg C\} \cup \{C_{\mathcal{T}}, \forall r.(\neg Q) \sqcup C, \neg A \sqcup \neg B \sqcup Q, \forall r.(\neg Q), \neg A\}$$
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Tableau for Unsatisfiability

$$C_\mathcal{T} = (\forall r.(\neg Q) \sqcup C) \sqcap (\neg A \sqcup \neg B \sqcup Q)$$
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$$L(b) = \{A, B\} \cup \{\neg Q\} \cup \{C_\mathcal{T}, \forall r.(\neg Q) \sqcup C, \neg A \sqcup \neg B \sqcup Q, \forall r.(\neg Q)\}$$

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$$C_{\mathcal{T}} = (\forall r.(\neg Q) \sqcup C) \sqcap (\neg A \sqcup \neg B \sqcup Q)$$
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$$C_{\mathcal{T}} = (\forall r.(\neg Q) \sqcup C) \sqcap (\neg A \sqcup \neg B \sqcup Q)$$
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$$L(b) = \{A, B\} \cup \{\neg Q\} \cup \{C_{\mathcal{T}}, \forall r.(\neg Q) \sqcup C, \neg A \sqcup \neg B \sqcup Q, \forall r.(\neg Q)\} \cup \{\neg A\}$$

Tableau for Unsatisfiability

$$C_{\mathcal{T}} = (\forall r.(\neg Q) \sqcup C) \sqcap (\neg A \sqcup \neg B \sqcup Q)$$
$$\mathcal{A} = \{r(a,b), A(b), B(b), \neg C(a)\}$$

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Tableau for Unsatisfiability

$$C_{\mathcal{T}} = (\forall r.(\neg Q) \sqcup C) \sqcap (\neg A \sqcup \neg B \sqcup Q)$$

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no further choice options
$\rightsquigarrow$ the KB is satisfiable

Optimized Structural Transformation

- we now want to define structural transformation formally
- we want to introduce just one new concept name per subexpression
- goal: rewrite TBox into an equisatisfiable TBox umzuschreiben containing only “simple” axioms

Polarity of Concepts

We define the polarity of a concept C inside a formula as follows:

- C occurs in C positively,
- C occurs in $\neg D$ positively (negatively) if C occurs in D negatively (positively),
- C occurs in $D \sqcap E$ or $D \sqcup E$ positively (negatively), if C occurs positively (negatively) in D or E,
- C occurs in $\exists r.D$ or $\forall r.D$ positively (negatively), if C occurs positively (negatively) in D,
- C occurs in $D \sqsubseteq E$ positively (negatively), if C occurs positively (negatively) in E or negatively (positively) in D.

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A concept occurs positively (negatively) in an ($\mathcal{ALC}$) TBox $\mathcal{T}$, if C occurs positively (negatively) in an axiom in $\mathcal{T}$.
$\rightsquigarrow$ a concept may occur both positively and negatively in an axiom

Optimized Transformation with Polarity

Let $\mathcal{T}$ be an $\mathcal{ALC}$ TBox. For every concept (sub-)expression C in $\mathcal{T}$, we introduce a fresh atomic concept A_C and define the function st(C) as follows:

$$\begin{array}{lll} \text{st}(A) = A & \text{st}(\neg C) = \neg A_C & \text{st}(\exists r.C) = \exists r.A_C \\ \text{st}(\top) = \top & \text{st}(C \sqcap D) = A_C \sqcap A_D & \text{st}(\forall r.C) = \forall r.A_C \\ \text{st}(\bot) = \bot & \text{st}(C \sqcup D) = A_C \sqcup A_D & \end{array}$$

The result of the structural transformation of a TBox $\mathcal{T}$ is a TBox $\mathcal{T}'$ with the following axioms:

- $A_C \sqsubseteq$ st(C) for every concept C occurring positively in $\mathcal{T}$,
- st(C) $\sqsubseteq A_C$ for every concept C occurring negatively in $\mathcal{T}$,
- $A_C \sqsubseteq A_D$ for every GCI $C \sqsubseteq D \in \mathcal{T}$.

Simplification of Axioms

we can now use known equivalences to simplify the axioms further:

$$\{C \sqsubseteq D \sqcap E\} \equiv \{C \sqsubseteq D, C \sqsubseteq E\}$$
$$\{C \sqcup D \sqsubseteq E\} \equiv \{C \sqsubseteq E, D \sqsubseteq E\}$$

Result of the Structural Transformation

by virtue of structural transformation and the known equivalences, we can rewrite an $\mathcal{ALC}$ KB into an equisatisfiable one, which contains only axioms of the following shape (A and B being atomic):

$$\begin{aligned} A_1 \sqcap \ldots \sqcap A_n &\sqsubseteq B_1 \sqcup \ldots \sqcup B_m \\ A &\sqsubseteq \exists r.B \\ A &\sqsubseteq \forall r.B \\ \exists r.A &\sqsubseteq B \\ \forall r.A &\sqsubseteq B \end{aligned}$$

Example: Optimized Structural Transformation

$$\overbrace{\underbrace{\forall r.(\overbrace{A \sqcap D}^{C_1})}_{C_3} \sqcap A \sqcap \underbrace{\exists s.(\overbrace{A \sqcap D}^{C_1})}_{C_4}}^{C_7} \sqsubseteq \underbrace{B \sqcup (\overbrace{\underbrace{\exists r.(\overbrace{C \sqcap E}^{C_2})}_{C_5} \sqcap F}^{C_8}) \sqcup \underbrace{\forall r.A}_{C_6}}_{C_9}$$

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$$A \sqcap D \sqsubseteq C_1$$

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$$\begin{aligned} A \sqcap D &\sqsubseteq C_1 \\ C_2 &\sqsubseteq C \sqcap E \end{aligned}$$

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$$\begin{aligned} A \sqcap D &\sqsubseteq C_1 \\ C_2 &\sqsubseteq C \sqcap E \\ \forall r.C_1 &\sqsubseteq C_3 \\ \exists s.C_1 &\sqsubseteq C_4 \end{aligned}$$

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C_2 &\sqsubseteq C \sqcap E \\
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C_5 &\sqsubseteq \exists r.C_3
\end{aligned}$$

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$$C_6 \sqsubseteq \forall r.A$$

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Example: Optimized Structural Transformation

we still can apply the simplification rules:

$$\overbrace{\underbrace{\forall r.(\overbrace{A \sqcap D}^{C_1})}_{C_3} \sqcap A \sqcap \underbrace{\exists s.(\overbrace{A \sqcap D}^{C_1})}_{C_4}}^{C_7} \sqsubseteq \underbrace{B \sqcup (\overbrace{\underbrace{\exists r.(\overbrace{C \sqcap E}^{C_2})}_{C_5} \sqcap F}^{C_8}) \sqcup \underbrace{\forall r.A}_{C_6}}_{C_9}$$

$$\begin{aligned} A \sqcap D &\sqsubseteq C_1 & C_6 &\sqsubseteq \forall r.A \\ C_2 &\sqsubseteq C & C_3 \sqcap A \sqcap C_4 &\sqsubseteq C_7 \\ \forall r.C_1 &\sqsubseteq C_3 & C_8 &\sqsubseteq C_5 \\ \exists s.C_1 &\sqsubseteq C_4 & C_9 &\sqsubseteq B \sqcup C_8 \sqcup C_6 \\ C_5 &\sqsubseteq \exists r.C_3 & C_7 &\sqsubseteq C_9 \\ C_2 &\sqsubseteq E & C_8 &\sqsubseteq F \end{aligned}$$

Agenda

- Motivation
- Recap: Translation into FOL
- Structural Transformation
- Translation into Clauses
- Summary

Translation into Clauses

a TBox with simplified axioms can now be translated into clauses (written as rules):

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If $m = 0$, the rule head contains $\bot(x)$.

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$$\begin{array}{rlr} A \sqsubseteq & \exists r.B & A(x) \to (\exists r.B)(x) \\ A \sqsubseteq & \forall r.B & A(x) \land r(x,y) \to B(y) \\ \exists r.A \sqsubseteq & B & r(x,y) \land A(y) \to B(x) \\ \forall r.A \sqsubseteq & B & \to (\exists r.\neg A)(x) \lor B(x) \end{array}$$

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Summary

- axioms can be simplified via structural transformation
- simplified axioms can be expressed as rules
- existential quantification in rule heads allowed
- often these rules allow to avoid nondeterminism