TECHNISCHE UNIVERSITÄT DRESDEN

FOUNDATIONS OF SEMANTIC WEB TECHNOLOGIES

Hypertableau II

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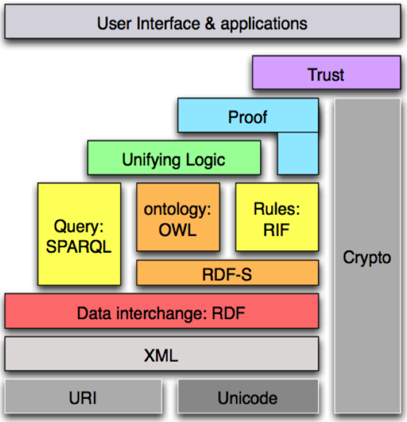
DRESDEN concept
Exzellenz aus Wissenschaft und Kultur

Content

Hypertableau II



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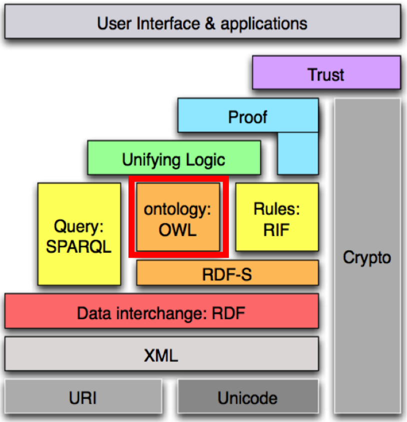

Agenda

- Recap
- The Hypertableau Rules
- Blocking in the Hypertableau Calculus
- Comparison Tableau and Hypertableau Calculus
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Result of the Structural Transformation

by virtue of structural transformation and the known equivalences, we can rewrite an $\mathcal{ALC}$ KB into an equisatisfiable one, which contains only axioms of the following shape (A and B being atomic):

$$\begin{aligned} A_1 \sqcap \ldots \sqcap A_n &\sqsubseteq B_1 \sqcup \ldots \sqcup B_m \\ A &\sqsubseteq \exists r.B \\ A &\sqsubseteq \forall r.B \\ \exists r.A &\sqsubseteq B \\ \forall r.A &\sqsubseteq B \end{aligned}$$

Translation into Clauses

a TBox with simplified axioms can now be translated into clauses (written as rules):

$$A_1 \sqcap \ldots \sqcap A_n \sqsubseteq B_1 \sqcup \ldots \sqcup B_m$$
$$A_1(x) \land \ldots \land A_n(x) \to B_1(x) \lor \ldots \lor B_m(x)$$

in case $m = 0$, the rule head contains $\bot(x)$.

$$\begin{array}{rlr} A \sqsubseteq & \exists r.B & A(x) \to (\exists r.B)(x) \\ A \sqsubseteq & \forall r.B & A(x) \land r(x,y) \to B(y) \\ \exists r.A \sqsubseteq & B & r(x,y) \land A(y) \to B(x) \\ \forall r.A \sqsubseteq & B & \to (\exists r.\neg A)(x) \lor B(x) \end{array}$$

Simplification via Structural Transformation

- structural transformation introduces new concepts for complex concept subexpressions
- taking polarity into account, it suffices to introduce subsumption instead of equivalences

Polarity in Structural Transformation

We define the polarity of a concept C inside a formula as follows:

- C occurs in C positively,
- C occurs in $\neg D$ positively (negatively) if C occurs in D negatively (positively),
- C occurs in $D \sqcap E$ or $D \sqcup E$ positively (negatively), if C occurs positively (negatively) in D or E,
- C occurs in $\exists r.D$ or $\forall r.D$ positively (negatively), if C occurs positively (negatively) in D,
- C occurs in $D \sqsubseteq E$ positively (negatively), if C occurs positively (negatively) in E or negatively (positively) in D.

A concept occurs positively (negatively) in an ($\mathcal{ALC}$) TBox $\mathcal{T}$, if C occurs positively (negatively) in an axiom in $\mathcal{T}$.
$\rightsquigarrow$ a concept may occur both positively and negatively in an axiom

Optimized Transformation with Polarity

Let $\mathcal{T}$ be an $\mathcal{ALC}$ TBox. For every concept (sub-)expression C in $\mathcal{T}$, we introduce a fresh atomic concept A_C and define the function st(C) as follows:

$$\begin{array}{lll} \text{st}(A) = A & \text{st}(\neg C) = \neg A_C & \text{st}(\exists r.C) = \exists r.A_C \\ \text{st}(\top) = \top & \text{st}(C \sqcap D) = A_C \sqcap A_D & \text{st}(\forall r.C) = \forall r.A_C \\ \text{st}(\bot) = \bot & \text{st}(C \sqcup D) = A_C \sqcup A_D & \end{array}$$

The result of the structural transformation of a TBox $\mathcal{T}$ is a TBox $\mathcal{T}'$ with the following axioms:

- $A_C \sqsubseteq$ st(C) for every concept C occurring positively in $\mathcal{T}$,
- st(C) $\sqsubseteq A_C$ for every concept C occurring negatively in $\mathcal{T}$,
- $A_C \sqsubseteq A_D$ for every GCI $C \sqsubseteq D \in \mathcal{T}$.

Simplification of Axioms

we can now use known equivalences to simplify the axioms further:

$$\{C \sqsubseteq D \sqcap E\} \equiv \{C \sqsubseteq D, C \sqsubseteq E\}$$
$$\{C \sqcup D \sqsubseteq E\} \equiv \{C \sqsubseteq E, D \sqsubseteq E\}$$

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Hypertableau Calculus: Basic Notions

- for a TBox $\mathcal{T}$, let cl$(\mathcal{T})$ be the corresponding set of clauses
- we assume that the ABox is non-empty
- we assume that the ABox contains only facts of the forms $A(a), \neg A(a), (\exists r.A)(a), (\exists r.\neg A)(a), r(a,b)$
- we write Vars$(\mathcal{T})$ to denote the set of variables in cl$(\mathcal{T})$
- we write Inds$(\mathcal{A})$ to denote the set of individual names in $\mathcal{A}$

The Hypertableau Calculus

HT-rule: for $A_1 \wedge \ldots \wedge A_n \rightarrow B_1 \vee \ldots \vee B_m \in \mathsf{cl}(\mathcal{T})$
and mapping $\sigma\colon \mathsf{Vars}(\mathcal{T}) \rightarrow \mathsf{Inds}(\mathcal{A})$ with
$\sigma(A_i) \in \mathcal{A}, \sigma(B_j) \notin \mathcal{A}$ for every $1 \leq i \leq n$,
pick a j with $1 \leq j \leq m$ and let $\mathcal{A} = \mathcal{A} \cup \{\sigma(B_j)\}$.

∃-rule: for $(\exists r.C)(v) \in \mathcal{A}$ where v is not blocked and
there is no r successor v' of v with $C(v') \in \mathcal{A}$,
let $\mathcal{A} = \mathcal{A} \cup \{r(v, v'), C(v')\}$ where v' is a new node.

- the tableau is now represented as ABox
- the tableau is contradictory if the ABox contains $\bot(v)$ or $A(v)$ and $(\neg A)(v)$ for some individual v (and concept A)

Example Hypertableau

Let the TBox $\mathcal{T}$ and ABox $\mathcal{A}$ be given:

$$\mathcal{T} = \{\exists r.A \sqsubseteq A\} \qquad \mathsf{cl}(\mathcal{T}) = \{r(x,y) \wedge A(y) \rightarrow A(x)\}$$

$$a_0 \xrightarrow{r} b_1 \xrightarrow{r} a_1 \xrightarrow{r} b_2 \cdots\cdots \cdots\cdots\rightarrow a_{n-1} \xrightarrow{r} b_n \xrightarrow{r} a_n$$

$$\mathcal{A} = \{\neg A(a_0), r(a_0,b_1), r(b_1,a_1), \ldots, r(a_{n-1},b_n), r(b_n,a_n), A(a_n)\}$$

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the hypertableau algorithm does not exhibit non-determinism here

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Blocking in the Hypertableau

by now, blocking mechanism still undefined:

Definition (Blocking)

An individual $v \in \text{Inds}(\mathcal{A})$ blocks an individual $v' \in \text{Inds}(\mathcal{A})$ in an ABox $\mathcal{A}$ directly, if:

1. v' is reachable from v,
2. $\{A \mid A(v) \in \mathcal{A}\} = \{A \mid A(v') \in \mathcal{A}\}$; and
3. there is no directly blocked node v'' such that v' is reachable from v''.

An individual $v' \in V$ is blocked if either

1. v' is directly blocked or
2. there is a directly blocked node v such that v' is reachable from v.

Blocking in the Hypertableau

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Blocking in the Hypertableau

- Like the original tableau, the ABox is seen as graph where every named individual is the root of a tree
- blocking depends only on atomic concepts
- can't we use subset blocking?

Example Hypertableau

$$\mathcal{T} = \{ \quad C \sqsubseteq \exists r.C$$
$$C \sqsubseteq \exists s.D$$
$$\exists s.D \sqsubseteq E$$
$$\top \sqsubseteq \forall r.(\neg E)\}$$

$$\mathcal{A} = \{(\exists t.C)(a)\}$$

Example Hypertableau

$$\mathcal{T} = \{ \quad C \sqsubseteq \exists r.C, \quad C \sqsubseteq \exists s.D, \quad \exists s.D \sqsubseteq E, \quad \top \sqsubseteq \forall r.(\neg E) \}$$

$$\mathsf{cl}(\mathcal{T}) = \{ \quad C(x) \to (\exists r.C)(x), \quad C(x) \to (\exists s.D)(x), \quad s(x,y) \land D(y) \to E(x), \quad r(x,y) \land E(y) \to \bot(x) \quad \}$$

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a

$\mathcal{A} = \{(\exists t.C)(a)\}$

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$$a \xrightarrow{t} v_0$$

$$\mathcal{A} = \{(\exists t.C)(a)\} \cup \{t(a, v_0), C(v_0)\}$$

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v_0 blocks v_1 (atomic concepts): $\mathcal{L}(v_1) = \{C\} \subseteq \mathcal{L}(v_1) = \{C\}$

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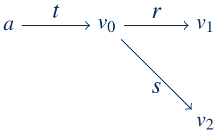


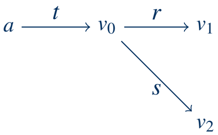
$$\begin{aligned} \mathcal{A} = {} & \{(\exists t.C)(a)\} \cup \{t(a,v_0), C(v_0)\} \cup \{(\exists r.C)(v_0)\} \cup \{(\exists s.D)(v_0)\} \\ & \cup \{r(v_0,v_1), C(v_1)\} \cup \{\exists r.C(v_1)\} \cup \{\exists s.D(v_1)\} \\ & \cup \{s(v_0,v_2), D(v_2)\} \end{aligned}$$

v_0 blocks v_1 (atomic concepts): $\mathcal{L}(v_1) = \{C\} \subseteq \mathcal{L}(v_1) = \{C\}$

Example Hypertableau

$$\mathcal{T} = \{ \quad C \sqsubseteq \exists r.C, \quad C \sqsubseteq \exists s.D, \quad \exists s.D \sqsubseteq E, \quad \top \sqsubseteq \forall r.(\neg E) \}$$

$$\mathsf{cl}(\mathcal{T}) = \{ \quad C(x) \to (\exists r.C)(x), \quad C(x) \to (\exists s.D)(x), \quad s(x,y) \wedge D(y) \to E(x), \quad r(x,y) \wedge E(y) \to \bot(x) \quad \}$$



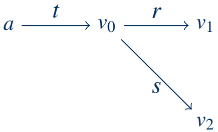
$$\mathcal{A} = \{(\exists t.C)(a)\} \cup \{t(a,v_0), C(v_0)\} \cup \{(\exists r.C)(v_0)\} \cup \{(\exists s.D)(v_0)\}$$
$$\cup \{r(v_0,v_1), C(v_1)\} \cup \{\exists r.C(v_1)\} \cup \{\exists s.D(v_1)\}$$
$$\cup \{s(v_0,v_2), D(v_2)\}$$

v_0 blocks v_1 (atomic concepts): $\mathcal{L}(v_1) = \{C\} \subseteq \mathcal{L}(v_1) = \{C\}$

Example Hypertableau

$$\mathcal{T} = \{ \quad C \sqsubseteq \exists r.C,\quad C \sqsubseteq \exists s.D,\quad \exists s.D \sqsubseteq E,\quad \top \sqsubseteq \forall r.(\neg E) \}$$

$$\mathsf{cl}(\mathcal{T}) = \{ \quad C(x) \to (\exists r.C)(x),\quad C(x) \to (\exists s.D)(x),\quad s(x,y) \wedge D(y) \to E(x),\quad r(x,y) \wedge E(y) \to \bot(x) \quad \}$$



$$\mathcal{A} = \{(\exists t.C)(a)\} \cup \{t(a, v_0), C(v_0)\} \cup \{(\exists r.C)(v_0)\} \cup \{(\exists s.D)(v_0)\}$$
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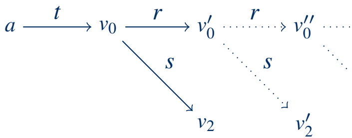
$$\cup \{s(v_0, v_2), D(v_2)\} \cup \{E(v_0)\}$$

v_0 blocks v_1 (atomic concepts): $\mathcal{L}(v_1) = \{C\} \subseteq \mathcal{L}(v_1) = \{C\} \cup \{E\}$
no further rule applicable

Example Hypertableau

When constructing the model, we run into troubles (recall: we replace the blocked individual by the blocking one, including the subtree rooted in the latter):

$$\mathsf{cl}(\mathcal{T}) = \{ \quad C(x) \rightarrow (\exists r.C)(x) \qquad C(x) \rightarrow (\exists s.D)(x)$$
$$s(x,y) \land D(y) \rightarrow E(x) \qquad r(x,y) \land E(y) \rightarrow \bot(x) \quad \}$$

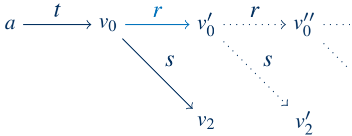


$$\mathcal{A} = \{(\exists t.C)(a), t(a,v_0), C(v_0), E(v_0), (\exists r.C)(v_0), (\exists s.D)(v_0),$$
$$s(v_0,v_2), D(v_2), r(v_0,v_0'), C(v_0'), E(v_0'), \exists r.C(v_0'), \exists s.D(v_0'),$$
$$s(v_0',v_2'), D(v_2'), r(v_0',v_0''), C(v_0''), E(v_0''), \exists r.C(v_0''), \exists s.D(v_0''), \ldots\}$$

Example Hypertableau

When constructing the model, we run into troubles (recall: we replace the blocked individual by the blocking one, including the subtree rooted in the latter):

$$\mathsf{cl}(\mathcal{T}) = \{ \quad C(x) \rightarrow (\exists r.C)(x) \quad C(x) \rightarrow (\exists s.D)(x) \quad s(x,y) \wedge D(y) \rightarrow E(x) \quad {\color{blue} r(x,y) \wedge E(y) \rightarrow \bot(x)} \quad \}$$



$$\mathcal{A} = \{(\exists t.C)(a), t(a,v_0), C(v_0), E(v_0), (\exists r.C)(v_0), (\exists s.D)(v_0), s(v_0,v_2), D(v_2), {\color{blue} r(v_0,v_0')}, C(v_0'), {\color{blue} E(v_0')}, \exists r.C(v_0'), \exists s.D(v_0'), s(v_0',v_2'), D(v_2'), r(v_0',v_0''), C(v_0''), E(v_0''), \exists r.C(v_0''), \exists s.D(v_0''), \ldots\}$$

Comparison Blocking in Tableau vs. Hypertableau Calculus

- subset blocking does not work in hypertableau
- including non-atomic concepts doesn't help
- axiom $\exists s.D \sqsubseteq E$ equivalent with $D \sqsubseteq \forall s^{-}.E$

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$\exists s.D \sqsubseteq E$ $\qquad\qquad$ $D \sqsubseteq \forall s^{-}.E$

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$\exists s.D \sqsubseteq E$	$D \sqsubseteq \forall s^{-}.E$
$\forall s.(\neg D) \sqcup E$	$\neg D \sqcup \forall s^{-}.E$

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- axiom $\exists s.D \sqsubseteq E$ equivalent with $D \sqsubseteq \forall s^-.E$

$\exists s.D \sqsubseteq E$	$D \sqsubseteq \forall s^-.E$
$\forall s.(\neg D) \sqcup E$	$\neg D \sqcup \forall s^-.E$
$s(x,y) \wedge D(y) \rightarrow E(x)$	$D(x) \wedge s^-(x,y) \rightarrow E(y)$

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$\exists s.D \sqsubseteq E$	$D \sqsubseteq \forall s^{-}.E$
$\forall s.(\neg D) \sqcup E$	$\neg D \sqcup \forall s^{-}.E$
$s(x,y) \wedge D(y) \rightarrow E(x)$	$D(x) \wedge s^{-}(x,y) \rightarrow E(y)$
	$D(x) \wedge s(y,x) \rightarrow E(y)$

Comparison Blocking in Tableau vs. Hypertableau Calculus

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- axiom $\exists s.D \sqsubseteq E$ equivalent with $D \sqsubseteq \forall s^{-}.E$

$$\exists s.D \sqsubseteq E \qquad\qquad D \sqsubseteq \forall s^{-}.E$$

$$\forall s.(\neg D) \sqcup E \qquad\qquad \neg D \sqcup \forall s^{-}.E$$

$$s(x,y) \wedge D(y) \rightarrow E(x) \qquad\qquad D(x) \wedge s^{-}(x,y) \rightarrow E(y)$$

$$D(x) \wedge s(y,x) \rightarrow E(y)$$

$$D(y) \wedge s(x,y) \rightarrow E(x)$$

Comparison Blocking in Tableau vs. Hypertableau Calculus

- subset blocking does not work in hypertableau
- including non-atomic concepts doesn't help
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$\exists s.D \sqsubseteq E$	$D \sqsubseteq \forall s^{-}.E$
$\forall s.(\neg D) \sqcup E$	$\neg D \sqcup \forall s^{-}.E$
$s(x,y) \wedge D(y) \rightarrow E(x)$	$D(x) \wedge s^{-}(x,y) \rightarrow E(y)$
	$D(x) \wedge s(y,x) \rightarrow E(y)$
	$D(y) \wedge s(x,y) \rightarrow E(x)$

- for inverse roles, even the the tableau calculus needs equality blocking
- for the axiom $\exists s.D \sqsubseteq E$ the tableau calculus requires a choice (GCI $\rightsquigarrow$ disjunction)

Agenda

- Recap
- The Hypertableau Rules
- Blocking in the Hypertableau Calculus
- Comparison Tableau and Hypertableau Calculus
- Summary

Remarks Hypertablau

- hypertableau requires equality blocking
- if number restrictions or functionality are involved, pairwise blocking is needed
- inverse roles disappear inside the rules (variable positions are swapped)
- translation of the rules is more complicated in practice (further heuristics are used to avoid disjunctions)
- for evaluating the rules, semi-naïve evaluation can be used
- dependency directed backtracking can be used just like in the tableau
- (in)equality for functionality / number restrictions:
  - Func(f) correspopnds to the rule $f(x, y_1) \wedge f(x, y_2) \rightarrow y_1 \approx y_2$
  - special rule $\approx$-rule for merging and pruning

Comparison Tableau vs. Hypertablau

- hypertableau has a more costly peprocessing
- nondeterminism can be often avoided
- blocking requires equality
- ABox is extended via rules analog to expanding the tableau
- good mechanism for evaluating rules
- implementation as ABox instead of graph structures makes blocking checks more difficult ($\leadsto$ optimization via hashing)
- both tableau and hypertableau can be extended to OWL 2 (but treatment of nominals difficult)
- hypertableau implemented in HermiT, tableau in FaCT++ and Pellet

Agenda

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Summary

- we have treated the basic calculi that are targeted at model construction
- reasoning for OWL profiles can be implemented with more efficient methods ($\rightsquigarrow$ consequence-based procedures)
- tableau and hypertableau are correct and terminating
  - corresponding proofs (particularly for more expressive logics) rather complicated
- in practice diverse optimizations used (other kinds of blocking, caching of model parts, heuristics, etc.)
- normally applicable for medium size knowledge bases, but depends a lot from the axioms' complexity
- worst-case for $\mathcal{ALC}$ KBs: ExpTime, for OWL 1 DL: NExpTime and for OWL 2 DL: 2-NExpTime

What We Haven't Talked About (in Detail)

- number restrictions (except functionality in the tableau)
- datatypes (specific algorithms)
- nominals and particularly the interplay with inverses and number restrictions
- optimized rule translation for hypertableau
- further optimizations: disjunction learning, told subsumers, etc.
- precise complexity considerations