TECHNISCHE UNIVERSITÄT DRESDEN

# FOUNDATIONS OF SEMANTIC WEB TECHNOLOGIES

## Semantics of SPARQL

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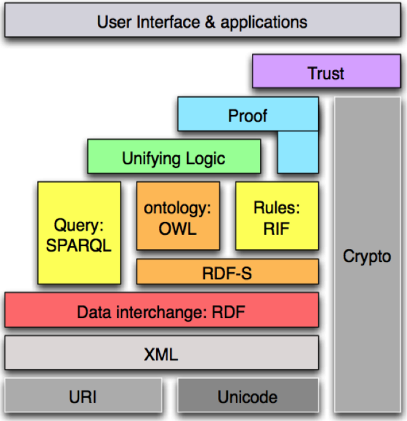
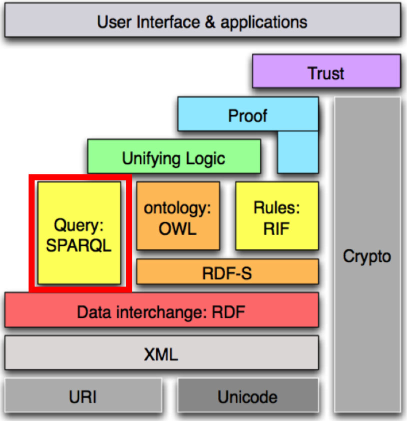
DRESDEN concept Exzellenz aus Wissenschaft und Kultur

# Content

# The SPARQL Query Language



# The SPARQL Query Language



# Agenda

1. Recap
2. Output Formats
3. SPARQL Semantics
4. Transformation of Queries into Algebra Objects
5. Evaluation of the SPARQL Algebra
6. Summary

# Agenda

# Recap: Introduced SPARQL Features

| **Basic Structure** |
|---|
| `PREFIX` |
| `WHERE` |

| **Graph Patterns** |
|---|
| Basic Graph Patterns |
| {. . .} |
| `OPTIONAL` |
| `UNION` |

| **Filter** |
|---|
| `BOUND` |
| `isURI` |
| `isBLANK` |
| `isLITERAL` |
| `STR` |
| `LANG` |
| `DATATYPE` |
| `sameTERM` |
| `langMATCHES` |
| `REGEX` |

| **Modifiers** |
|---|
| `ORDER BY` |
| `LIMIT` |
| `OFFSET` |
| `DISTINCT` |

| **Output Formats** |
|---|
| `SELECT` |

# Agenda

# Output Format SELECT

So far all results have been tables (solution sequences): Output format `SELECT`

Syntax: `SELECT <VariableList>` or `SELECT *`

## Advantage

Simple sequential processing of the results

## Disadvantage

Structure/relationships between the objects in the results is lost

# Output Format CONSTRUCT

CONSTRUCT creates an RDF graph for the results

## Example Query

```
PREFIX ex: <http://example.org/>
CONSTRUCT { ?person  ex:mailbox  ?email .
            ?person  ex:telephone  ?tel . }
WHERE { ?person  ex:email   ?email .
        ?person  ex:tel     ?tel . }
```

# Output Format CONSTRUCT

CONSTRUCT creates an RDF graph for the results

## Example Query

```
PREFIX ex: <http://example.org/>
CONSTRUCT { ?person  ex:mailbox  ?email .
            ?person  ex:telephone  ?tel . }
WHERE { ?person  ex:email   ?email .
        ?person  ex:tel     ?tel . }
```

## Advantage

Structured result data with relationships between the elements

## Disadvantages

- Sequential processing of the results is harder
- No treatment of unbound variables (triples are omitted)

# CONSTRUCT Templates with Blank Nodes

## Data

```
@prefix  foaf:  <http://xmlns.com/foaf/0.1/> .
_:a foaf:firstname  "Alice" ;
    foaf:surname "Hacker" .
_:b foaf:firstname  "Bob" ;
    foaf:surname     "Hacker" .
```

# CONSTRUCT Templates with Blank Nodes

## Data

```
@prefix  foaf:  <http://xmlns.com/foaf/0.1/> .
_:a foaf:firstname  "Alice" ;
    foaf:surname "Hacker" .
_:b foaf:firstname  "Bob" ;
    foaf:surname     "Hacker" .
```

## Query

```
PREFIX foaf:   <http://xmlns.com/foaf/0.1/>
PREFIX vcard: <http://www.w3.org/2001/vcard-rdf/3.0#>
CONSTRUCT {
  ?x  vcard:N           _:v .
  _:v vcard:givenName  ?gname ;
      vcard:familyName ?fname
} WHERE {
  ?x foaf:firstname ?gname .
  ?x foaf:surname   ?fname  }
```

# CONSTRUCT Templates with Blank Nodes

## Resulting RDF graph

```
@prefix vcard: <http://www.w3.org/2001/vcard-rdf/3.0#> .
_:v1 vcard:N  _:x .
_:x  vcard:givenName  "Alice" ;
     vcard:familyName "Hacker" .
_:v2 vcard:N  _:z .
_:z  vcard:givenName  "Bob" ;
     vcard:familyName "Hacker" .
```

# Further Output Formats: ASK & DESCRIBE

SPARQL supports two additional output formats:

- `ASK` only checks whether the query has at least one answer (true/false result)
- `DESCRIBE` (informative) returns an RDF description for each resulting URI (application dependent)

## Example Query

```
DESCRIBE ?x WHERE { ?x <http://ex.org/emplID> "123" }
```

## Possible Result (prefix declarations omitted):

```
_:a       exOrg:emplID    "123" ;
          foaf:mbox_sha1sum   "ABCD1234" ;
          vcard:N
          [ vcard:Family       "Smith" ;
            vcard:Given        "John"   ] .
foaf:mbox_sha1sum  a  owl:InverseFunctionalProperty .
```

# Agenda

# Semantics of Query Languages

So far only informal presentation of SPARQL features

- User: “Which answers can I expect for my query?”
- Developer: “Which behaviour is expected from my SPARQL implementation?”
- Marketing: “Is our product already conformant with the SPARQL standard?”

$\leadsto$ Formal semantics should clarify these questions . . .

# Logic-based Semantics

Semantics of formal logics:

- Model-theoretic semantics: Which interpretations do satisfy my knowledge base?
- Proof-theoretic semantics: Which derivations can be build from my knowledge base?
- ...

# Logic-based Semantics

Semantics of formal logics:

- Model-theoretic semantics: Which interpretations do satisfy my knowledge base?
- Proof-theoretic semantics: Which derivations can be build from my knowledge base?
- . . .

# Semantics of Programming Languages

- Axiomatic semantics: Which logical statements hold for my program?
- Operational semantics: What happens during the processing of my program?
- Denotational semantics: How can we describe the input/output function of the program in an abstract way?

# Semantics of Programming Languages

- Axiomatic semantics: Which logical statements hold for my program?
- Operational semantics: What happens during the processing of my program?
- Denotational semantics: How can we describe the input/output function of the program in an abstract way?

What to do with query languages?

# Semantics of Query Languages (1)

## Query Entailment

- Query as description of allowed results
- Data as set of logical assumptions (axiom set/theory)
- Results as logical entailment

$\rightsquigarrow$ OWL DL and RDF(S) as query languages

$\rightsquigarrow$ conjunctive queries

# Semantics of Query Languages (2)

## Query Algebra

- Query as instruction for computing the results
- Queried data as input
- Results as output

$\rightsquigarrow$ Relational algebra for SQL

$\rightsquigarrow$ SPARQL Algebra

# Agenda

# Translation into SPARQL Algebra

```
{ ?book ex:price ?price .
  FILTER (?price < 15)
  OPTIONAL { ?book ex:title ?title }
  { ?book ex:author ex:Shakespeare } UNION
  { ?book ex:author ex:Marlowe }
}
```

Semantics of a SPARQL query:

1. Transformation of the query into an algebra expression
2. Evaluation of the algebra expression

# Translation into SPARQL Algebra

```
{ ?book ex:price ?price
  FILTER (?price < 15)
  OPTIONAL { ?book ex:title ?title }
  { ?book ex:author ex:Shakespeare } UNION
  { ?book ex:author ex:Marlowe }
}
```

Attention: Filters apply to the whole group in which they occur

# Translation into SPARQL Algebra

```
{ ?book ex:price ?price
  OPTIONAL { ?book ex:title ?title }
  { ?book ex:author ex:Shakespeare } UNION
  { ?book ex:author ex:Marlowe }
  FILTER (?price < 15)
}
```

1. Expand abbreviated IRIs

# Translation into SPARQL Algebra

```
{ ?book <http://ex.org/price> ?price
  OPTIONAL { ?book <http://ex.org/title> ?title }
  { ?book <http://ex.org/author>
                  <http://ex.org/Shakespeare> } UNION
  { ?book <http://ex.org/author>
                  <http://ex.org/Marlowe> }
  FILTER (?price < 15)
}
```

# Translation into SPARQL Algebra

```
{ ?book <http://ex.org/price> ?price
  OPTIONAL { ?book <http://ex.org/title> ?title }
  { ?book <http://ex.org/author>
                  <http://ex.org/Shakespeare> } UNION
  { ?book <http://ex.org/author>
                  <http://ex.org/Marlowe> }
  FILTER (?price < 15)
}
```

2. Replace triple patterns with operator Bgp(·)

# Translation into SPARQL Algebra

```
{ Bgp(?book <http://ex.org/price> ?price)
  OPTIONAL {Bgp(?book <http://ex.org/title> ?title)}
  {Bgp(?book <http://ex.org/author>
                <http://ex.org/Shakespeare>)} UNION
  {Bgp(?book <http://ex.org/author>
                <http://ex.org/Marlowe>)}
  FILTER (?price < 15)
}
```

# Translation into SPARQL Algebra

```
{ Bgp(?book <http://ex.org/price> ?price)
  OPTIONAL {Bgp(?book <http://ex.org/title> ?title)}
  {Bgp(?book <http://ex.org/author>
                  <http://ex.org/Shakespeare>)} UNION
  {Bgp(?book <http://ex.org/author>
                  <http://ex.org/Marlowe>)}
  FILTER (?price < 15)
}
```

3. Introduce the LeftJoin(·) operator for optional parts

# Translation into SPARQL Algebra

```
{ LeftJoin(Bgp(?book <http://ex.org/price> ?price),
           Bgp(?book <http://ex.org/title> ?title),
           true)
  {Bgp(?book <http://ex.org/author>
                 <http://ex.org/Shakespeare>)} UNION
  {Bgp(?book <http://ex.org/author>
                 <http://ex.org/Marlowe>)}
  FILTER (?price < 15)
}
```

# Translation into SPARQL Algebra

```
{ LeftJoin(Bgp(?book <http://ex.org/price> ?price),
           Bgp(?book <http://ex.org/title> ?title),
           true)
  {Bgp(?book <http://ex.org/author>
                  <http://ex.org/Shakespeare>)} UNION
  {Bgp(?book <http://ex.org/author>
                  <http://ex.org/Marlowe>)}
  FILTER (?price < 15)
}
```

4. Combine alternative graph patterns with Union(·) operator

⇝ Refers to neighbouring patterns and has higher precedence than conjunction (left associative)

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# Translation into SPARQL Algebra

```
{ LeftJoin(Bgp(?book <http://ex.org/price> ?price),
           Bgp(?book <http://ex.org/title> ?title),
           true)
  Union(Bgp(?book <http://ex.org/author>
                             <http://ex.org/Shakespeare>),
        Bgp(?book <http://ex.org/author>
                             <http://ex.org/Marlowe>))
  FILTER (?price < 15)
}
```

# Translation into SPARQL Algebra

```
{ LeftJoin(Bgp(?book <http://ex.org/price> ?price),
           Bgp(?book <http://ex.org/title> ?title),
           true)
  Union(Bgp(?book <http://ex.org/author>
                   <http://ex.org/Shakespeare>),
        Bgp(?book <http://ex.org/author>
                   <http://ex.org/Marlowe>))
  FILTER (?price < 15)
}
```

5. Apply Join(·) operator to join non-filter elements

# Translation into SPARQL Algebra

```
{ Join(
    LeftJoin(Bgp(?book <http://ex.org/price> ?price),
             Bgp(?book <http://ex.org/title> ?title),
             true),
    Union(Bgp(?book <http://ex.org/author>
                        <http://ex.org/Shakespeare>),
          Bgp(?book <http://ex.org/author>
                        <http://ex.org/Marlowe>)))
    FILTER (?price < 15)
}
```

# Translation into SPARQL Algebra

```
{ Join(
    LeftJoin(Bgp(?book <http://ex.org/price> ?price),
             Bgp(?book <http://ex.org/title> ?title),
             true),
    Union(Bgp(?book <http://ex.org/author>
                    <http://ex.org/Shakespeare>),
          Bgp(?book <http://ex.org/author>
                    <http://ex.org/Marlowe>)))
    FILTER (?price < 15)
}
```

6. Translate a group with filters with the Filter(·) operator

# Translation into SPARQL Algebra

```
Filter(?price < 15,
    Join(
     LeftJoin(Bgp(?book <http://ex.org/price> ?price),
              Bgp(?book <http://ex.org/title> ?title),
              true),
     Union(Bgp(?book <http://ex.org/author>
                    <http://ex.org/Shakespeare>),
          Bgp(?book <http://ex.org/author>
                    <http://ex.org/Marlowe>))))
```

# Translation into SPARQL Algebra

```
Filter(?price < 15,
    Join(
     LeftJoin(Bgp(?book <http://ex.org/price> ?price),
          Bgp(?book <http://ex.org/title> ?title),
          true),
     Union(Bgp(?book <http://ex.org/author>
               <http://ex.org/Shakespeare>),
          Bgp(?book <http://ex.org/author>
               <http://ex.org/Marlowe>))))
```

- Online translation tool:
  `http://sparql.org/query-validator.html`

# Agenda

# Semantics of the SPARQL Algebra Operations

Now we have an algebra object, but what do the algebra operations mean?

| | |
|---|---|
| Bgp($P$) | match/evaluate pattern $P$ |
| Join($M_1$, $M_2$) | conjunctive join of solutions $M_1$ and $M_2$ |
| Union($M_1$, $M_2$) | union of solutions $M_1$ with $M_2$ |
| LeftJoin($M_1$, $M_2$, $F$) | optional join of $M_1$ with $M_2$ with filter constraint $F$ (`true` if no filter given) |
| Filter($F$, $M$) | filter solutions $M$ with constraint $F$ |
| $Z$ | empty pattern (identity for join) |

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Now we have an algebra object, but what do the algebra operations mean?

| | |
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| Filter($F$, $M$) | filter solutions $M$ with constraint $F$ |
| $Z$ | empty pattern (identity for join) |

- Only Bgp($\cdot$) matches or evaluates graph patterns

$\leadsto$ We can use entailment checking rather than graph matching

# Definition of the SPARQL Operators

How can we define that more formally?

Output:

- “solution set” (formatting irrelevant)

Input:

- Queried (active) graph
- Partial results from previous evaluation steps
- Different parameters according to the operation

$\rightsquigarrow$ How can we formally describe the “results”?

# SPARQL Results

Intuition: Results coded as tables of variable assignments

**Result:**

List of solutions (solution sequence)

$\rightsquigarrow$ each solution corresponds to one table row

# SPARQL Results

Intuition: Results coded as tables of variable assignments

## Result:

List of solutions (solution sequence)

$\rightsquigarrow$ each solution corresponds to one table row

## Solution:

Partial function

- Domain: relevant variables
- Range: IRIs $\cup$ blank nodes $\cup$ RDF literals

$\rightsquigarrow$ Unbound variables are those that have no assigned value (partial function)

# Evaluation of Basic Graph Patterns

## Definition (Solution)

Let $P$ be a basic graph pattern. A partial function $\mu$ is a solution for Bgp($P$) over the queried (active) graph $G$ if:

1. the domain of $\mu$ is exactly the set of variables in $P$,
2. there exists an assignment $\sigma$ from blank nodes in $P$ to IRIs, blank nodes, or RDF literals such that:
3. the RDF graph $\mu(\sigma(P))$ is a subgraph of $G$.

# Evaluation of Basic Graph Patterns

- The result of evaluating Bgp($P$) over $G$ is written $[\![\mathrm{Bgp}(P)]\!]_{\mathsf{G}}$
- The result is a multi set of solutions $\mu$
- The multiplicity of each solution $\mu$ corresponds to the number of different assignments $\sigma$

# Multi Sets

## Definition (Multi Set)

A multi set over a set $S$ is a total function $M\colon S \to \mathbb{N}^+ \cup \{\omega\}$

- $\mathbb{N}^+$ denotes the positive natural numbers
- $\omega > n$ for all $n \in \mathbb{N}^+$
- $M(s)$ is the multiplicity of $s \in S$
- $\omega$: countably infinite number of occurrences

- We represent a multi se over the set $S$ also with the set $\{(s, M(s)) \mid s \in S\}$
- We write $(s, n) \in M$ if $M(s) = n$
- We assume that $M(s) = 0$ if $s \notin S$
- Alternative notation: $\dot{\{}a, b, b\dot{\}}$ corresponds to the multi set $M$ over the set $\{a, b\}$ with $M(a) = 1$ and $M(b) = 2$

# Solution Mapping Example

```
ex:Birte ex:gives [
   a ex:Lecture ;
   ex:hasTopic "SPARQL" ] .
ex:Sebastian ex:gives [
   a ex:Lecture ;
   ex:hasTopic "DLs and OWL" ] .
```

Bgp(`?who ex:gives _:x .  _:x ex:hasTopic ?what`)

# Solution Mapping Example

```
ex:Birte ex:gives _:a .
_:a rdf:type ex:Lecture .
_:a ex:hasTopic "SPARQL" .
ex:Sebastian ex:gives _:b .
_:b rdf:type ex:Lecture .
_:b ex:hasTopic "DLs and OWL" .
```

Bgp(`?who ex:gives _:x .  _:x ex:hasTopic ?what`)

# Solution Mapping Example

```
ex:Birte ex:gives _:a .
_:a rdf:type ex:Lecture .
_:a ex:hasTopic "SPARQL" .
ex:Sebastian ex:gives _:b .
_:b rdf:type ex:Lecture .
_:b ex:hasTopic "DLs and OWL" .
```

Bgp(`?who ex:gives _:x . _:x ex:hasTopic ?what`)

$\mu_1$: ?who $\mapsto$ ex : Birte, ?what $\mapsto$ "SPARQL"

$\sigma_1$: _:x $\mapsto$ _:a

# Solution Mapping Example

```
ex:Birte ex:gives _:a .
_:a rdf:type ex:Lecture .
_:a ex:hasTopic "SPARQL" .
ex:Sebastian ex:gives _:b .
_:b rdf:type ex:Lecture .
_:b ex:hasTopic "DLs and OWL" .
```

Bgp(`?who ex:gives _:x .  _:x ex:hasTopic ?what`)

$\mu_1$: ?who $\mapsto$ ex : Birte, ?what $\mapsto$ "SPARQL"

$\sigma_1$: _:x $\mapsto$ _:a

$\mu_2$: ?who $\mapsto$ ex : Sebastian, ?what $\mapsto$ "DLs and OWL"

$\sigma_2$: _:x $\mapsto$ _:b

# Solution Mapping Example

```
ex:Birte ex:gives _:a .
_:a rdf:type ex:Lecture .
_:a ex:hasTopic "SPARQL" .
ex:Sebastian ex:gives _:b .
_:b rdf:type ex:Lecture .
_:b ex:hasTopic "DLs and OWL" .
```

Bgp(`?who ex:gives _:x .  _:x ex:hasTopic ?what`)

$\mu_1$: ?who $\mapsto$ ex : Birte, ?what $\mapsto$ "SPARQL"

$\sigma_1$: _:x $\mapsto$ _:a

$\mu_2$: ?who $\mapsto$ ex : Sebastian, ?what $\mapsto$ "DLs and OWL"

$\sigma_2$: _:x $\mapsto$ _:b

Two solutions each with multiplicity 1

# Exercise Solution Sets

```
ex:Birte ex:gives [
   a ex:Lecture ;
   ex:hasTopic "SPARQL" ] .
ex:Birte ex:gives [
   a ex:Lecture ;
   ex:hasTopic "SPARQL Algebra" ] .
```

Bgp(`?who ex:gives _:x .  _:x ex:hasTopic _:y`)

# Solution

# Solution

```
ex:Birte ex:gives _:a .
_:a rdf:type ex:Lecture .
_:a ex:hasTopic "SPARQL" .
ex:Birte ex:gives _:b .
_:b rdf:type ex:Lecture .
_:b ex:hasTopic "SPARQL Algebra" .
```

Bgp(`?who ex:gives _:x .  _:x ex:hasTopic _:y`)

# Solution

```
ex:Birte ex:gives _:a .
_:a rdf:type ex:Lecture .
_:a ex:hasTopic "SPARQL" .
ex:Birte ex:gives _:b .
_:b rdf:type ex:Lecture .
_:b ex:hasTopic "SPARQL Algebra" .
```

Bgp(`?who ex:gives _:x .  _:x ex:hasTopic _:y`)

$\mu_1$ : ?who ↦ ex : Birte,

$\sigma_1$ : _:x ↦ _:a    _:y ↦ "SPARQL"

# Solution

```
ex:Birte ex:gives _:a .
_:a rdf:type ex:Lecture .
_:a ex:hasTopic "SPARQL" .
ex:Birte ex:gives _:b .
_:b rdf:type ex:Lecture .
_:b ex:hasTopic "SPARQL Algebra" .
```

Bgp(`?who ex:gives _:x . _:x ex:hasTopic _:y`)

$\mu_1$: ?who $\mapsto$ ex : Birte,

$\sigma_1$: _:x $\mapsto$ _:a  _:y $\mapsto$ "SPARQL"

$\mu_2$: ?who $\mapsto$ ex : Birte,

$\sigma_2$: _:x $\mapsto$ _:b  _:y $\mapsto$ "SPARQL Algebra"

# Solution

```
ex:Birte ex:gives _:a .
_:a rdf:type ex:Lecture .
_:a ex:hasTopic "SPARQL" .
ex:Birte ex:gives _:b .
_:b rdf:type ex:Lecture .
_:b ex:hasTopic "SPARQL Algebra" .
```

Bgp(`?who ex:gives _:x . _:x ex:hasTopic _:y`)

$$\begin{array}{lll} \mu_1: & \texttt{?who} \mapsto \texttt{ex:Birte}, & \\ \sigma_1: & \texttt{\_:x} \mapsto \texttt{\_:a} & \texttt{\_:y} \mapsto \texttt{"SPARQL"} \\ \mu_2: & \texttt{?who} \mapsto \texttt{ex:Birte}, & \\ \sigma_2: & \texttt{\_:x} \mapsto \texttt{\_:b} & \texttt{\_:y} \mapsto \texttt{"SPARQL Algebra"} \end{array}$$

One solution with multiplicity 2

# Union of Solutions (1)

**Definition (Compatibility)**

Two solutions $\mu_1$ and $\mu_2$ are compatible if
$\mu_1(x) = \mu_2(x)$ for all $x$, for which $\mu_1$ and $\mu_2$ are defined

# Union of Solutions (1)

## Definition (Compatibility)

Two solutions $\mu_1$ and $\mu_2$ are compatible if
$\mu_1(x) = \mu_2(x)$ for all $x$, for which $\mu_1$ and $\mu_2$ are defined

$\mu_1 : \texttt{?x} \mapsto \texttt{ex:a}, \texttt{?y} \mapsto \texttt{ex:b}$
$\mu_2 : \texttt{?y} \mapsto \texttt{ex:b}, \texttt{?z} \mapsto \texttt{ex:c}$

# Union of Solutions (1)

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$\mu_1$: $\texttt{?x} \mapsto \texttt{ex:a}, \texttt{?y} \mapsto \texttt{ex:b}$
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$\mu_2$: $\texttt{?y} \mapsto \texttt{ex:b}, \texttt{?z} \mapsto \texttt{ex:c}$ ✓

$\mu_1$: $\texttt{?x} \mapsto \texttt{ex:a}, \texttt{?y} \mapsto \texttt{ex:b}$
$\mu_2$: $\texttt{?x} \mapsto \texttt{ex:b}, \texttt{?z} \mapsto \texttt{ex:c}$ $\lightning$

# Union of Solutions (1)

## Definition (Compatibility)

Two solutions $\mu_1$ and $\mu_2$ are compatible if
$\mu_1(x) = \mu_2(x)$ for all $x$, for which $\mu_1$ and $\mu_2$ are defined

$\mu_1$: $\mathtt{?x} \mapsto \mathtt{ex:a}, \mathtt{?y} \mapsto \mathtt{ex:b}$
$\mu_2$: $\mathtt{?y} \mapsto \mathtt{ex:b}, \mathtt{?z} \mapsto \mathtt{ex:c}$ ✓

$\mu_1$: $\mathtt{?x} \mapsto \mathtt{ex:a}, \mathtt{?y} \mapsto \mathtt{ex:b}$
$\mu_2$: $\mathtt{?x} \mapsto \mathtt{ex:b}, \mathtt{?z} \mapsto \mathtt{ex:c}$ ↯

$\mu_1$: $\mathtt{?x} \mapsto \mathtt{ex:a}$
$\mu_2$: $\mathtt{?y} \mapsto \mathtt{ex:b}$

# Union of Solutions (1)

## Definition (Compatibility)

Two solutions $\mu_1$ and $\mu_2$ are compatible if
$\mu_1(x) = \mu_2(x)$ for all $x$, for which $\mu_1$ and $\mu_2$ are defined

$\mu_1$: $\texttt{?x} \mapsto \texttt{ex:a}, \texttt{?y} \mapsto \texttt{ex:b}$
$\mu_2$: $\texttt{?y} \mapsto \texttt{ex:b}, \texttt{?z} \mapsto \texttt{ex:c}$ ✓

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# Union of Solutions (2)

Union of two compatible solutions $\mu_1$ and $\mu_2$:

$$(\mu_1 \cup \mu_2)(x) = \begin{cases} \mu_1(x) & \text{if } x \in \text{dom}(\mu_1) \\ \mu_2(x) & \text{otherwise} \end{cases}$$

$\rightsquigarrow$ simple intuition: union of matching table rows

- Next lecture: Evaluation of the main algebra operators

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# Agenda

## Summary

- SPARQL queries are translated into algebra objects
- The BGPs generate solutions
- Other algebra operators combine solutions
- Details in the next lecture