TECHNISCHE UNIVERSITÄT DRESDEN

FOUNDATIONS OF SEMANTIC WEB TECHNOLOGIES

OWL and Rules

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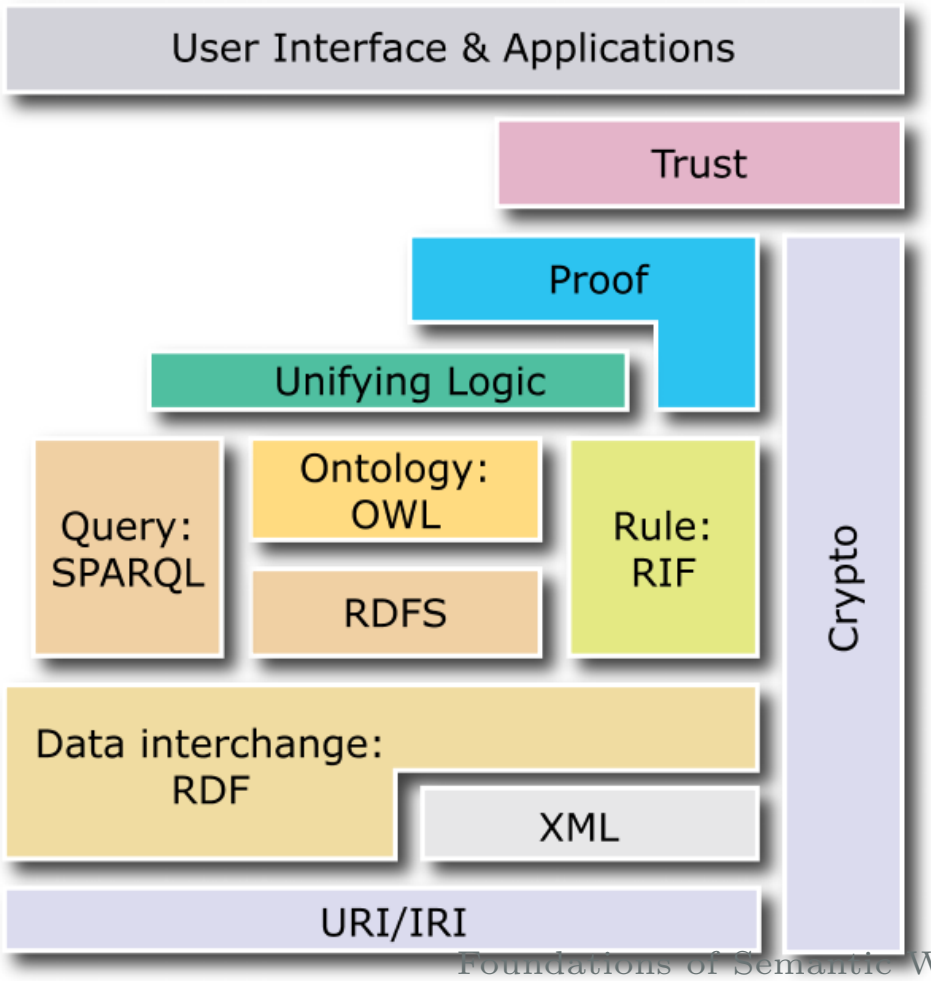
Dresden, 16 July 2013

DRESDEN concept
Exzellenz aus Wissenschaft und Kultur

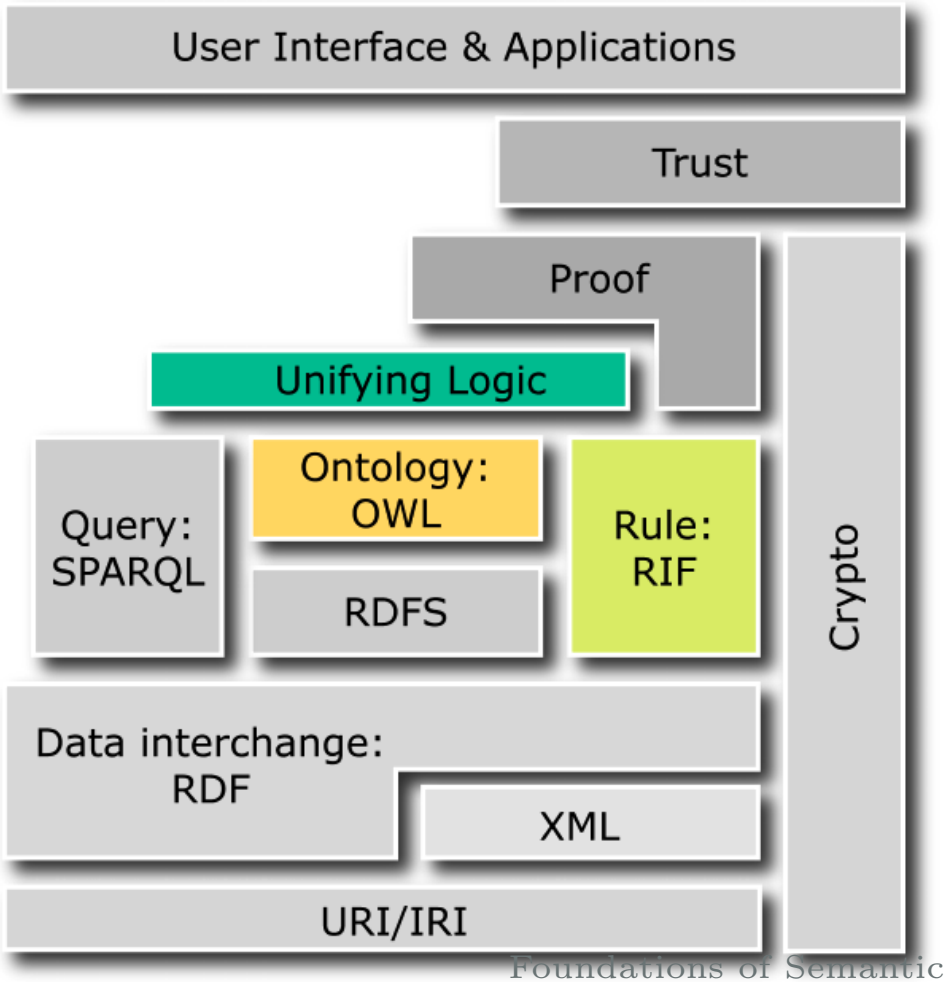
Introduction

Semantic Web Stack / Layer Cake



- Each layer builds on the layers below
- Standardization in progress and driven by W3C
- Hypertext Web technologies and some Semantic Web technologies already standardized

Two Different Paradigms



- Ontologies: OWL
- Rules: RIF, SWRL

Investigation towards a Unifying Logic

OWL 2 DL ($\mathcal{SROIQ}$(D)+...)

Basic Language Constructs

- Classes (concepts) – unary predicates

  Person, Woman, Mother, Uncle

- Properties (roles) – binary predicates

  hasChild, hasParent, hasWife

- Individuals – constants

  Mary, John, Bill

OWL 2 DL ($\mathcal{SROIQ}$(D)+...)

Axioms

- Assertions of named individuals to (complex) classes and properties

  $$\text{Woman}(\text{Mary}) \qquad \text{hasMother}(\text{Bill},\text{Mary})$$

- (Sub)class and property hierarchies ($\sqsubseteq$)

  $$\text{Woman} \sqsubseteq \text{Person} \qquad \text{hasMother} \sqsubseteq \text{hasParent}$$

- Equivalent classes ($\equiv$ – shortcut for $\sqsubseteq$ and $\sqsupseteq$)

  $$\text{Person} \equiv \text{Human}$$

OWL 2 DL ($\mathcal{SROIQ}$(D)+...)

Complex Classes

- Class intersection ($\sqcap$)

$$\text{Mother} \equiv \text{Person} \sqcap \text{Woman}$$

- Class union ($\sqcup$)

$$\text{Parent} \equiv \text{Mother} \sqcup \text{Father}$$

- Class complement ($\neg$)

$$\text{ChildlessPerson} \equiv \text{Person} \sqcap \neg\text{Parent}$$

OWL 2 DL ($\mathcal{SROIQ}$(D)+...)

Complex Classes

- `owl:Thing` ($\top$) and `owl:Nothing` ($\bot$)

$$\text{Man} \sqcap \text{Woman} \sqsubseteq \bot$$

- Existential quantification ($\exists$)

$$\text{Parent} \equiv \exists \text{hasChild}.\text{Person} \qquad \exists \text{hasWife}.\top \sqsubseteq \text{Man}$$

- Universal quantification ($\forall$)

$$\text{NoDaughters} \equiv \forall \text{hasChild}.\text{Male} \qquad \top \sqsubseteq \forall \text{hasWife}.\text{Woman}$$

OWL 2 DL ($\mathcal{SROIQ}$(D)+...)

Complex Classes

- Qualified cardinality constraints ($\leq$ and $\geq$)

  $\leq 2\text{hasChild.Parent}(\text{John})$ $\quad$ $\geq 4\text{hasChild.}\top(\text{John})$

- Nominals/enumerations of individuals (`owl:oneOf`)

  $\{\text{William}\} \equiv \{\text{Bill}\}$ $\quad$ $\text{SiblingsOfJohn} \equiv \{\text{Mary, Tom}\}$

- Self

  $\text{NarcisticPerson} \equiv \exists \text{loves.Self}$

OWL 2 DL ($\mathcal{SROIQ}$(D)+...)

Property Characteristics

- Inverse – hasChild$^-$ instead of hasParent
- Symmetric – hasSpouse
- Asymmetric – hasChild
- Disjoint – hasParent and hasChild
- Reflexive – hasRelative
- Irreflexive – hasChild
- Functional – hasSpouse
- Inverse functional – hasSpouse
- Transitive – hasDescendent

OWL 2 DL ($\mathcal{SROIQ}$(D)+...)

And also ...

- Property chains

$$\text{hasParent} \circ \text{hasParent} \sqsubseteq \text{hasGrandparent}$$

- Top property (U universal role)
- Bottom property (not part of $\mathcal{SROIQ}$(D))
- Datatypes (with facets) (D)
- Keys (not part of $\mathcal{SROIQ}$(D))

OWL 2 Profiles

fragments of OWL 2 for better computational properties

- OWL 2 EL:
  - Corresponds to $\mathcal{SROEL}(\mathrm{D})$ /$\mathcal{EL}^{++}$
  - Allows $\sqcap$, $\exists$, $\top$, $\bot$, nominals, property chains and hierarchies, and datatypes
  - Also allows: reflexive and transitive properties, keys
  - Used in large biomedicine ontologies, such as SNOMED CT, or GALEN (containing complex structural descriptions)

OWL 2 Profiles

- OWL 2 QL:
  - Corresponds to DL-Lite
  - Left of $\sqsubseteq$ only allows: $\exists$ limited to $\top$
  - Right of $\sqsubseteq$ only allows: $\sqcap$, $\neg$, $\exists$
  - Allows property inclusions but not property chains
  - Closely related to database technology, query answering can be realized by rewriting queries
- OWL 2 RL:
  - Corresponds to DLP, a rule fragment of OWL 2 DL
  - Well-suited for enriching RDF data
  - Details follow later

What we can(not) do with OWL

- Describe schema level knowledge: class hierarchy, properties about classes, relationships between classes, etc.
- Consistency and class subsumption checking
- Classifying individuals to classes
- Assert the existence of unknown individuals (i.e., those that must exist but cannot be named)
- Cannot specify arbitrary relationships between instances/individuals; due to the inherent tree structure of DLs
- Cannot express n-ary relationships between individuals with $n > 2$; DL extensions with n-ary predicates exist, but not part of OWL

Rules

- Prominent alternative to OWL modeling:
  - Rule-based expert systems
  - Logic Programming/Prolog
  - F-Logic [Kifer et al., 1995]
  - W3C Rule Interchange Format RIF (standard since 2010)
- Often argued to be more intuitive for modeling:

$$\text{worksAt(x,y)},\text{university(y)},\ \text{supervises(x,z)},\text{PhDstudent(z)} \rightarrow \text{ProfessorOf(x, z)}$$

Rules

- Rules can be divided into 3 categories:
  - First-order rules: logical implication F $\rightarrow$ G
    - Closely related to RIF-BLD (Basic Logic Dialect)
    - "Open world", declarative (first-order) semantics, monotonic
  - Logic Programming/PROLOG rules:
    - Close to first-order rules but with optional procedural aspects and possible built-ins
    - Covered by RIF-FLD (Framework for Logic Dialects)
    - "Closed world", (semi-)declarative, non-monotonic
  - Production rules:
    - IF condition THEN action
    - Roughly corresponds to RIF-PRD (Production Rule Dialect)
    - Semantics varies, sometimes defined as ad hoc computational mechanisms

First-order rules (Horn clauses)

- $\overbrace{A_1 \wedge A_2 \wedge \cdots \wedge A_k}^{\text{body}} \rightarrow \overbrace{H}^{\text{head}}$
  - Each A_i and H is a first-order atomic formula $P(t_1, \ldots, t_m)$ with P a predicate symbol with arity m
  - Each t_j is a term: a variable or an expression $f(s_1, \ldots, s_k)$ where f is a function symbol of arity k and $s_1, \ldots, s_k$ are terms
  - No quantifiers; no negation
- Datalog rules: first-order rules without function symbols
  - First used for deductive databases
  - Complexity: PTime data complexity (ExpTime combined)
  - Suitable for large datasets (and relatively small/fixed rule set)

What we can(not) do with Rules

- Specify and infer arbitrary relationships between individuals, including n-ary relationships with $n > 2$
- Many people find rules more natural for modeling
- Non-monotonic extensions are very well-studied, more than that of OWL (ASP solvers, etc.)
- Rules are usually only applied to known constants
- Cannot express the existence of unknown/unnamed individuals (unlike OWL)

Can't we bring them together?

Our Agenda . . .

This tutorial provides a condensed exposition of the recent efforts to answer the previous main question by focusing on the following issues:

- What kind of rules are readily expressible in OWL?
- What is DL-safety notion for rules? Why does it allow one to combine rules and OWL ontologies without losing decidability?
- Can we integrate DL-safe rules seamlessly within OWL framework by some small syntactic extension to OWL?
- Can we add non-monotonic flavor to such integration between DLs and rules?

Rules readily expressible in OWL

Reasoning Needs

z newsFrom rome .
rome locadedIn italy .

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$$\text{newsFrom}(x, y) \wedge \text{locatedIn}(y, z) \rightarrow \text{newsFrom}(x, z)$$

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In OWL:

$$\text{newsFrom} \circ \text{locatedIn} \sqsubseteq \text{newsFrom}$$

(using owl:propertyChainAxiom)

Translating OWL Axioms

Which OWL axioms can be encoded as rules?

Let's see some examples

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$$A \sqsubseteq B \text{ becomes } A(x) \rightarrow B(x)$$
$$R \sqsubseteq S \text{ becomes } R(x, y) \rightarrow S(x, y)$$

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$$R \sqsubseteq S \text{ becomes } R(x, y) \rightarrow S(x, y)$$

$$A \sqcap \exists R.\exists S.B \sqsubseteq C \text{ becomes } A(x) \wedge R(x, y) \wedge S(y, z) \wedge B(z) \rightarrow C(x)$$

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$$A \sqcap \exists R.\exists S.B \sqsubseteq C \text{ becomes } A(x) \wedge R(x, y) \wedge S(y, z) \wedge B(z) \rightarrow C(x)$$

$$A \sqsubseteq \forall R.B \text{ becomes } A(x) \wedge R(x, y) \rightarrow B(y)$$

Rules in OWL

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$$A \sqsubseteq \neg B \text{ becomes } A(x) \wedge B(x) \rightarrow f$$

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$$A \sqsubseteq \neg B \text{ becomes } A(x) \wedge B(x) \rightarrow f$$

$$\top \sqsubseteq \leq 1R.\top \text{ becomes } R(x, y) \wedge R(x, z) \rightarrow y = z$$

Rules in OWL

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$$\top \sqsubseteq \leq 1R.\top \text{ becomes } R(x, y) \wedge R(x, z) \rightarrow y = z$$

$$A \sqcap \exists R.\{b\} \sqsubseteq C \text{ becomes } A(x) \wedge R(x, b) \rightarrow C(x)$$

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Rules in OWL

Which OWL axioms can be encoded as rules?

$$\{a\} \equiv \{b\} \text{ becomes } \rightarrow a = b$$

$$A \sqcap B \sqsubseteq \bot \text{ becomes } A(x) \wedge B(x) \rightarrow f$$

$$A \sqsubseteq B \sqcap C \text{ becomes } A(x) \rightarrow B(x) \text{ and } A(x) \rightarrow C(x)$$
$$A \sqcup B \sqsubseteq C \text{ becomes } A(x) \rightarrow C(x) \text{ and } B(x) \rightarrow C(x)$$

Rules in OWL

A DL axiom α can be translated into rules if, after translating α into a first-order predicate logic expression α', and after normalizing this expression into a set of clauses M, each formula in M is a Horn clause (i.e., a rule).

Issue: How complicated a translation is allowed?

Rules in OWL

A DL axiom α can be translated into rules if, after translating α into a first-order predicate logic expression α', and after normalizing this expression into a set of clauses M, each formula in M is a Horn clause (i.e., a rule).

Issue: How complicated a translation is allowed?

Naive translation: e.g.,

$$\mathrm{R} \circ \mathrm{S} \sqsubseteq \mathrm{T} \text{ becomes } \mathrm{R}(\mathrm{x}, \mathrm{y}) \wedge \mathrm{S}(\mathrm{y}, \mathrm{z}) \rightarrow \mathrm{T}(\mathrm{x}, \mathrm{z})$$

This essentially results in OWL RL

Which rules can be translated into OWL axioms?

- Rolification
- Examples
- Formal definition: Rule Graphs

Rolification

$$\text{Elephant(x)} \wedge \text{Mouse(y)} \rightarrow \text{biggerThan(x, y)}$$

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Rolification of a concept A: $A \sqsubseteq \exists R_A.\text{Self}$

$$\begin{aligned} \text{Elephant} &\sqsubseteq \exists R_{\text{Elephant}}.\text{Self} \\ \text{Mouse} &\sqsubseteq \exists R_{\text{Mouse}}.\text{Self} \\ R_{\text{Elephant}} \circ U \circ R_{\text{Mouse}} &\sqsubseteq \text{biggerThan} \end{aligned}$$

Rolification

$$\mathrm{A(x) \wedge R(x,\, y) \rightarrow S(x,\, y)} \text{ becomes } \mathrm{R_A \circ R \sqsubseteq S}$$
$$\mathrm{A(y) \wedge R(x,\, y) \rightarrow S(x,\, y)} \text{ becomes } \mathrm{R \circ R_A \sqsubseteq S}$$
$$\mathrm{A(x) \wedge B(y) \wedge R(x,\, y) \rightarrow S(x,\, y)} \text{ becomes } \mathrm{R_A \circ R \circ R_B \sqsubseteq S}$$

Rolification

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$$\mathrm{A(x) \wedge B(y) \wedge R(x,\, y) \rightarrow S(x,\, y)} \text{ becomes } \mathrm{R_A \circ R \circ R_B \sqsubseteq S}$$

$$\mathrm{Woman(x) \wedge marriedTo(x,\, y) \wedge Man(y) \rightarrow hasHusband(x,\, y)}$$
$$\mathrm{R_{Woman} \circ marriedTo \circ R_{Man} \sqsubseteq hasHusband}$$

careful - role regularity needs to be preserved

$$\mathrm{hasHusband \sqsubseteq marriedTo}$$

Rolification

$$\begin{aligned} \text{worksAt}(x, y) \wedge \text{University}(y) \wedge \text{supervises}(x, z) \\ \wedge \text{PhDStudent}(z) \rightarrow \text{professorOf}(x, z) \end{aligned}$$

$$R_{\exists \text{worksAt.University}} \circ \text{supervises} \circ R_{\text{PhDStudent}} \sqsubseteq \text{professorOf}$$

Rules in OWL 2

$$\mathrm{Man}(x) \wedge \mathrm{hasBrother}(x, y) \wedge \mathrm{hasChild}(y, z) \rightarrow \mathrm{Uncle}(x)$$

$$\mathrm{Man} \sqcap \exists \mathrm{hasBrother}.\exists \mathrm{hasChild}.\top \sqsubseteq \mathrm{Uncle}$$

$$\mathrm{NutAllergic}(x) \wedge \mathrm{NutProdcut}(y) \rightarrow \mathrm{dislikes}(x, y)$$

$$\mathrm{NutAllergic} \sqsubseteq \exists \mathrm{nutAllergic}.\mathrm{Self}$$

$$\mathrm{NutProduct} \sqsubseteq \exists \mathrm{nutProduct}.\mathrm{Self}$$

$$\mathrm{nutAllergic} \circ \mathrm{U} \circ \mathrm{nutProduct} \sqsubseteq \mathrm{dislikes}$$

$$\mathrm{dislikes}(x, z) \wedge \mathrm{Dish}(y) \wedge \mathrm{contains}(y, z) \rightarrow \mathrm{dislikes}(x, y)$$

$$\mathrm{Dish} \sqsubseteq \exists \mathrm{dish}.\mathrm{Self}$$

$$\mathrm{dislikes} \circ \mathrm{contains}^{-} \circ \mathrm{dish} \sqsubseteq \mathrm{dislikes}$$

So how can we pinpoint this?

- Tree-shaped bodies (variables)
- First argument of the conclusion is the root

$$C(x) \wedge R(x, a) \wedge S(x, y) \wedge D(y) \wedge T(y, a) \rightarrow E(x)$$

$$C \sqcap \exists R.\{a\} \sqcap \exists S.(D \sqcap \exists T.\{a\}) \sqsubseteq E$$

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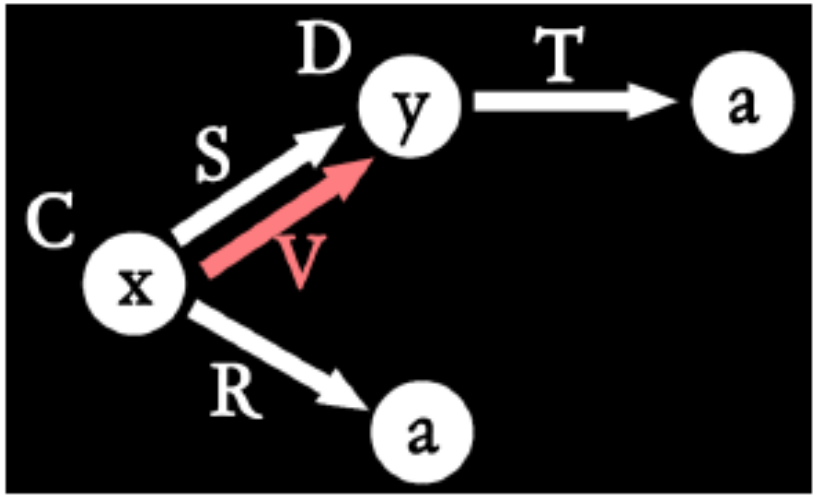
$$C(x) \wedge R(x,a) \wedge S(x,y) \wedge D(y) \wedge T(y,a) \rightarrow V(x,y)$$

$$C \sqcap \exists R.\{a\} \sqsubseteq \exists R_1.\mathrm{Self}$$
$$D \sqcap \exists T.\{a\} \sqsubseteq \exists R_2.\mathrm{Self}$$
$$R_1 \circ S \circ R_2 \sqsubseteq V$$



So how can we pinpoint this?

Rule graph: $C(x) \wedge R(x, a) \wedge S(x, y) \wedge D(y) \wedge T(y, a) \rightarrow P(x, y)$

Graph analysis: determine whether a rule is expressible within a given profile
Automatic Transformation

DLs Rules: $\mathcal{EL}^{++}$

$$\mathrm{R}_1(\mathrm{x},\mathrm{y}) \wedge \mathrm{C}_1(\mathrm{y}) \wedge \mathrm{R}_2(\mathrm{y},\mathrm{w}) \wedge \mathrm{R}_3(\mathrm{y},\mathrm{z}) \wedge \mathrm{C}_2(\mathrm{z}) \wedge \mathrm{R}_4(\mathrm{x},\mathrm{x}) \rightarrow \mathrm{C}_3(\mathrm{x})$$

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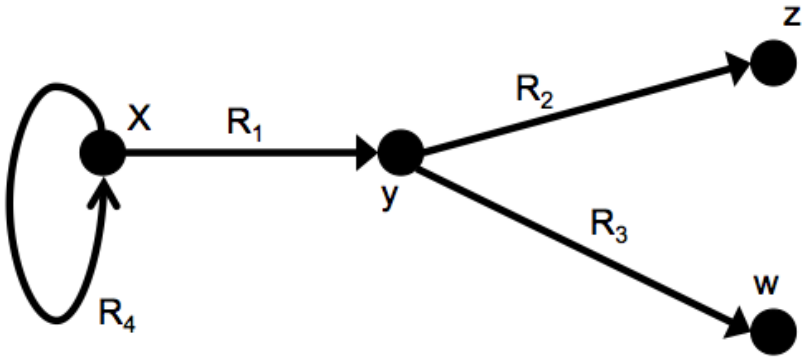
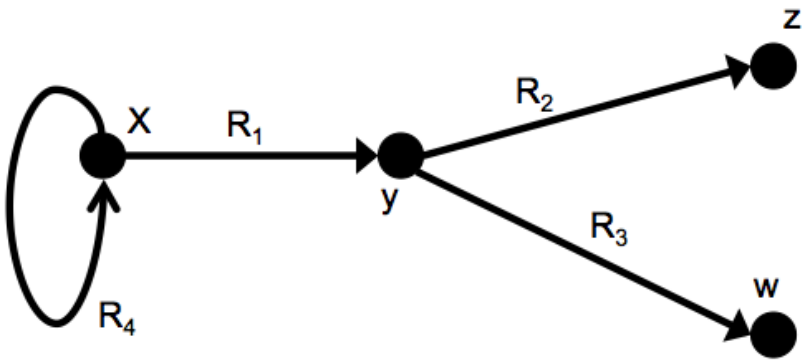


DLs Rules: $\mathcal{EL}^{++}$

$$\mathrm{R_1(x,y) \wedge C_1(y) \wedge R_2(y,w) \wedge R_3(y,z) \wedge C_2(z) \wedge R_4(x,x) \rightarrow C_3(x)}$$



$$\mathrm{\exists R_1.(C_1 \sqcap \exists R_2.\top \sqcap \exists R_3.C_2) \sqcap \exists R_4.Self \sqsubseteq C_3}$$

DLs Rules: $\mathcal{SROIQ}$

$$\mathrm{R}_1(\mathrm{y},\mathrm{x}) \wedge \mathrm{C}_1(\mathrm{y}) \wedge \mathrm{R}_2(\mathrm{w},\mathrm{y}) \wedge \mathrm{R}_3(\mathrm{y},\mathrm{z}) \wedge \mathrm{C}_2(\mathrm{z}) \wedge \mathrm{R}_4(\mathrm{x},\mathrm{x}) \rightarrow \mathrm{C}_3(\mathrm{x})$$

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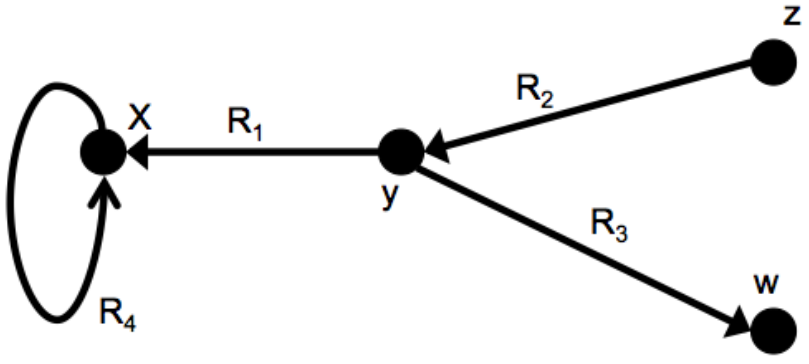
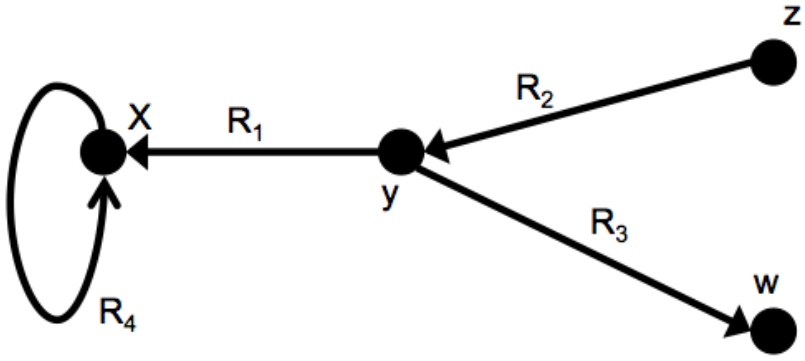


DLs Rules: $\mathcal{SROIQ}$

$$\mathrm{R_1(y,x) \wedge C_1(y) \wedge R_2(w,y) \wedge R_3(y,z) \wedge C_2(z) \wedge R_4(x,x) \rightarrow C_3(x)}$$



$$\mathrm{\exists R_1^-.(C_1 \sqcap \exists R_2^-.\top \sqcap \exists R_3.C_2) \sqcap \exists R_4.Self \sqsubseteq C_3}$$

Nominal Schemas

What we learned about rules expressible in OWL

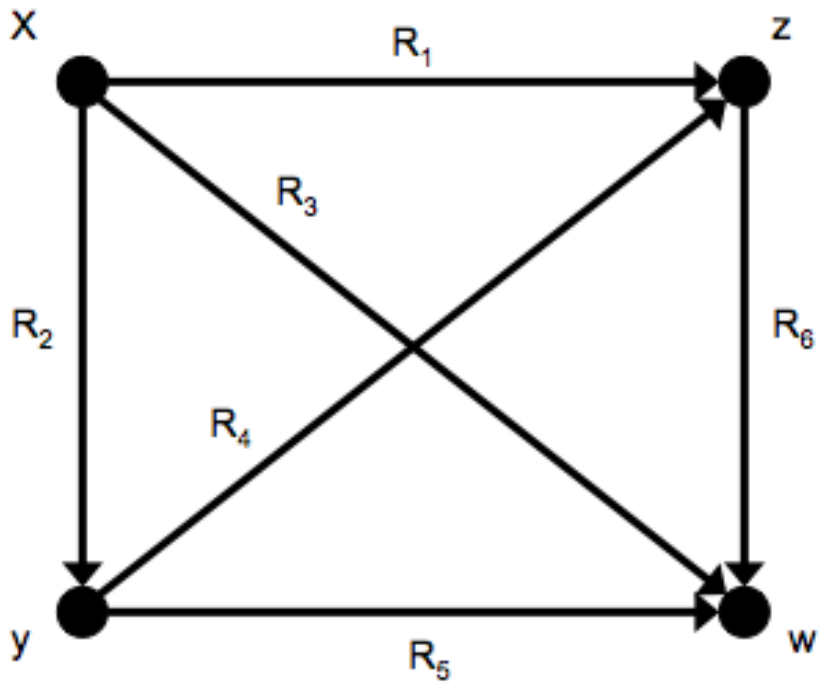
- Rules with tree-shaped body can be expressed in DL,
- role conjunction allows DLs to express some rules with non-tree-shaped body, but
- many rules are not covered.

Clique of 4

$$\mathrm{R}_1(\mathrm{x},\mathrm{y}) \wedge \mathrm{R}_2(\mathrm{x},\mathrm{z}) \wedge \mathrm{R}_3(\mathrm{x},\mathrm{w}) \wedge \mathrm{R}_4(\mathrm{y},\mathrm{z}) \wedge \mathrm{R}_5(\mathrm{y},\mathrm{w}) \wedge \mathrm{R}_6(\mathrm{w},\mathrm{z}) \rightarrow \mathrm{C}(\mathrm{x})$$

Clique of 4

$$R_1(x, y) \wedge R_2(x, z) \wedge R_3(x, w) \wedge R_4(y, z) \wedge R_5(y, w) \wedge R_6(w, z) \rightarrow C(x)$$



Nominal Schemas

A Better Uncle for OWL
[Krötzsch et al; WWW 2011]

$$\text{hasParent}(x, y) \wedge \text{married}(y, z) \wedge \text{hasParent}(x, z) \rightarrow C(x)$$

$$\exists \text{hasParent}.\exists \text{married}.\{z\} \sqcap \exists \text{hasParent}.\{z\} \sqsubseteq C$$

Nominal Schemas

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[Krötzsch et al; WWW 2011]

$\text{hasParent}(x, y) \wedge \text{married}(y, z) \wedge \text{hasParent}(x, z) \rightarrow C(x)$

$\exists \text{hasParent}.\exists \text{married}.\{z\} \sqcap \exists \text{hasParent}.\{z\} \sqsubseteq C$

{z} only binds to known/named individuals!

Covers DL-safe datalog (arbitrary arity of predicates)

Complex Rules to OWL

Theorem

Any rule R containing m different free variables, where $m > 3$, can be directly expressed in DL using n nominal schemas s.t. $n \leq m - 2$.

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- Problem: although the DL-part and rule-part are both decidable, the combination is undecidable!
- Work around: weaken the rule semantics?

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- In the rule:

$$\mathrm{R}(\mathrm{u},\mathrm{x}) \wedge \mathrm{A}(\mathrm{x}) \wedge \mathrm{S}(\mathrm{u},\mathrm{y}) \wedge \mathrm{T}(\mathrm{x},\mathrm{z}) \wedge \mathrm{T}(\mathrm{y},\mathrm{z}) \wedge \mathrm{R}(\mathrm{u},\mathrm{z}) \wedge \mathrm{S}(\mathrm{x},\mathrm{y}) \rightarrow \mathrm{B}(\mathrm{u})$$

the variables $\mathrm{u},\mathrm{x},\mathrm{y},\mathrm{z}$ refer only to named individuals.

DL-safety

- Decidability guaranteed if rules only operate on named individuals
  - Named individuals are finite.
- DL-safety: variables in the rules refer to only named individuals in the OWL ontology.
- In the rule:

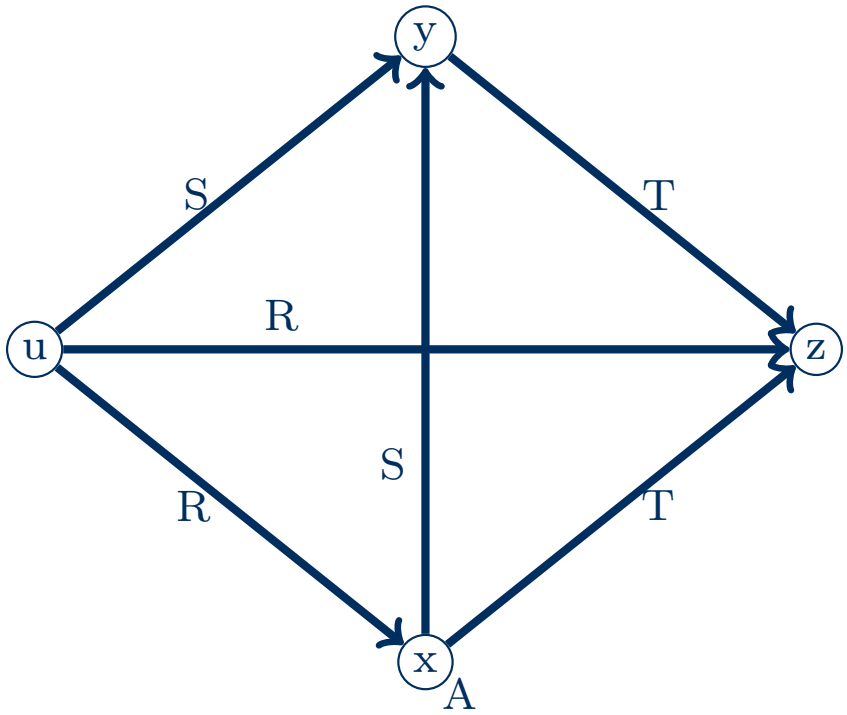
$$\mathrm{R}(\mathrm{u},\mathrm{x}) \wedge \mathrm{A}(\mathrm{x}) \wedge \mathrm{S}(\mathrm{u},\mathrm{y}) \wedge \mathrm{T}(\mathrm{x},\mathrm{z}) \wedge \mathrm{T}(\mathrm{y},\mathrm{z}) \wedge \mathrm{R}(\mathrm{u},\mathrm{z}) \wedge \mathrm{S}(\mathrm{x},\mathrm{y}) \rightarrow \mathrm{B}(\mathrm{u})$$

  the variables $\mathrm{u}, \mathrm{x}, \mathrm{y}, \mathrm{z}$ refer only to named individuals.
- Approach taken by DL-safe SWRL.

Revisiting DL-safety: can it be relaxed?

$$R(u, x) \wedge A(x) \wedge S(u, y) \wedge T(x, z) \wedge T(y, z) \wedge R(u, z) \wedge S(x, y) \rightarrow B(u)$$

Rule body forms complicated graph:



Revisiting DL-safety: can it be relaxed?

$$\mathrm{R}(\mathrm{u},\mathrm{x}) \wedge \mathrm{A}(\mathrm{x}) \wedge \mathrm{S}(\mathrm{u},\mathrm{y}) \wedge \mathrm{T}(\mathrm{x},\mathrm{z}) \wedge \mathrm{T}(\mathrm{y},\mathrm{z}) \wedge \mathrm{R}(\mathrm{u},\mathrm{z}) \wedge \mathrm{S}(\mathrm{x},\mathrm{y}) \rightarrow \mathrm{B}(\mathrm{u})$$

If y and z refer to named individuals, say a, b, it represents:

$$\mathrm{R}(\mathrm{u},\mathrm{x}) \wedge \mathrm{A}(\mathrm{x}) \wedge \mathrm{S}(\mathrm{u},\mathrm{a}) \wedge \mathrm{T}(\mathrm{x},\mathrm{a}) \wedge \mathrm{T}(\mathrm{a},\mathrm{a}) \wedge \mathrm{R}(\mathrm{u},\mathrm{a}) \wedge \mathrm{S}(\mathrm{x},\mathrm{a}) \rightarrow \mathrm{B}(\mathrm{u})$$
$$\mathrm{R}(\mathrm{u},\mathrm{x}) \wedge \mathrm{A}(\mathrm{x}) \wedge \mathrm{S}(\mathrm{u},\mathrm{a}) \wedge \mathrm{T}(\mathrm{x},\mathrm{b}) \wedge \mathrm{T}(\mathrm{a},\mathrm{b}) \wedge \mathrm{R}(\mathrm{u},\mathrm{b}) \wedge \mathrm{S}(\mathrm{x},\mathrm{a}) \rightarrow \mathrm{B}(\mathrm{u})$$
$$\mathrm{R}(\mathrm{u},\mathrm{x}) \wedge \mathrm{A}(\mathrm{x}) \wedge \mathrm{S}(\mathrm{u},\mathrm{b}) \wedge \mathrm{T}(\mathrm{x},\mathrm{a}) \wedge \mathrm{T}(\mathrm{b},\mathrm{a}) \wedge \mathrm{R}(\mathrm{u},\mathrm{a}) \wedge \mathrm{S}(\mathrm{x},\mathrm{b}) \rightarrow \mathrm{B}(\mathrm{u})$$
$$\mathrm{R}(\mathrm{u},\mathrm{x}) \wedge \mathrm{A}(\mathrm{x}) \wedge \mathrm{S}(\mathrm{u},\mathrm{b}) \wedge \mathrm{T}(\mathrm{x},\mathrm{b}) \wedge \mathrm{T}(\mathrm{b},\mathrm{b}) \wedge \mathrm{R}(\mathrm{u},\mathrm{b}) \wedge \mathrm{S}(\mathrm{x},\mathrm{b}) \rightarrow \mathrm{B}(\mathrm{u})$$

Revisiting DL-safety: can it be relaxed?

$$R(u,x) \wedge A(x) \wedge S(u,y) \wedge T(x,z) \wedge T(y,z) \wedge R(u,z) \wedge S(x,y) \rightarrow B(u)$$

$$R(u,x) \wedge A(x) \wedge S(u,a) \wedge T(x,a) \wedge T(a,a) \wedge R(u,a) \wedge S(x,a) \rightarrow B(u)$$
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- Only y and z need to be DL-safe.
- Expressible in OWL EL.

Nominal schemas: DL-safety only to some variables

- In the rule

$$\mathrm{R}(\mathrm{u},\mathrm{x}) \wedge \mathrm{A}(\mathrm{x}) \wedge \mathrm{S}(\mathrm{u},\mathrm{y}) \wedge \mathrm{T}(\mathrm{x},\mathrm{z}) \wedge \mathrm{T}(\mathrm{y},\mathrm{z}) \wedge \mathrm{R}(\mathrm{u},\mathrm{z}) \wedge \mathrm{S}(\mathrm{x},\mathrm{y}) \rightarrow \mathrm{B}(\mathrm{u})$$

  only y and z need to be DL-safe to be expressible in OWL EL.
- Expressing it in OWL EL need multiple axioms:

$$\exists \mathrm{R}.\{\mathrm{a}\} \sqcap \exists \mathrm{R}.(\mathrm{A} \sqcap \exists \mathrm{S}.\{\mathrm{a}\} \sqcap \exists \mathrm{T}.\{\mathrm{a}\}) \sqcap \exists \mathrm{S}.(\{\mathrm{a}\} \sqcap \exists \mathrm{T}.\{\mathrm{a}\}) \sqsubseteq \mathrm{B}$$
$$\exists \mathrm{R}.\{\mathrm{b}\} \sqcap \exists \mathrm{R}.(\mathrm{A} \sqcap \exists \mathrm{S}.\{\mathrm{a}\} \sqcap \exists \mathrm{T}.\{\mathrm{b}\}) \sqcap \exists \mathrm{S}.(\{\mathrm{a}\} \sqcap \exists \mathrm{T}.\{\mathrm{b}\}) \sqsubseteq \mathrm{B}$$
$$\exists \mathrm{R}.\{\mathrm{a}\} \sqcap \exists \mathrm{R}.(\mathrm{A} \sqcap \exists \mathrm{S}.\{\mathrm{b}\} \sqcap \exists \mathrm{T}.\{\mathrm{a}\}) \sqcap \exists \mathrm{S}.(\{\mathrm{b}\} \sqcap \exists \mathrm{T}.\{\mathrm{a}\}) \sqsubseteq \mathrm{B}$$
$$\exists \mathrm{R}.\{\mathrm{b}\} \sqcap \exists \mathrm{R}.(\mathrm{A} \sqcap \exists \mathrm{S}.\{\mathrm{b}\} \sqcap \exists \mathrm{T}.\{\mathrm{b}\}) \sqcap \exists \mathrm{S}.(\{\mathrm{b}\} \sqcap \exists \mathrm{T}.\{\mathrm{b}\}) \sqsubseteq \mathrm{B}$$

- With nominal schemas, the above 4 axioms can be condensed:
  $\exists \mathrm{R}.\{\mathrm{z}\} \sqcap \exists \mathrm{R}.(\mathrm{A} \sqcap \exists \mathrm{S}.\{\mathrm{y}\} \sqcap \exists \mathrm{T}.\{\mathrm{z}\}) \sqcap \exists \mathrm{S}.(\{\mathrm{y}\} \sqcap \exists \mathrm{T}.\{\mathrm{z}\}) \sqsubseteq \mathrm{B}$

Nominal schemas: syntax and semantics

- Nominal schemas: a "variable nominal" construct in the form of {x} where x is a variable.

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- If an axiom α contains n different nominal schemas (each may occur more than once) while the ontology has m different named individuals, then α represents m^n different axioms, each is obtained by substituting nominal schemas with named individuals.
- All those m^n axioms are called groundings of α.

What can we express with nominal schemas?

- Let $\mathcal{SROEL\!V}(\times, \sqcap) = \mathcal{SROEL}(\times, \sqcap)$ + nominal schemas.

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- Let $\mathcal{SROEL}\mathcal{V}(\times, \sqcap) = \mathcal{SROEL}(\times, \sqcap)$ + nominal schemas.
- $\mathcal{SROEL}\mathcal{V}(\sqcap, \times)$ covers:
  - DL-safe Datalog rules with predicates of arbitrary arity is also in $\mathcal{SROEL}\mathcal{V}$.
  - OWL 2 EL without datatypes
  - DL-safe OWL 2 RL without datatypes — but, preserves only ABox entailments (the main inference task for OWL 2 RL)
  - most of OWL 2 QL

Complexity bounds

- Reasoning for $\mathcal{SROIQV} = \mathcal{SROIQ}$ + nominal schemas, is theoretically no worse than $\mathcal{SROIQ}$ [Krötzsch et al., WWW11]

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- Reasoning for $\mathcal{SROIQV} = \mathcal{SROIQ}$ + nominal schemas, is theoretically no worse than $\mathcal{SROIQ}$ [Krötzsch et al., WWW11]
- Reasoning for $\mathcal{SROELV}$ is:
  - still polynomial (like $\mathcal{SROEL}$) if the number of occurrences of different nominal schemas in an axiom is bounded by a fixed constant;
  - in general, it is exponential (c.f., combined complexity of Datalog is ExpTime)

An Efficient Implementation for Nominal Schemas?

Naive grounding

- Naive reasoning directly from the semantics:

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  - Ground each axiom containing nominal schemas into exponentially many axioms without nominal schemas.
  - The resulting ontology is in standard DL or OWL and equivalent to the original one.
  - Use any existing reasoning algorithm for the corresponding standard DL on the resulting ontology.

Naive grounding

- Naive reasoning directly from the semantics:
  - Ground each axiom containing nominal schemas into exponentially many axioms without nominal schemas.
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  - Use any existing reasoning algorithm for the corresponding standard DL on the resulting ontology.
- Practically inefficient.

Naive grounding example

$$\exists \text{hParent}.\exists \text{married}.\{\text{z}\} \sqcap \exists \text{hParent}.\{\text{z}\} \sqsubseteq \text{C}$$

Naive grounding example

$$\exists \text{hParent}.\exists \text{married}.\{\text{z}\} \sqcap \exists \text{hParent}.\{\text{z}\} \sqsubseteq \text{C}$$

Full grounding:

$$\exists \text{hParent}.\exists \text{married}.\{\text{a}\} \sqcap \exists \text{hParent}.\{\text{a}\} \sqsubseteq \text{C}$$
$$\exists \text{hParent}.\exists \text{married}.\{\text{b}\} \sqcap \exists \text{hParent}.\{\text{b}\} \sqsubseteq \text{C}$$
$$\exists \text{hParent}.\exists \text{married}.\{\text{c}\} \sqcap \exists \text{hParent}.\{\text{c}\} \sqsubseteq \text{C}$$

where a, b, and c are the only individuals in the knowledge base

Naive grounding example

$$\exists R_1.(\exists R_4.\{z\} \sqcap \exists R_5.(\{w\} \sqcap \exists R_6.\{z\})) \sqcap \exists R_2.\{z\} \sqcap \exists R_3.\{w\} \sqsubseteq C$$

Naive grounding example

$$\exists R_1.(\exists R_4.\{z\} \sqcap \exists R_5.(\{w\} \sqcap \exists R_6.\{z\})) \sqcap \exists R_2.\{z\} \sqcap \exists R_3.\{w\} \sqsubseteq C$$

Full grounding:

$$\exists R_1.(\exists R_4.\{a\} \sqcap \exists R_5.(\{a\} \sqcap \exists R_6.\{a\})) \sqcap \exists R_2.\{a\} \sqcap \exists R_3.\{a\} \sqsubseteq C$$
$$\exists R_1.(\exists R_4.\{a\} \sqcap \exists R_5.(\{b\} \sqcap \exists R_6.\{a\})) \sqcap \exists R_2.\{a\} \sqcap \exists R_3.\{b\} \sqsubseteq C$$
$$\exists R_1.(\exists R_4.\{a\} \sqcap \exists R_5.(\{c\} \sqcap \exists R_6.\{a\})) \sqcap \exists R_2.\{a\} \sqcap \exists R_3.\{c\} \sqsubseteq C$$
$$\exists R_1.(\exists R_4.\{a\} \sqcap \exists R_5.(\{b\} \sqcap \exists R_6.\{a\})) \sqcap \exists R_2.\{a\} \sqcap \exists R_3.\{b\} \sqsubseteq C$$
$$\exists R_1.(\exists R_4.\{b\} \sqcap \exists R_5.(\{c\} \sqcap \exists R_6.\{b\})) \sqcap \exists R_2.\{a\} \sqcap \exists R_3.\{c\} \sqsubseteq C$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

where a, b, and c are the only individuals in the knowledge base

Defining Optimizations

- Delayed grounding: [Adila et al. at RR 2012]
- Ordered Resolution: [Cong et al. at JIST 2012]

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