DEDUCTION SYSTEMS

Answer-Set Programming Motivation and Introduction

* slides adapted from Torsten Schaub [Gebser et al.(2012)]

Sarah Gaggl

Dresden, 15 May 2014
Outline

1 Motivation
   - Declarative Problem Solving
   - ASP in a Nutshell
   - ASP Paradigm

2 Introduction
   - Syntax
   - Semantics
   - Examples
   - Language Constructs
   - Modeling
Informatics

- Problem
  - Computer
- Solution
  - Output
Informatics

“What is the problem?” versus “How to solve the problem?”
Traditional programming

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Declarative problem solving

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Declarative problem solving

- Problem
  - Modeling
  - Representation
- Solution
  - Interpreting
  - Output
- Solving
Answer Set Programming

in a Nutshell

- ASP is an approach to **declarative problem solving**, combining
  - a rich yet simple modeling language
  - with high-performance solving capacities
Answer Set Programming in a Nutshell

- ASP is an approach to **declarative problem solving**, combining
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- ASP has its roots in
  - (deductive) databases
  - logic programming (with negation)
  - (logic-based) knowledge representation and (nonmonotonic) reasoning
  - constraint solving (in particular, SATisfiability testing)
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- ASP is versatile as reflected by the ASP solver clasp, winning first places at ASP, CASC, MISC, PB, and SAT competitions
- ASP embraces many emerging application areas
Answer Set Programming
in a Hazelnutshell

- ASP is an approach to declarative problem solving, combining
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tailed to Knowledge Representation and Reasoning
Answer Set Programming
in a Hazelnutshell

- ASP is an approach to **declarative problem solving**, combining
  - a rich yet simple modeling language
  - with high-performance solving capacities
tailored to Knowledge Representation and Reasoning

**ASP = DB+LP+KR+SAT**
KR’s shift of paradigm

Theorem Proving based approach (eg. Prolog)

1. Provide a representation of the problem
2. A solution is given by a derivation of a query
KR’s shift of paradigm

Theorem Proving based approach  (eg. Prolog)
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Model Generation based approach  (eg. SATisfiability testing)
1. Provide a representation of the problem
2. A solution is given by a model of the representation
LP-style playing with blocks

Prolog program

\[
\begin{align*}
on(a,b). \\
on(b,c). \\
above(X,Y) & : \neg \ on(X,Y). \\
above(X,Y) & : \ on(X,Z), \ above(Z,Y).
\end{align*}
\]
LP-style playing with blocks

Prolog program

```prolog
on(a, b).
on(b, c).

above(X, Y) :- on(X, Y).
above(X, Y) :- on(X, Z), above(Z, Y).
```

Prolog queries

```prolog
?- above(a, c).
true.
```
LP-style playing with blocks

**Prolog program**

```
on(a,b).
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above(X,Y) :- on(X,Y).
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**Prolog queries**

```
?- above(a,c).
ture.

?- above(c,a).
no.
```
LP-style playing with blocks

**Prolog program**

```prolog
on(a, b).
on(b, c).

above(X, Y) :- on(X, Y).
above(X, Y) :- on(X, Z), above(Z, Y).
```

**Prolog queries (testing entailment)**

```prolog
?- above(a, c).
true.

?- above(c, a).
no.
```
LP-style playing with blocks

Shuffled Prolog program

\[
\text{on}(a, b). \\
\text{on}(b, c). \\
\text{above}(X, Y) :- \text{above}(X, Z), \text{on}(Z, Y). \\
\text{above}(X, Y) :- \text{on}(X, Y).
\]
LP-style playing with blocks

Shuffled Prolog program

on(a, b).
on(b, c).

above(X, Y) :- above(X, Z), on(Z, Y).
above(X, Y) :- on(X, Y).

Prolog queries

?- above(a, c).
LP-style playing with blocks

Shuffled Prolog program

\[
\text{on}(a,b).
\text{on}(b,c).
\]
\[
\text{above}(X,Y) \leftarrow \text{above}(X,Z), \text{on}(Z,Y).
\text{above}(X,Y) \leftarrow \text{on}(X,Y).
\]

Prolog queries (answered via fixed execution)

\[
?- \text{above}(a,c).
\]

Fatal Error: local stack overflow.
SAT-style playing with blocks

**Formula**

\[
\begin{align*}
on(a, b) \\
\land \non(b, c) \\
\land (\non(X, Y) \to above(X, Y)) \\
\land (\non(X, Z) \land above(Z, Y) \to above(X, Y))
\end{align*}
\]
SAT-style playing with blocks

Formula

\[
\begin{align*}
on(a, b) \\
\land on(b, c) \\
\land (on(X, Y) \rightarrow above(X, Y)) \\
\land (on(X, Z) \land above(Z, Y) \rightarrow above(X, Y))
\end{align*}
\]

Herbrand model

\[
\{\quad \begin{align*}
on(a, b), & \quad on(b, c), & \quad on(a, c), & \quad on(b, b), \\
above(a, b), & \quad above(b, c), & \quad above(a, c), & \quad above(b, b), & \quad above(c, b)
\end{align*}\quad \}
\]
SAT-style playing with blocks

**Formula**

\[
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on(a, b) \\
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\land (on(X, Z) \land above(Z, Y) \rightarrow above(X, Y)) 
\end{align*}
\]

**Herbrand model (among 426!)**

\[
\{ \quad on(a, b), \quad on(b, c), \quad on(a, c), \quad on(b, b), \\
above(a, b), \quad above(b, c), \quad above(a, c), \quad above(b, b), \quad above(c, b) \quad \}
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SAT-style playing with blocks

Formula

\begin{align*}
on(a, b) & \\
\land & \non(b, c) \\
\land & (\non(X, Y) \rightarrow \above(X, Y)) \\
\land & (\non(X, Z) \land \above(Z, Y) \rightarrow \above(X, Y))
\end{align*}

Herbrand model (among 426!)

\begin{align*}
\{ & \non(a, b), \non(b, c), \non(a, c), \non(b, b), \\
& \above(a, b), \above(b, c), \above(a, c), \above(b, b), \above(c, b) \}
\end{align*}
SAT-style playing with blocks

Formula

\[
on(a, b) \\
\land \ on(b, c) \\
\land \ (on(X, Y) \rightarrow above(X, Y)) \\
\land \ (on(X, Z) \land above(Z, Y) \rightarrow above(X, Y))
\]

Herbrand model (among 426!)

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\above(a, b), \ above(b, c), \ above(a, c), \ above(b, b), \ above(c, b) \ \}
\]
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-answer Set Programming (ASP)
ASP-style playing with blocks

Logic program

on(a,b).
on(b,c).

above(X,Y) :- on(X,Y).
above(X,Y) :- on(X,Z), above(Z,Y).
ASP-style playing with blocks

**Logic program**

```
on(a, b).
on(b, c).

above(X, Y) :- on(X, Y).
above(X, Y) :- on(X, Z), above(Z, Y).
```

**Stable Herbrand model**

```
{ on(a, b), on(b, c), above(b, c), above(a, b), above(a, c) }
```
ASP-style playing with blocks

Logic program

\[
\text{on}(a, b).
\]
\[
\text{on}(b, c).
\]
\[
\text{above}(X, Y) :- \text{on}(X, Y).
\]
\[
\text{above}(X, Y) :- \text{on}(X, Z), \text{above}(Z, Y).
\]

Stable Herbrand model (and no others)

\{
\text{on}(a, b), \text{on}(b, c), \text{above}(b, c), \text{above}(a, b), \text{above}(a, c)
\}
ASP-style playing with blocks

Logic program

\[
\begin{align*}
on(a, b) . \\
on(b, c) . \\
\text{above}(X, Y) & \leftarrow \text{above}(Z, Y), \ on(X, Z) . \\
\text{above}(X, Y) & \leftarrow \ on(X, Y) .
\end{align*}
\]

Stable Herbrand model (and no others)

\[
\{ \text{on}(a, b), \ \text{on}(b, c), \ \text{above}(b, c), \ \text{above}(a, b), \ \text{above}(a, c) \}
\]
## ASP versus LP

<table>
<thead>
<tr>
<th></th>
<th>ASP</th>
<th>Prolog</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model generation</td>
<td>Model generation</td>
<td>Query orientation</td>
</tr>
<tr>
<td>Bottom-up</td>
<td>Bottom-up</td>
<td>Top-down</td>
</tr>
<tr>
<td>Modeling language</td>
<td>Modeling language</td>
<td>Programming language</td>
</tr>
<tr>
<td>Rule-based format</td>
<td>Rule-based format</td>
<td></td>
</tr>
<tr>
<td>Instantiation</td>
<td>Instantiation</td>
<td>Unification</td>
</tr>
<tr>
<td>Flat terms</td>
<td>Flat terms</td>
<td>Nested terms</td>
</tr>
<tr>
<td>NP((NP))</td>
<td>(Turing +) (NP((NP)))</td>
<td>Turing</td>
</tr>
</tbody>
</table>

(Turing +) \(NP(\(NP\))\)
## ASP versus SAT

<table>
<thead>
<tr>
<th></th>
<th>ASP</th>
<th>SAT</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model generation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Bottom-up</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constructive Logic</td>
<td>Classical Logic</td>
<td></td>
</tr>
<tr>
<td>Closed (and open) world reasoning</td>
<td>Open world reasoning</td>
<td></td>
</tr>
<tr>
<td>Modeling language</td>
<td></td>
<td>—</td>
</tr>
<tr>
<td>Complex reasoning modes</td>
<td>Satisfiability testing</td>
<td></td>
</tr>
<tr>
<td>Satisfiability</td>
<td>Satisfiability</td>
<td></td>
</tr>
<tr>
<td>Enumeration/Projection</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Optimization</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Intersection/Union</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>(Turing +) $NP^{(NP)}$</td>
<td>$NP$</td>
<td>—</td>
</tr>
</tbody>
</table>
Two sides of a coin

- **ASP as High-level Language**
  - Express problem instance(s) as sets of facts
  - Encode problem (class) as a set of rules
  - Read off solutions from stable models of facts and rules

- **ASP as Low-level Language**
  - Compile a problem into a logic program
  - Solve the original problem by solving its compilation
What is ASP good for?

- Combinatorial search problems in the realm of $P$, $NP$, and $NP^{NP}$ (some with substantial amount of data), like
  - Automated Planning
  - Code Optimization
  - Composition of Renaissance Music
  - Database Integration
  - Decision Support for NASA shuttle controllers
  - Model Checking
  - Product Configuration
  - Robotics
  - System Biology
  - System Synthesis
  - (industrial) Team-building
  - and many many more
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What does ASP offer?

- Integration of DB, KR, and SAT techniques
- Succinct, elaboration-tolerant problem representations
  - Rapid application development tool
- Easy handling of dynamic, knowledge intensive applications
  - Including: data, frame axioms, exceptions, defaults, closures, etc
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Agenda

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   - ASP Paradigm

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   - Syntax
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Problem solving in ASP: Syntax

- Problem
- Logic Program
- Stable Models
- Solution

Modeling → Logic Program → Solving → Stable Models → Interpreting
Normal logic programs

- A (normal) logic program over a set $A$ of atoms is a finite set of rules
- A (normal) rule, $r$, is of the form

$$a_0 \leftarrow a_1, \ldots, a_m, \text{not } a_{m+1}, \ldots, \text{not } a_n$$

where $0 \leq m \leq n$ and each $a_i \in A$ is an atom for $0 \leq i \leq n$
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  where $0 \leq m \leq n$ and each $a_i \in A$ is an atom for $0 \leq i \leq n$.

- Notation

  $$\begin{align*}
  \text{head}(r) &= a_0 \\
  \text{body}(r) &= \{a_1, \ldots, a_m, \neg a_{m+1}, \ldots, \neg a_n\} \\
  \text{body}(r)^+ &= \{a_1, \ldots, a_m\} \\
  \text{body}(r)^- &= \{a_{m+1}, \ldots, a_n\}
  \end{align*}$$
Normal logic programs

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\text{body}(r)^+ &= \{a_1, \ldots, a_m\} \\
\text{body}(r)^- &= \{a_{m+1}, \ldots, a_n\}
\end{align*}$$

- A program is called positive if $\text{body}(r)^- = \emptyset$ for all its rules.
Rough notational convention

We sometimes use the following notation interchangeably in order to stress the respective view:

<table>
<thead>
<tr>
<th>Source code</th>
<th>true, false</th>
<th>if</th>
<th>and</th>
<th>or</th>
<th>iff</th>
<th>default</th>
<th>classical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Logic program formula</td>
<td>:- ,</td>
<td>¬</td>
<td>¬</td>
<td>¬</td>
<td>¬</td>
<td>not</td>
<td>not</td>
</tr>
<tr>
<td>Formula</td>
<td>⊥, T</td>
<td>→</td>
<td>∧</td>
<td>→</td>
<td>∼</td>
<td>¬</td>
<td>¬</td>
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Problem solving in ASP: Semantics

Diagram:
- Problem
  - Modeling
    - Logic Program
      - Solving
        - Stable Models
          - Interpreting
            - Solution
Formal Definition

Stable models of positive programs
Formal Definition

Stable models of positive programs

- A set of atoms $X$ is closed under a positive program $P$ iff for any $r \in P$, $head(r) \in X$ whenever $body(r)^+ \subseteq X$
  - $X$ corresponds to a model of $P$ (seen as a formula)
Formal Definition

Stable models of positive programs

• A set of atoms $X$ is **closed under** a positive program $P$ iff for any $r \in P$, $\text{head}(r) \in X$ whenever $\text{body}(r)^+ \subseteq X$
  – $X$ corresponds to a model of $P$ (seen as a formula)

• The **smallest** set of atoms which is closed under a positive program $P$ is denoted by $Cn(P)$
  – $Cn(P)$ corresponds to the $\subseteq$-smallest model of $P$ (ditto)
Formal Definition

Stable models of positive programs

- A set of atoms $X$ is **closed under** a positive program $P$ iff for any $r \in P$, $head(r) \subseteq X$ whenever $body(r)^+ \subseteq X$.
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- The **smallest** set of atoms which is closed under a positive program $P$ is denoted by $Cn(P)$.
  - $Cn(P)$ corresponds to the $\subseteq$-smallest model of $P$ (ditto).

- The set $Cn(P)$ of atoms is the **stable model** of a positive program $P$. 
Some “logical” remarks

- Positive rules are also referred to as definite clauses
  - Definite clauses are disjunctions with exactly one positive atom:
    \[ a_0 \lor \neg a_1 \lor \cdots \lor \neg a_m \]
  - A set of definite clauses has a (unique) smallest model
Some “logical” remarks

- Positive rules are also referred to as **definite clauses**
  - Definite clauses are disjunctions with exactly one positive atom:
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  - A set of definite clauses has a (unique) smallest model

- **Horn clauses** are clauses with **at most** one positive atom
  - Every definite clause is a Horn clause but not vice versa
  - Non-definite Horn clauses can be regarded as integrity constraints
  - A set of Horn clauses has a smallest model or none
Some “logical” remarks

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  - A set of Horn clauses has a smallest model or none

- This smallest model is the intended semantics of such sets of clauses
  - Given a positive program \( P \), \( Cn(P) \) corresponds to the smallest model of the set of definite clauses corresponding to \( P \)
Basic idea
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Consider the logical formula $\Phi$ and its three (classical) models:

\[
\{p, q\}, \{q, r\}, \text{ and } \{p, q, r\}
\]
Basic idea

Consider the logical formula \( \Phi \) and its three (classical) models:

\[
\{p, q\}, \{q, r\}, \text{ and } \{p, q, r\}
\]

\( \Phi \equiv q \land (q \land \neg r \to p) \)

\[
\begin{align*}
p & \leftrightarrow 1 \\
q & \leftrightarrow 1 \\
r & \leftrightarrow 0
\end{align*}
\]
Basic idea

Consider the logical formula $\Phi$ and its three (classical) models:

$\{p, q\}, \{q, r\}, \text{ and } \{p, q, r\}$

$\Phi = q \land (q \land \neg r \rightarrow p)$
Basic idea

Consider the logical formula $\Phi$ and its three (classical) models:

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Formula $\Phi$ has one stable model, often called answer set:

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Informally, a set $X$ of atoms is a **stable model** of a logic program $P$

- if $X$ is a (classical) model of $P$ and
- if all atoms in $X$ are **justified** by some rule in $P$

(rooted in intuitionistic logics HT (Heyting, 1930) and G3 (Gödel, 1932))
Basic idea

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Formal Definition

Stable model of normal programs

- The Gelfond-Lifschitz Reduct [Gelfond and Lifschitz (1991)], $P^X$, of a program $P$ relative to a set $X$ of atoms is defined by

\[ P^X = \{ \text{head}(r) \leftarrow \text{body}(r)^+ \mid r \in P \text{ and } \text{body}(r)^- \cap X = \emptyset \} \]
Formal Definition

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Stable model of normal programs

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- A set $X$ of atoms is a stable model of a program $P$, if $Cn(P^X) = X$

- Note: $Cn(P^X)$ is the $\subseteq$–smallest (classical) model of $P^X$
- Note: Every atom in $X$ is justified by an “applying rule from $P$”
A closer look at $P^X$

- In other words, given a set $X$ of atoms from $P$,

  $P^X$ is obtained from $P$ by deleting
  1. each rule having $not \ a$ in its body with $a \in X$
  and then
  2. all negative atoms of the form $not \ a$
     in the bodies of the remaining rules
A closer look at $P^X$

- In other words, given a set $X$ of atoms from $P$,

  $P^X$ is obtained from $P$ by deleting
  1. each rule having $\text{not } a$ in its body with $a \in X$
  and then
  2. all negative atoms of the form $\text{not } a$
     in the bodies of the remaining rules

- Note: Only negative body literals are evaluated w.r.t. $X$
A first example

\[ P = \{ p \leftarrow p, \quad q \leftarrow \text{not } p \} \]
A first example

\[ P = \{ p \leftarrow p, \ q \leftarrow \text{not } p \} \]

<table>
<thead>
<tr>
<th>( X )</th>
<th>( Cn(P^X) )</th>
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</thead>
<tbody>
<tr>
<td>( \emptyset )</td>
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<tr>
<td>( {p} )</td>
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\[ P = \{ p \leftarrow p, \; q \leftarrow \text{not } p \} \]

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<tbody>
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<tr>
<td>( {q} )</td>
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<tr>
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<td>( p \leftarrow p )</td>
<td>( \emptyset )</td>
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</table>
A first example

\[ P = \{ p \leftrightarrow p, \ q \leftrightarrow \text{not } p \} \]

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<tr>
<td>( { q } )</td>
<td>( p \leftrightarrow p )</td>
<td>( { q } )</td>
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<tr>
<td>( { p, q } )</td>
<td>( p \leftrightarrow p )</td>
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</table>
A first example

\[ P = \{ p \leftarrow p, \ q \leftarrow \text{not } p \} \]

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<tr>
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<tr>
<td>({p, q})</td>
<td>(p \leftarrow p)</td>
<td>(\emptyset)</td>
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</table>
A first example

\[ P = \{ p \iff p, \ q \iff \neg p \} \]

<table>
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<tr>
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<td>${q}$</td>
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<tr>
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<tr>
<td>${q}$</td>
<td>$p \iff p$</td>
<td>${q}$</td>
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<tr>
<td>${p, q}$</td>
<td>$p \iff p$</td>
<td>$\emptyset$</td>
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</table>
A second example

\[ P = \{ p \leftarrow \text{not } q, \ q \leftarrow \text{not } p \} \]
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\[ P = \{ p \leftarrow \text{not } q, \ q \leftarrow \text{not } p \} \]

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A second example

\[ P = \{p \leftarrow \text{not } q, \ q \leftarrow \text{not } p\} \]

<table>
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<tr>
<th>(X)</th>
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<tr>
<td>(\emptyset)</td>
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<td>(q \leftarrow)</td>
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<tr>
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<td>(p \leftarrow)</td>
<td>({p}) (\checkmark)</td>
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<tr>
<td></td>
<td>(q \leftarrow)</td>
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<tr>
<td>({q})</td>
<td>(q \leftarrow)</td>
<td>({q}) (\checkmark)</td>
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<tr>
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</tr>
<tr>
<td>({p, q})</td>
<td></td>
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A third example

\[ P = \{ p \leftarrow \text{not } p \} \]
A third example

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<table>
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<td>(\emptyset)</td>
<td>(p)</td>
<td>({ p })</td>
</tr>
<tr>
<td>{p}</td>
<td></td>
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Some properties

- A logic program may have zero, one, or multiple stable models!
Some properties

- A logic program may have zero, one, or multiple stable models!
- If $X$ is an stable model of a logic program $P$, then $X$ is a model of $P$ (seen as a formula)
- If $X$ and $Y$ are stable models of a normal program $P$, then $X \not\subset Y$
Let $P$ be a logic program

- Let $\mathcal{T}$ be a set of (variable-free) terms
- Let $\mathcal{A}$ be a set of (variable-free) atoms constructable from $\mathcal{T}$
Programs with Variables

Let $P$ be a logic program

- Let $\mathcal{T}$ be a set of variable-free terms (also called Herbrand universe)
- Let $\mathcal{A}$ be a set of (variable-free) atoms constructable from $\mathcal{T}$
  (also called alphabet or Herbrand base)
Programs with Variables

Let $P$ be a logic program

- Let $\mathcal{T}$ be a set of (variable-free) terms
- Let $\mathcal{A}$ be a set of (variable-free) atoms constructable from $\mathcal{T}$

- **Ground Instances of $r \in P$:** Set of variable-free rules obtained by replacing all variables in $r$ by elements from $\mathcal{T}$:

$$\text{ground}(r) = \{ r\theta \mid \theta : \text{var}(r) \rightarrow \mathcal{T}, \text{var}(r\theta) = \emptyset \}$$

where $\text{var}(r)$ stands for the set of all variables occurring in $r$; $	heta$ is a (ground) substitution
Programs with Variables

Let $P$ be a logic program

- Let $\mathcal{T}$ be a set of (variable-free) terms
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- Ground Instances of $r \in P$: Set of variable-free rules obtained by replacing all variables in $r$ by elements from $\mathcal{T}$:

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  where $\text{var}(r)$ stands for the set of all variables occurring in $r$; $\theta$ is a (ground) substitution

- Ground Instantiation of $P$: $\text{ground}(P) = \bigcup_{r \in P}\text{ground}(r)$
An Example

\[ P = \{ r(a, b) \leftarrow, r(b, c) \leftarrow, t(X, Y) \leftarrow r(X, Y) \} \]
\[ T = \{ a, b, c \} \]
\[ A = \{ r(a, a), r(a, b), r(a, c), r(b, a), r(b, b), r(b, c), r(c, a), r(c, b), r(c, c), t(a, a), t(a, b), t(a, c), t(b, a), t(b, b), t(b, c), t(c, a), t(c, b), t(c, c) \} \]
An Example

\[ P = \{ r(a, b) \leftarrow, r(b, c) \leftarrow, t(X, Y) \leftarrow r(X, Y) \} \]

\[ \mathcal{T} = \{a, b, c\} \]

\[ \mathcal{A} = \{ r(a, a), r(a, b), r(a, c), r(b, a), r(b, b), r(b, c), r(c, a), r(c, b), r(c, c), t(a, a), t(a, b), t(a, c), t(b, a), t(b, b), t(b, c), t(c, a), t(c, b), t(c, c) \} \]

\[ \text{ground}(P) = \left\{ \begin{array}{l}
    r(a, b) \leftarrow, \\
    r(b, c) \leftarrow, \\
    t(a, a) \leftarrow r(a, a), t(b, a) \leftarrow r(b, a), t(c, a) \leftarrow r(c, a), \\
    t(a, b) \leftarrow r(a, b), t(b, b) \leftarrow r(b, b), t(c, b) \leftarrow r(c, b), \\
    t(a, c) \leftarrow r(a, c), t(b, c) \leftarrow r(b, c), t(c, c) \leftarrow r(c, c) \\
\end{array} \right\} \]
An Example

\[ P = \{ \text{ } r(a, b) \leftarrow, \ r(b, c) \leftarrow, \ t(X, Y) \leftarrow r(X, Y) \ \} \]

\[ T = \{ a, b, c \} \]

\[ A = \{ \ r(a, a), \ r(a, b), \ r(a, c), \ r(b, a), \ r(b, b), \ r(b, c), \ r(c, a), \ r(c, b), \ r(c, c), \ t(a, a), \ t(a, b), \ t(a, c), \ t(b, a), \ t(b, b), \ t(b, c), \ t(c, a), \ t(c, b), \ t(c, c) \ \} \]

\[ \text{ground}(P) = \{ \ r(a, b) \leftarrow, \ r(b, c) \leftarrow, \ t(a, b) \leftarrow \} \]

- Intelligent Grounding aims at reducing the ground instantiation
Stable models of programs with Variables

Let $P$ be a normal logic program with variables
Stable models of programs with Variables

Let $P$ be a normal logic program with variables

- A set $X$ of (ground) atoms is a stable model of $P$, if $\text{Cn}(\text{ground}(P)^X) = X$
Problem solving in ASP: Extended Syntax

Problem

Logic Program

Modeling

Solution

Interpreting

Stable Models

Solving
Language Constructs

• Variables (over the Herbrand Universe)
  \[ p(X) :- q(X) \]
  over constants \{ a, b, c \}
  stands for \[ p(a) :- q(a), p(b) :- q(b), p(c) :- q(c) \]

• Conditional Literals
  \[ p :- q(X) : r(X) \]
  given \[ r(a), r(b), r(c) \]
  stands for \[ p :- q(a), q(b), q(c) \]

• Disjunction
  \[ p(X) | q(X) :- r(X) \]

• Integrity Constraints
  \[ :- q(X), p(X) \]

• Choice
  \[ 2 \{ p(X,Y) : q(X) \} 7 :- r(Y) \]

• Aggregates
  \[ s(Y) :- r(Y), 2 \#count \{ p(X,Y) : q(X) \} 7 \]
  also: \#sum, \#avg, \#min, \#max, \#even, \#odd
Language Constructs

• Variables (over the Herbrand Universe)
  - \( p(X) :- q(X) \) over constants \( \{a, b, c\} \) stands for
    \[ p(a) :- q(a), p(b) :- q(b), p(c) :- q(c) \]

• Conditional Literals
  - \( p :- q(X) : r(X) \) given \( r(a), r(b), r(c) \) stands for
    \[ p :- q(a), q(b), q(c) \]

• Disjunction
  - \( p(X) \mid q(X) :- r(X) \)

• Integrity Constraints
  - ::- q(X), p(X)

• Choice
  - \( 2 \{ p(X,Y) : q(X) \} 7 :- r(Y) \)

• Aggregates
  - \( s(Y) :- r(Y), 2 \# \text{count} \{ p(X,Y) : q(X) \} 7 \)
  - also: \#\text{sum}, \#\text{avg}, \#\text{min}, \#\text{max}, \#\text{even}, \#\text{odd}
Language Constructs

- **Conditional Literals**
  
  \[ p : \neg q(X) : r(X) \quad \text{given} \quad r(a), \ r(b), \ r(c) \]
  
  stands for
  
  \[ p : \neg q(a), \ q(b), \ q(c) \]
Language Constructs

- **Disjunction**
  
  \[ p(X) | q(X) :− r(X) \]
Language Constructs

- Integrity Constraints
  - \[ p(X) :- q(X), p(X) \]
Language Constructs

- **Variables (over the Herbrand Universe)**
  - $p(X) :- q(X)$
  - Over constants \{a, b, c\}
  - \[ p(a) :- q(a), p(b) :- q(b), p(c) :- q(c) \]

- **Conditional Literals**
  - $p :- q(X) : r(X)$
  - Given \( r(a), r(b), r(c) \)
  - \[ p :- q(a), q(b), q(c) \]

- **Disjunction**
  - $p(X) | q(X) :- r(X)$

- **Integrity Constraints**
  - $:- q(X), p(X)$

- **Choice**
  - $2 \{ p(X, Y) : q(X) \} 7 :- r(Y)$

- **Aggregates**
  - $s(Y) :- r(Y), 2 \#\text{count} \{ p(X, Y) : q(X) \} 7 : r(Y)$
  - Also: $\#\text{sum}, \#\text{avg}, \#\text{min}, \#\text{max}, \#\text{even}, \#\text{odd}$
• Variables (over the Herbrand Universe)
  \[ p(X) :- q(X) \]
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Language Constructs

- **Variables** (over the Herbrand Universe)
  - \( p(X) :- q(X) \) over constants \( \{a, b, c\} \) stands for
    - \( p(a) :- q(a), p(b) :- q(b), p(c) :- q(c) \)

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  - \( p :- q(X) : r(X) \) given \( r(a), r(b), r(c) \) stands for
    - \( p :- q(a), q(b), q(c) \)

- **Integrity Constraints**
  - \( :- q(X), p(X) \)

- **Choice**
  - \( 2 \{ p(X, Y) : q(X) \} 7 :- r(Y) \)

- **Aggregates**
  - \( s(Y) :- r(Y), 2 \#\text{count} \{ p(X, Y) : q(X) \} 7 \)
  - also: \#sum, \#avg, \#min, \#max, \#even, \#odd
Modeling

- For solving a problem class $\mathbf{C}$ for a problem instance $\mathbf{I}$, encode
  1. the problem instance $\mathbf{I}$ as a set $P_I$ of facts and
  2. the problem class $\mathbf{C}$ as a set $P_C$ of rules
such that the solutions to $\mathbf{C}$ for $\mathbf{I}$ can be (polynomially) extracted from the stable models of $P_I \cup P_C$
Modeling

- For solving a problem class \( C \) for a problem instance \( I \), encode
  - the problem instance \( I \) as a set \( P_I \) of facts and
  - the problem class \( C \) as a set \( P_C \) of rules
such that the solutions to \( C \) for \( I \) can be (polynomially) extracted from the stable models of \( P_I \cup P_C \)

- \( P_I \) is (still) called problem instance
- \( P_C \) is often called the problem encoding
Modeling

- For solving a problem class $C$ for a problem instance $I$, encode
  1. the problem instance $I$ as a set $P_I$ of facts and
  2. the problem class $C$ as a set $P_C$ of rules
  such that the solutions to $C$ for $I$ can be (polynomially) extracted from the stable models of $P_I \cup P_C$

- $P_I$ is (still) called problem instance
- $P_C$ is often called the problem encoding

- An encoding $P_C$ is uniform, if it can be used to solve all its problem instances
  That is, $P_C$ encodes the solutions to $C$ for any set $P_I$ of facts
Example 3-Colorability

- Vertices are represented with predicates $\text{node}(X)$;
- Edges are represented with predicates $\text{edge}(X, Y)$.

Question: Is there a valid assignment of three colors for an input graph $G$ such that no two adjacent vertices have the same color?
node(1..6).

Graph coloring
Graph coloring

node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).
Graph coloring

\[
\text{node}(1..6).
\]

\[
\begin{align*}
\text{edge}(1,2). & \quad \text{edge}(1,3). & \quad \text{edge}(1,4). \\
\text{edge}(2,4). & \quad \text{edge}(2,5). & \quad \text{edge}(2,6). \\
\text{edge}(3,1). & \quad \text{edge}(3,4). & \quad \text{edge}(3,5). \\
\text{edge}(4,1). & \quad \text{edge}(4,2). \\
\text{edge}(5,3). & \quad \text{edge}(5,4). & \quad \text{edge}(5,6). \\
\text{edge}(6,2). & \quad \text{edge}(6,3). & \quad \text{edge}(6,5). \\
\end{align*}
\]

\[
\text{col}(r). \quad \text{col}(b). \quad \text{col}(g).
\]
Graph coloring

\[
\begin{align*}
\text{node} & (1..6). \\
\text{edge} & (1,2). \quad \text{edge} (1,3). \quad \text{edge} (1,4). \\
\text{edge} & (2,4). \quad \text{edge} (2,5). \quad \text{edge} (2,6). \\
\text{edge} & (3,1). \quad \text{edge} (3,4). \quad \text{edge} (3,5). \\
\text{edge} & (4,1). \quad \text{edge} (4,2). \\
\text{edge} & (5,3). \quad \text{edge} (5,4). \quad \text{edge} (5,6). \\
\text{edge} & (6,2). \quad \text{edge} (6,3). \quad \text{edge} (6,5). \\
\text{col} & (r). \quad \text{col} (b). \quad \text{col} (g).
\end{align*}
\]
Graph coloring

node(1..6).

dge(1,2). edge(1,3). edge(1,4).
edge(2,4). edge(2,5). edge(2,6).
edge(3,1). edge(3,4). edge(3,5).
edge(4,1). edge(4,2).
edge(5,3). edge(5,4). edge(5,6).
edge(6,2). edge(6,3). edge(6,5).

col(r). col(b). col(g).

Problem instance
Graph coloring

\[
\begin{align*}
\text{node} & \,(1..6). \\
\text{edge} & \, (1,2). \quad \text{edge} \, (1,3). \quad \text{edge} \, (1,4). \\
\text{edge} & \, (2,4). \quad \text{edge} \, (2,5). \quad \text{edge} \, (2,6). \\
\text{edge} & \, (3,1). \quad \text{edge} \, (3,4). \quad \text{edge} \, (3,5). \\
\text{edge} & \, (4,1). \quad \text{edge} \, (4,2). \\
\text{edge} & \, (5,3). \quad \text{edge} \, (5,4). \quad \text{edge} \, (5,6). \\
\text{edge} & \, (6,2). \quad \text{edge} \, (6,3). \quad \text{edge} \, (6,5). \\
\text{col} & \, (r). \quad \text{col} \, (b). \quad \text{col} \, (g). \\
\end{align*}
\]

\[
\begin{align*}
1 \{ \text{color}(X,C) : \text{col}(C) \} & \, 1 :- \, \text{node}(X). 
\end{align*}
\]
Graph coloring

node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

col(r).  col(b).  col(g).

1 { color(X,C) : col(C) } 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
Graph coloring

node(1..6).

edge(1,2). edge(1,3). edge(1,4).
edge(2,4). edge(2,5). edge(2,6).
edge(3,1). edge(3,4). edge(3,5).
edge(4,1). edge(4,2).
edge(5,3). edge(5,4). edge(5,6).
edge(6,2). edge(6,3). edge(6,5).

col(r). col(b). col(g).

1 { color(X,C) : col(C) } 1 :- node(X).
:- edge(X,Y), color(X,C), color(Y,C).

Problem encoding
Graph coloring

\[
\begin{align*}
\text{node}(1..6). \\
\text{edge}(1,2). & \quad \text{edge}(1,3). & \quad \text{edge}(1,4). \\
\text{edge}(2,4). & \quad \text{edge}(2,5). & \quad \text{edge}(2,6). \\
\text{edge}(3,1). & \quad \text{edge}(3,4). & \quad \text{edge}(3,5). \\
\text{edge}(4,1). & \quad \text{edge}(4,2). \\
\text{edge}(5,3). & \quad \text{edge}(5,4). & \quad \text{edge}(5,6). \\
\text{edge}(6,2). & \quad \text{edge}(6,3). & \quad \text{edge}(6,5). \\
\text{col}(r). & \quad \text{col}(b). & \quad \text{col}(g). \\
\end{align*}
\]

1 \{ color(X,C) : col(C) \} 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
color.lp

node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

col(r).  col(b).  col(g).

1 { color(X,C) : col(C) } 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
ASP solving process

1. Problem
2. Logic Program
3. Grounder
4. Solver
5. Stable Models
6. Solution

Modeling → Logic Program → Grounder → Solver → Stable Models → Interpreting
Graph coloring: Grounding

$ gringo --text color.lp
Graph coloring: Grounding

```
$ gringo --text color.lp

node(1). node(2). node(3). node(4). node(5). node(6).
edge(1,2). edge(1,3). edge(1,4). edge(2,4). edge(2,5). edge(2,6).
edge(3,1). edge(3,4). edge(3,5). edge(4,1). edge(4,2). edge(5,3).
edge(5,4). edge(5,6). edge(6,2). edge(6,3). edge(6,5).

col(r). col(b). col(g).

1 {color(1,r), color(1,b), color(1,g)} 1.
1 {color(2,r), color(2,b), color(2,g)} 1.
1 {color(3,r), color(3,b), color(3,g)} 1.
1 {color(4,r), color(4,b), color(4,g)} 1.
1 {color(5,r), color(5,b), color(5,g)} 1.
1 {color(6,r), color(6,b), color(6,g)} 1.

:- color(1,r), color(2,r).
:- color(1,b), color(2,b).
:- color(1,g), color(2,g).
:- color(1,r), color(3,r).
:- color(1,b), color(3,b).
:- color(1,g), color(3,g).
:- color(1,r), color(4,r).
:- color(1,b), color(4,b).
:- color(1,g), color(4,g).
:- color(1,r), color(5,r).
:- color(1,b), color(5,b).
:- color(1,g), color(5,g).
:- color(2,r), color(4,r).
:- color(2,b), color(4,b).
:- color(2,g), color(4,g).
:- color(2,r), color(5,r).
:- color(2,b), color(5,b).
:- color(2,g), color(5,g).
:- color(3,r), color(4,r).
:- color(3,b), color(4,b).
:- color(3,g), color(4,g).
:- color(3,r), color(5,r).
:- color(3,b), color(5,b).
:- color(3,g), color(5,g).
:- color(4,r), color(5,r).
:- color(4,b), color(5,b).
:- color(4,g), color(5,g).
```
ASP solving process

Problem

Modeling

Logic Program

Grounder

Solving

Solver

Stable Models

Solution

Interpreting
Graph coloring: Solving

$\text{gringo color.lp | clasp 0}$
Graph coloring: Solving

```
$ gringo color.lp | clasp 0

clasp version 2.1.0
Reading from stdin
Solving...
Answer: 1
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,g) color(4,b) color(3,r) color(2,r) color(1,g)
Answer: 2
edge(1,2) ... col(r) ... node(1) ... color(6,r) color(5,g) color(4,r) color(3,b) color(2,b) color(1,g)
Answer: 3
edge(1,2) ... col(r) ... node(1) ... color(6,g) color(5,b) color(4,g) color(3,r) color(2,r) color(1,b)
Answer: 4
edge(1,2) ... col(r) ... node(1) ... color(6,r) color(5,b) color(4,r) color(3,g) color(2,g) color(1,b)
Answer: 5
edge(1,2) ... col(r) ... node(1) ... color(6,g) color(5,r) color(4,g) color(3,b) color(2,b) color(1,r)
Answer: 6
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,r) color(4,b) color(3,g) color(2,g) color(1,r)
SATISFIABLE
```

Models : 6
Time : 0.002s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time : 0.000s
Reasoning Modes

- Satisfiability
- Enumeration†
- Projection†
- Intersection‡
- Union‡
- Optimization

- and combinations of them

† without solution recording
‡ without solution enumeration
References

- Martin Gebser, Benjamin Kaufmann Roland Kaminski, and Torsten Schaub. 

- Michael Gelfond and Vladimir Lifschitz. 

  - See also: [http://potassco.sourceforge.net](http://potassco.sourceforge.net)