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The SPARQL Query Language
The SPARQL Query Language
Agenda

1 Recap
2 Output Formats
3 SPARQL Semantics
4 Transformation of Queries into Algebra Objects
5 Evaluation of the SPARQL Algebra
6 Summary
Agenda

1 Recap

2 Output Formats

3 SPARQL Semantics

4 Transformation of Queries into Algebra Objects

5 Evaluation of the SPARQL Algebra

6 Summary
Recap: Introduced SPARQL Features

<table>
<thead>
<tr>
<th>Basic Structure</th>
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<tr>
<td>PREFIX</td>
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<tr>
<td>WHERE</td>
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<table>
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<tr>
<th>Graph Patterns</th>
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<tr>
<td>Basic Graph Patterns</td>
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<tr>
<td>{ ... }</td>
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<td>OFFSET</td>
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<tr>
<th>Output Formats</th>
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<tbody>
<tr>
<td>SELECT</td>
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</table>
Agenda

1. Recap
2. Output Formats
3. SPARQL Semantics
4. Transformation of Queries into Algebra Objects
5. Evaluation of the SPARQL Algebra
6. Summary
Output Format \texttt{SELECT}

So far all results have been tables (solution sequences): \texttt{Output format SELECT}

\textbf{Syntax:} \texttt{SELECT <VariableList> or SELECT *}

\textbf{Advantage}

Simple sequential processing of the results

\textbf{Disadvantage}

Structure/relationships between the objects in the results is lost
**Output Format** **CONSTRUCT**

**CONSTRUCT** creates an RDF graph for the results

**Example Query**

```sql
PREFIX ex: <http://example.org/>
CONSTRUCT { ?person ex:mailbox ?email .
    ?person ex:telephone ?tel . }
WHERE { ?person ex:email ?email .
    ?person ex:tel ?tel . }
```
Output Format **CONSTRUCT**

**CONSTRUCT** creates an RDF graph for the results

**Example Query**

```PREFIX ex: <http://example.org/> PREFIX ex: <http://example.org/>
CONSTRUCT { ?person ex:mailbox ?email .
    ?person ex:telephone ?tel . }
CONSTRUCT { ?person ex:mailbox ?email .
    ?person ex:telephone ?tel . }
WHERE { ?person ex:email ?email .
    ?person ex:tel ?tel . }
WHERE { ?person ex:email ?email .
    ?person ex:tel ?tel . }
```

**Advantage**

Structured result data with relationships between the elements

**Disadvantages**

- Sequential processing of the results is harder
- No treatment of unbound variables (triples are omitted)
CONSTRUCT Templates with Blank Nodes

Data

@prefix foaf: <http://xmlns.com/foaf/0.1/> .
_:a foaf:firstname "Alice" ;
    foaf:surname "Hacker" .
_:b foaf:firstname "Bob" ;
    foaf:surname "Hacker" .
CONSTRUCT Templates with Blank Nodes

Data

@prefix foaf: <http://xmlns.com/foaf/0.1/> .
_:a foaf:firstname "Alice" ;
    foaf:surname "Hacker" .
_:b foaf:firstname "Bob" ;
    foaf:surname "Hacker" .

Query

PREFIX foaf: <http://xmlns.com/foaf/0.1/>
PREFIX vcard: <http://www.w3.org/2001/vcard-rdf/3.0#>
CONSTRUCT {
    ?x vcard:N _:v .
    _:v vcard:givenName ?gname ;
    vcard:familyName ?fname
} WHERE {
    ?x foaf:surname ?fname
}
CONSTRUCT Templates with Blank Nodes

Resulting RDF graph

```plaintext
@prefix vcard: <http://www.w3.org/2001/vcard-rdf/3.0#> .
_:v1 vcard:N _:x .
_:x vcard:givenName "Alice" ;
     vcard:familyName "Hacker" .
_:v2 vcard:N _:z .
_:z vcard:givenName "Bob" ;
     vcard:familyName "Hacker" .
```
Further Output Formats: **ASK & DESCRIBE**

SPARQL supports two additional output formats:

- **ASK** only checks whether the query has at least one answer (true/false result)
- **DESCRIBE** (informative) returns an RDF description for each resulting URI (application dependent)

**Example Query**

```sparql
DESCRIBE ?x WHERE { ?x <http://ex.org/emplID> "123" }
```

**Possible Result (prefix declarations omitted):**

```sparql
_:a exOrg:emplID "123" ;
   foaf:mbox_sha1sum "ABCD1234" ;
   vcard:N
   [ vcard:Family "Smith" ;
     vcard:Given "John" ] .
foaf:mbox_sha1sum a owl:InverseFunctionalProperty .
```
Agenda

1. Recap
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Semantics of Query Languages

So far only informal presentation of SPARQL features

- User: “Which answers can I expect for my query?”
- Developer: “Which behaviour is expected from my SPARQL implementation?”
- Marketing: “Is our product already conformant with the SPARQL standard?”

⇒ Formal semantics should clarify these questions . . .
Logic-based Semantics

Semantics of formal logics:

- Model-theoretic semantics: Which interpretations do satisfy my knowledge base?
- Proof-theoretic semantics: Which derivations can be build from my knowledge base?
- ...
Logic-based Semantics

Semantics of formal logics:

- Model-theoretic semantics: Which interpretations do satisfy my knowledge base?
- Proof-theoretic semantics: Which derivations can be build from my knowledge base?
- …
Semantics of Programming Languages

- Axiomatic semantics: Which logical statements hold for my program?
- Operational semantics: What happens during the processing of my program?
- Denotational semantics: How can we describe the input/output function of the program in an abstract way?
Semantics of Programming Languages

- Axiomatic semantics: Which logical statements hold for my program?
- Operational semantics: What happens during the processing of my program?
- Denotational semantics: How can we describe the input/output function of the program in an abstract way?

What to do with query languages?
Semantics of Query Languages (1)

Query Entailment

- Query as description of allowed results
- Data as set of logical assumptions (axiom set/theory)
- Results as logical entailment

$\Rightarrow$ OWL DL and RDF(S) as query languages
$\Rightarrow$ conjunctive queries
Semantics of Query Languages (2)

Query Algebra

- Query as instruction for computing the results
- Queried data as input
- Results as output

⇝ Relational algebra for SQL
⇝ SPARQL Algebra
Agenda

1. Recap
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Translation into SPARQL Algebra

```
  FILTER (?price < 15)
  OPTIONAL { ?book ex:title ?title }
  { ?book ex:author ex:Shakespeare } UNION
  { ?book ex:author ex:Marlowe }
}
```

Semantics of a SPARQL query:

1. Transformation of the query into an algebra expression
2. Evaluation of the algebra expression
Translation into SPARQL Algebra

```sparql
{ ?book ex:price ?price
  FILTER (?price < 15)
  OPTIONAL { ?book ex:title ?title } 
  { ?book ex:author ex:Shakespeare } UNION
  { ?book ex:author ex:Marlowe }
}
```

**Attention:** Filters apply to the whole group in which they occur
Translation into SPARQL Algebra

{  ?book ex:price ?price
    OPTIONAL {  ?book ex:title ?title }
    {  ?book ex:author ex:Shakespeare } UNION
    {  ?book ex:author ex:Marlowe }
    FILTER (?price < 15)
}

1 Expand abbreviated IRIs
Translation into SPARQL Algebra

```sparql
    <http://ex.org/Shakespeare> } UNION 
    <http://ex.org/Marlowe> } 
  FILTER (?price < 15)
}
```
Translation into SPARQL Algebra

    <http://ex.org/Shakespeare> } UNION
    <http://ex.org/Marlowe> } }
FILTER (?price < 15)
}

2. Replace triple patterns with operator Bgp(·)
Translation into SPARQL Algebra

{  
  \texttt{Bgp(?book \: \langle http://ex.org/price \rangle \: \: ?price) }
  
  OPTIONAL { \texttt{Bgp(?book \: \langle http://ex.org/title \rangle \: \: ?title) } }
  
  \texttt{Bgp(?book \: \langle http://ex.org/author \rangle}
  
  \texttt{\: \langle http://ex.org/Shakespeare \rangle) } \text{ UNION }
  
  \texttt{Bgp(?book \: \langle http://ex.org/author \rangle}
  
  \texttt{\: \langle http://ex.org/Marlowe \rangle) }
  
  \texttt{FILTER (?price < 15) }
  
}
Translation into SPARQL Algebra

```sparql
  OPTIONAL { Bgp(?book <http://ex.org/title> ?title) } 
  { Bgp(?book <http://ex.org/author> 
    <http://ex.org/Shakespeare>) } UNION 
  { Bgp(?book <http://ex.org/author> 
    <http://ex.org/Marlowe>) } 
  FILTER (?price < 15) }
```

3. Introduce the LeftJoin(·) operator for optional parts
Translation into SPARQL Algebra

```sparql
{ leftjoin(bgp(?book <http://ex.org/price> ?price),
            true)

  { bgp(?book <http://ex.org/author>
        <http://ex.org/Shakespeare>}) union

  { bgp(?book <http://ex.org/author>
        <http://ex.org/Marlowe>)}

  filter (?price < 15)
}
```
Translation into SPARQL Algebra

{ LeftJoin(Bgp(?book <http://ex.org/price> ?price),
            true)

            {Bgp(?book <http://ex.org/author>
            <http://ex.org/Shakespeare>}) UNION

            {Bgp(?book <http://ex.org/author>
            <http://ex.org/Marlowe>)}

FILTER (?price < 15)
}

4. Combine alternative graph patterns with Union(·) operator

 النبي Refers to neighbouring patterns and has higher precedence than conjunction (left associative)
Translation into SPARQL Algebra

```sparql
{  
    LeftJoin(Bgp(?book <http://ex.org/price> ?price),
             true)
    Union(Bgp(?book <http://ex.org/author>
              <http://ex.org/Shakespeare>),
           Bgp(?book <http://ex.org/author>
                <http://ex.org/Marlowe>))
    FILTER (?price < 15)
}
```
Translation into SPARQL Algebra

```
{  LeftJoin(Bgp(?book <http://ex.org/price> ?price),
             true)
   Union(Bgp(?book <http://ex.org/author>
                  <http://ex.org/Shakespeare>),
             Bgp(?book <http://ex.org/author>
                  <http://ex.org/Marlowe>))
   FILTER (?price < 15)
}
```

5. Apply Join(·) operator to join non-filter elements
Translation into SPARQL Algebra

{ 
  Join(
    LeftJoin(Bgp(?book <http://ex.org/price> ?price),
             true),
    Union(Bgp(?book <http://ex.org/author>
                  <http://ex.org/Shakespeare>),
           Bgp(?book <http://ex.org/author>
                <http://ex.org/Marlowe>)))
  FILTER (?price < 15)
}

Translation into SPARQL Algebra

```sparql
{  
  Join(
    LeftJoin(Bgp(?book <http://ex.org/price> ?price),
             true),
    Union(Bgp(?book <http://ex.org/author> <http://ex.org/Shakespeare>),
  FILTER (?price < 15)
}
```

6. Translate a group with filters with the Filter(·) operator
Translation into SPARQL Algebra

Translation into SPARQL Algebra

Filter(\(\text{price} < 15\),
Join(
  LeftJoin(Bgp(\(\text{book} <http://ex.org/price> \text{price}\)),
              Bgp(\(\text{book} <http://ex.org/title> \text{title}\)),
              \text{true}),
  Union(Bgp(\(\text{book} <http://ex.org/author> <http://ex.org/Shakespeare>\)),
          Bgp(\(\text{book} <http://ex.org/author> <http://ex.org/Marlowe>\))))

- Online translation tool:
  http://sparql.org/query-validator.html
Agenda

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Semantics of the SPARQL Algebra

Operations

Now we have an algebra object, but what do the algebra operations mean?

<table>
<thead>
<tr>
<th>Operation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bgp($P$)</td>
<td>match/evaluate pattern $P$</td>
</tr>
<tr>
<td>Join($M_1$, $M_2$)</td>
<td>conjunctive join of solutions $M_1$ and $M_2$</td>
</tr>
<tr>
<td>Union($M_1$, $M_2$)</td>
<td>union of solutions $M_1$ with $M_2$</td>
</tr>
<tr>
<td>LeftJoin($M_1$, $M_2$, $F$)</td>
<td>optional join of $M_1$ with $M_2$ with filter constraint $F$ (true if no filter given)</td>
</tr>
<tr>
<td>Filter($F$, $M$)</td>
<td>filter solutions $M$ with constraint $F$</td>
</tr>
<tr>
<td>$Z$</td>
<td>empty pattern (identity for join)</td>
</tr>
</tbody>
</table>
Semantics of the SPARQL Algebra

Operations

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<td>Union($M_1, M_2$)</td>
<td>union of solutions $M_1$ with $M_2$</td>
</tr>
<tr>
<td>LeftJoin($M_1, M_2, F$)</td>
<td>optional join of $M_1$ with $M_2$ with filter constraint $F$ (true if no filter given)</td>
</tr>
<tr>
<td>Filter($F, M$)</td>
<td>filter solutions $M$ with constraint $F$</td>
</tr>
<tr>
<td>$Z$</td>
<td>empty pattern (identity for join)</td>
</tr>
</tbody>
</table>

- Only Bgp(·) matches or evaluates graph patterns

~ We can use entailment checking rather than graph matching
Definition of the SPARQL Operators

How can we define that more formally?

Output:
- “solution set” (formatting irrelevant)

Input:
- Queried (active) graph
- Partial results from previous evaluation steps
- Different parameters according to the operation

How can we formally describe the “results”? 
SPARQL Results

Intuition: Results coded as tables of variable assignments

Result:
List of solutions (solution sequence)

→ each solution corresponds to one table row
SPARQL Results

Intuition: Results coded as tables of variable assignments

Result:
List of solutions (solution sequence)

⇝ each solution corresponds to one table row

Solution:
Partial function
- Domain: relevant variables
- Range: IRIs $\cup$ blank nodes $\cup$ RDF literals

⇝ Unbound variables are those that have no assigned value (partial function)
Evaluation of Basic Graph Patterns

Definition (Solution)

Let $P$ be a basic graph pattern. A partial function $\mu$ is a solution for $\text{Bgp}(P)$ over the queried (active) graph $G$ if:

1. the domain of $\mu$ is exactly the set of variables in $P$,
2. there exists an assignment $\sigma$ from blank nodes in $P$ to IRIs, blank nodes, or RDF literals such that:
3. the RDF graph $\mu(\sigma(P))$ is a subgraph of $G$. 
Evaluation of Basic Graph Patterns

- The result of evaluating $\text{Bgp}(P)$ over $G$ is written $\left[ \text{Bgp}(P) \right]_G$
- The result is a multi set of solutions $\mu$
- The multiplicity of each solution $\mu$ corresponds to the number of different assignments $\sigma$
Multi Sets

Definition (Multi Set)

A multi set over a set $S$ is a total function $M : S \rightarrow \mathbb{N}^+ \cup \{\omega\}$

- $\mathbb{N}^+$ denotes the positive natural numbers
- $\omega > n$ for all $n \in \mathbb{N}^+$
- $M(s)$ is the multiplicity of $s \in S$
- $\omega$: countably infinite number of occurrences

- We represent a multi set over the set $S$ also with the set $\{(s, M(s)) \mid s \in S\}$
- We write $(s, n) \in M$ if $M(s) = n$
- We assume that $M(s) = 0$ if $s \notin S$
- Alternative notation: $\{a, b, b\}$ corresponds to the multi set $M$ over the set $\{a, b\}$ with $M(a) = 1$ and $M(b) = 2$
Solution Mapping Example

ex:Birte ex:gives [  
  a ex:Lecture ;  
  ex:hasTopic "SPARQL" ] .

ex:Sebastian ex:gives [  
  a ex:Lecture ;  
  ex:hasTopic "DLs and OWL" ] .

Bgp(?who ex:gives _:x .  
  _:x ex:hasTopic ?what)
Solution Mapping Example

ex:Birte ex:gives _:a .
_:a rdf:type ex:Lecture .
_:a ex:hasTopic "SPARQL" .
ex:Sebastian ex:gives _:b .
_:b rdf:type ex:Lecture .
_:b ex:hasTopic "DLs and OWL" .

Bgp(?who ex:gives _:x .  _:x ex:hasTopic ?what)
Solution Mapping Example

\[
\text{ex:Birte ex:gives } \_ : \text{a}.
\_ : \text{a} \text{ rdf:type ex:Lecture}.
\_ : \text{a} \text{ ex:hasTopic "SPARQL".}
\text{ex:Sebastian ex:gives } \_ : \text{b}.
\_ : \text{b} \text{ rdf:type ex:Lecture}.
\_ : \text{b} \text{ ex:hasTopic "DLs and OWL".}
\]

\[
\text{Bgp}(?\text{who ex:gives } \_ : x. \_ : x \text{ ex:hasTopic } ?\text{what})
\]

\[
\mu_1: \quad ?\text{who } \mapsto \text{ex:Birte, } \quad ?\text{what } \mapsto \text{"SPARQL"}
\]

\[
\sigma_1: \quad \_ : x \mapsto \_ : a
\]
Solution Mapping Example

```
ex:Birte ex:gives _:_a .
:_:_a rdf:type ex:Lecture .
:_:_a ex:hasTopic "SPARQL" .
ex:Sebastian ex:gives _:_b .
:_:_b rdf:type ex:Lecture .
:_:_b ex:hasTopic "DLs and OWL" .
```

\[ \text{Bgp}(\text{?who ex:gives } _:_x . \ _:_x ex:hasTopic \ ?what) \]

\[
\mu_1: \begin{align*}
\text{?who} & \mapsto \text{ex:Birte,} \\
\text{?what} & \mapsto \text{"SPARQL"}
\end{align*}
\]

\[
\sigma_1: \begin{align*}
\text{?x} & \mapsto \text{_:_a}
\end{align*}
\]

\[
\mu_2: \begin{align*}
\text{?who} & \mapsto \text{ex:Sebastian,} \\
\text{?what} & \mapsto \text{"DLs and OWL"}
\end{align*}
\]

\[
\sigma_2: \begin{align*}
\text{?x} & \mapsto \text{_:_b}
\end{align*}
\]
Solution Mapping Example

ex:Birte ex:gives _:a .
_:a rdf:type ex:Lecture .
_:a ex:hasTopic "SPARQL" .
ex:Sebastian ex:gives _:b .
_:b rdf:type ex:Lecture .
_:b ex:hasTopic "DLs and OWL" .

Bgp(?who ex:gives _:x . _:x ex:hasTopic ?what)

\[ \mu_1: \quad ?\text{who} \mapsto \text{ex:Birte}, \quad ?\text{what} \mapsto "\text{SPARQL}" \]

\[ \sigma_1: \quad _\text{:x} \mapsto _\text{:a} \]

\[ \mu_2: \quad ?\text{who} \mapsto \text{ex:Sebastian}, \quad ?\text{what} \mapsto "\text{DLs and OWL}" \]

\[ \sigma_2: \quad _\text{:x} \mapsto _\text{:b} \]

Two solutions each with multiplicity 1
Exercise Solution Sets

```
ex:Birte ex:gives [ 
  a ex:Lecture ; 
  ex:hasTopic "SPARQL" ] . 
ex:Birte ex:gives [ 
  a ex:Lecture ; 
  ex:hasTopic "SPARQL Algebra" ] . 
```

```
Bgp(?who ex:gives _:_x . _:_x ex:hasTopic _:_y)
```
Solution
Solution

```
ex:Birte ex:gives _:a .
_:a rdf:type ex:Lecture .
_:a ex:hasTopic "SPARQL" .
ex:Birte ex:gives _:b .
_:b rdf:type ex:Lecture .
_:b ex:hasTopic "SPARQL Algebra" .

Bgp(?who ex:gives _:x . _:x ex:hasTopic _:y)
```
Solution

_ :a rdf:type ex:Lecture .
_ :a ex:hasTopic "SPARQL" .
_ :b rdf:type ex:Lecture .
_ :b ex:hasTopic "SPARQL Algebra" .

Bgp(?who ex:gives _ :x . _ :x ex:hasTopic _ :y)

\[ \mu_1: \quad \text{?who} \mapsto \text{ex: Birte} , \]
\[ \sigma_1: \quad _ :x \mapsto _ :a \quad _ :y \mapsto \text{"SPARQL"} \]
Solution

ex:Birte ex:gives _:_a .
_:_a rdf:type ex:Lecture .
_:_a ex:hasTopic "SPARQL" .
ex:Birte ex:gives _:_b .
_:_b rdf:type ex:Lecture .
_:_b ex:hasTopic "SPARQL Algebra" .

Bgp(?who ex:gives _:_x . _:_x ex:hasTopic _:_y)

\[ \mu_1: \quad ?who \mapsto ex: Birte, \]

\[ \sigma_1: \quad _:_x \mapsto _:_a \quad _:_y \mapsto "SPARQL" \]

\[ \mu_2: \quad ?who \mapsto ex: Birte, \]

\[ \sigma_2: \quad _:_x \mapsto _:_b \quad _:_y \mapsto "SPARQL Algebra" \]
Solution

ex:Birte ex:gives _:a .
_:a rdf:type ex:Lecture .
_:a ex:hasTopic "SPARQL" .
ex:Birte ex:gives _:b .
_:b rdf:type ex:Lecture .
_:b ex:hasTopic "SPARQL Algebra" .

Bgp(?who ex:gives _:x . _:x ex:hasTopic _:y)

\[\mu_1: \text{?who} \mapsto \text{ex:Birte,}\]
\[\sigma_1: _:x \mapsto _:a _:y \mapsto "\text{SPARQL}"\]
\[\mu_2: \text{?who} \mapsto \text{ex:Birte,}\]
\[\sigma_2: _:x \mapsto _:b _:y \mapsto "\text{SPARQL Algebra}"\]

One solution with multiplicity 2
Union of Solutions (1)

**Definition (Compatibility)**

Two solutions $\mu_1$ and $\mu_2$ are compatible if $\mu_1(x) = \mu_2(x)$ for all $x$, for which $\mu_1$ and $\mu_2$ are defined.
Two solutions $\mu_1$ and $\mu_2$ are compatible if $\mu_1(x) = \mu_2(x)$ for all $x$, for which $\mu_1$ and $\mu_2$ are defined.

$\mu_1: ?x \mapsto \text{ex}: a, ?y \mapsto \text{ex}: b$

$\mu_2: ?y \mapsto \text{ex}: b, ?z \mapsto \text{ex}: c$
Union of Solutions (1)

Definition (Compatibility)

Two solutions $\mu_1$ and $\mu_2$ are compatible if $\mu_1(x) = \mu_2(x)$ for all $x$, for which $\mu_1$ and $\mu_2$ are defined.

$\mu_1$: $?x \mapsto \text{ex}: a$, $?y \mapsto \text{ex}: b$
$\mu_2$: $?y \mapsto \text{ex}: b$, $?z \mapsto \text{ex}: c$  ✓
Definition (Compatibility)

Two solutions $\mu_1$ and $\mu_2$ are compatible if $\mu_1(x) = \mu_2(x)$ for all $x$, for which $\mu_1$ and $\mu_2$ are defined.

$$\mu_1: \ ?x \mapsto ex: a, \ ?y \mapsto ex: b$$
$$\mu_2: \ ?y \mapsto ex: b, \ ?z \mapsto ex: c \quad \checkmark$$

$$\mu_1: \ ?x \mapsto ex: a, \ ?y \mapsto ex: b$$
$$\mu_2: \ ?x \mapsto ex: b, \ ?z \mapsto ex: c$$
Union of Solutions (1)

**Definition (Compatibility)**

Two solutions $\mu_1$ and $\mu_2$ are compatible if

$$\mu_1(x) = \mu_2(x)$$

for all $x$, for which $\mu_1$ and $\mu_2$ are defined.

\[
\begin{align*}
\mu_1 &: \ ?x \mapsto \text{ex : a}, \ ?y \mapsto \text{ex : b} \\
\mu_2 &: \ ?y \mapsto \text{ex : b}, \ ?z \mapsto \text{ex : c} & \checkmark \\
\end{align*}
\]

\[
\begin{align*}
\mu_1 &: \ ?x \mapsto \text{ex : a}, \ ?y \mapsto \text{ex : b} \\
\mu_2 &: \ ?x \mapsto \text{ex : b}, \ ?z \mapsto \text{ex : c} & \copyright
\end{align*}
\]
Union of Solutions (1)

Definition (Compatibility)

Two solutions $\mu_1$ and $\mu_2$ are compatible if $\mu_1(x) = \mu_2(x)$ for all $x$, for which $\mu_1$ and $\mu_2$ are defined.

$\mu_1$: $\ ?x \mapsto \text{ex} : a$, $\ ?y \mapsto \text{ex} : b$
$\mu_2$: $\ ?y \mapsto \text{ex} : b$, $\ ?z \mapsto \text{ex} : c$ ✓

$\mu_1$: $\ ?x \mapsto \text{ex} : a$, $\ ?y \mapsto \text{ex} : b$
$\mu_2$: $\ ?x \mapsto \text{ex} : b$, $\ ?z \mapsto \text{ex} : c$ ❌

$\mu_1$: $\ ?x \mapsto \text{ex} : a$
$\mu_2$: $\ ?y \mapsto \text{ex} : b$
Union of Solutions (1)

**Definition (Compatibility)**

Two solutions $\mu_1$ and $\mu_2$ are compatible if $\mu_1(x) = \mu_2(x)$ for all $x$, for which $\mu_1$ and $\mu_2$ are defined.

\[
\begin{align*}
\mu_1 &: ?x \mapsto \text{ex} : a, \ ?y \mapsto \text{ex} : b \\
\mu_2 &: ?y \mapsto \text{ex} : b, \ ?z \mapsto \text{ex} : c \quad \checkmark
\end{align*}
\]

\[
\begin{align*}
\mu_1 &: ?x \mapsto \text{ex} : a, \ ?y \mapsto \text{ex} : b \\
\mu_2 &: ?x \mapsto \text{ex} : b, \ ?z \mapsto \text{ex} : c \quad \times
\end{align*}
\]

\[
\begin{align*}
\mu_1 &: ?x \mapsto \text{ex} : a \\
\mu_2 &: ?y \mapsto \text{ex} : b \quad \checkmark
\end{align*}
\]
Union of Solutions (2)

Union of two compatible solutions $\mu_1$ and $\mu_2$:

$$(\mu_1 \cup \mu_2)(x) = \begin{cases} 
\mu_1(x) & \text{if } x \in \text{dom}(\mu_1) \\
\mu_2(x) & \text{otherwise}
\end{cases}$$

⇝ simple intuition: union of matching table rows

- Next lecture: Evaluation of the main algebra operators
Summary

- SPARQL queries are translated into algebra objects
- The BGPs generate solutions
- Other algebra operators combine solutions
- Details in the next lecture