TECHNISCHE UNIVERSITÄT DRESDEN

FOUNDATIONS OF SEMANTIC WEB TECHNOLOGIES

OWL & Description Logics

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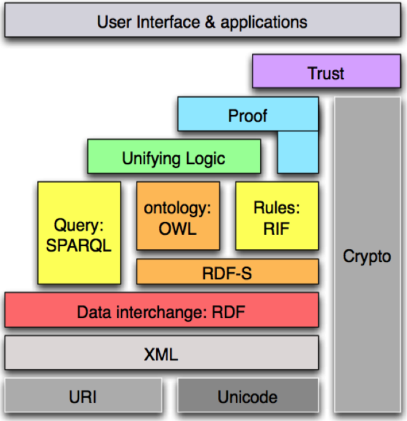
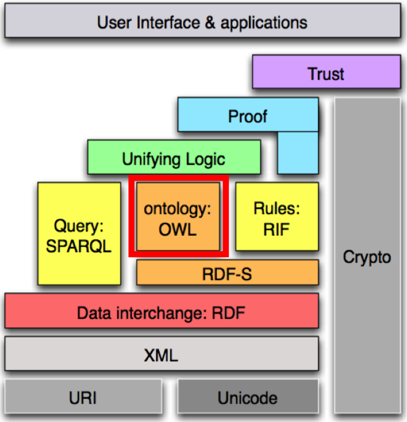
DRESDEN concept Exzellenz aus Wissenschaft und Kultur

Content

OWL & Description Logics



OWL & Description Logics



Agenda

- Motivation
- Introduction Description Logics
- The Description Logic $\mathcal{ALC}$
- Extensions of $\mathcal{ALC}$
- Inference Problems

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Description Logics

- description logics (DLs) are one of the current KR paradigms
- have significantly influenced the standardization of Semantic Web languages
  - OWL is essentially based on DLs
- numerous reasoners

Quonto	JFact	FaCT++	RacerPro
Owlgres	Pellet	SHER	snorocket
OWLIM	Jena	Oracle Prime	QuOnto
Trowl	HermiT	condor	CB
	ELK	konclude	RScale

OWL Tools

Tools and editors for OWL:

- Protégé, `http://protege.stanford.edu` (free editor)
- TopBraid Composer, `http://www.topquadrant.com` (commercial editor)
- OWL API, `http://owlapi.sourceforge.net/` (free Java library)
- OWL Syntax Converter, `http://mowl-power.cs.man.ac.uk:8080/converter/` (conversion/validation service)

Description Logics

- Origin of DLs: semantic networks and frame-based systems
- Downside of the former: only intuitive semantics - diverging interpretations
- DLs provide a formal semantics on logical grounds
- Can be seen as decidable fragments of first-order logic (FOL), closely related to modal logics
- Significant portion of DL-related research devoted to clarifying the computational effort of reasoning tasks in terms of their worst-case complexity
- Despite high complexities, even for expressive DLs exist optimized reasoning algorithms with good average case behavior

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DL building blocks

- individuals: `birte`, `cs63.800`, `sebastian`, etc.
  - ⇝ constants in FOL, resources in RDF
- concept names: `Person`, `Course`, `Student`, etc.
  - ⇝ unary predicates in FOL, classes in RDF
- role names: `hasFather`, `attends`, `worksWith`, etc.
  - ⇝ binary predicates in FOL, properties in RDF
  - – can be subdivided into abstract and concrete roles (object und data properties)

the set of all individual, concept and role names is called signature or vocabulary

Constituents of a DL Knowledge Base

TBox $\mathcal{T}$	information about concepts and their taxonomic dependencies
ABox $\mathcal{A}$	informationen about individuals, their concept and role memberships

in more expressive DLs also:

RBox $\mathcal{R}$	information about roles and their mutual dependencies

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Complex Concepts

$\mathcal{ALC}$, Attribute Language with Complement, is the simplest DL that is Boolean closed

we define (complex) $\mathcal{ALC}$ concepts as follows:

- every concept name is a concept,
- $\top$ and $\bot$ are concepts,
- for concepts C and D, $\neg C$, $C \sqcap D$, and $C \sqcup D$ are concepts,
- for a role r and a concept$C$, $\exists r.C$ and $\forall r.C$ are concepts

Example: $\texttt{Student} \sqcap \forall \texttt{attendsCourse}.\texttt{MasterCourse}$
Intuitively: describes the concept comprising all students that attend only master courses

Concept Constructors vs. OWL

- $\top$ corresponds to `owl:Thing`
- $\bot$ corresponds to `owl:Nothing`
- $\sqcap$ corresponds to `owl:intersectionOf`
- $\sqcup$ corresponds to `owl:unionOf`
- $\neg$ corresponds to `owl:complementOf`
- $\forall$ corresponds to `owl:allValuesFrom`
- $\exists$ corresponds to `owl:someValuesFrom`

Concept Axioms

For concepts C, D, a general concept inclusion (GCI) axiom has the form

$$C \sqsubseteq D$$

- $C \equiv D$ is an abbreviation for $C \sqsubseteq D$ and $D \sqsubseteq C$
- a TBox (terminological Box) consists of a set of GCIs

TBox $\mathcal{T}$

ABox

an $\mathcal{ALC}$ ABox assertion can be of one of the following forms

- $C(a)$, called concept assertion
- $r(a, b)$, called role assertion

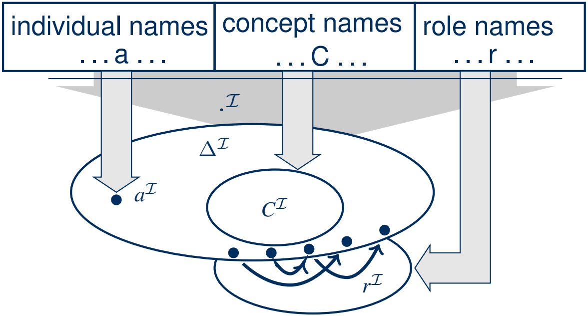
an ABox consists of a set of ABox assertions

ABox $\mathcal{A}$

The Description Logic $\mathcal{ALC}$

- $\mathcal{ALC}$ is a syntactic variant of the modal logic **K** with multiple modalities
- semantics defined in a model-theoretic way, that is, via interpretations
- can be expressed in first-order predicate logic
- a DL interpretation $\mathcal{I}$ consists of a domain $\Delta^\mathcal{I}$ and a function $\cdot^\mathcal{I}$, that maps
  - individual names a to domain elements $a^\mathcal{I} \in \Delta^\mathcal{I}$
  - concept names C to sets of domain elements $C^\mathcal{I} \subseteq \Delta^\mathcal{I}$
  - role names r to sets of pairs of domain elements $r^\mathcal{I} \subseteq \Delta^\mathcal{I} \times \Delta^\mathcal{I}$

Schematic Representation of an Interpretation



Interpretation of Complex Concepts

the interpretation of complex concepts is defined inductively:

Name	Syntax	Semantics
top	$\top$	$\Delta^{\mathcal{I}}$
bottom	$\bot$	$\emptyset$
negation	$\neg C$	$\Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
conjunction	$C \sqcap D$	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$
disjunction	$C \sqcup D$	$C^{\mathcal{I}} \cup D^{\mathcal{I}}$
universal quantifier	$\forall r.C$	$\{x \in \Delta^{\mathcal{I}} \mid (x,y) \in r^{\mathcal{I}}$ implies $y \in C^{\mathcal{I}}\}$
existential quantifier	$\exists r.C$	$\{x \in \Delta^{\mathcal{I}} \mid$ there is some $y \in \Delta^{\mathcal{I}}$, such that $(x,y) \in r^{\mathcal{I}}$ and $y \in C^{\mathcal{I}}\}$

Interpretation of Axioms

interpretation can be extended to axioms:

name	syntax	semantic	notation
inclusion	$C \sqsubseteq D$	holds if $C^\mathcal{I} \subseteq D^\mathcal{I}$	$\mathcal{I} \models C \sqsubseteq D$
equivalence	$C \equiv D$	holds if $C^\mathcal{I} = D^\mathcal{I}$	$\mathcal{I} \models C \equiv D$
concept assertion	$C(a)$	holds if $a^\mathcal{I} \in C^\mathcal{I}$	$\mathcal{I} \models C(a)$
role assertion	$r(a, b)$	holds if $(a^\mathcal{I}, b^\mathcal{I}) \in r^\mathcal{I}$	$\mathcal{I} \models r(a, b)$

Logical Entailment in Knowledge Bases

- Let $\mathcal{I}$ be an interpretation, $\mathcal{T}$ a TBox, $\mathcal{A}$ an Abox and $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ a knowledge base
- $\mathcal{I}$ is a model for $\mathcal{T}$, if $\mathcal{I} \models$ ax for every axiom ax in $\mathcal{T}$, written $\mathcal{I} \models \mathcal{T}$
- $\mathcal{I}$ is a model for $\mathcal{A}$, if $\mathcal{I} \models$ ax for every assertion ax in $\mathcal{A}$, written $\mathcal{I} \models \mathcal{A}$
- $\mathcal{I}$ is a model for $\mathcal{K}$, if $\mathcal{I} \models \mathcal{T}$ and $\mathcal{I} \models \mathcal{A}$
- An axiom ax follows from $\mathcal{K}$, written $\mathcal{K} \models$ ax, if every model $\mathcal{I}$ of $\mathcal{K}$ is also a model of ax.

Semantics via Translation into FOL

translation of TBox axioms into first-order predicate logics through the mapping π with C, D complex classes, r a role and A an atomic class:

$$\pi(C \sqsubseteq D) = \forall x.(\pi_x(C) \rightarrow \pi_x(\mathrm{D})) \qquad \pi(C \equiv D) = \forall x.(\pi_x(C) \leftrightarrow \pi_x(\mathrm{D}))$$

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$$\begin{aligned}
\pi_x(A) &= A(x) \\
\pi_x(\neg C) &= \neg \pi_x(C) \\
\pi_x(C \sqcap D) &= \pi_x(C) \wedge \pi_x(\mathrm{D}) \\
\pi_x(C \sqcup D) &= \pi_x(C) \vee \pi_x(\mathrm{D}) \\
\pi_x(\forall r.C) &= \forall y.(r(x,y) \rightarrow \pi_y(C)) \\
\pi_x(\exists r.C) &= \exists y.(r(x,y) \wedge \pi_y(C))
\end{aligned}$$

Semantics via Translation into FOL

translation of TBox axioms into first-order predicate logics through the mapping π with C, D complex classes, r a role and A an atomic class:

$$\pi(C \sqsubseteq D) = \forall x.(\pi_x(C) \rightarrow \pi_x(\mathrm{D})) \qquad \pi(C \equiv D) = \forall x.(\pi_x(C) \leftrightarrow \pi_x(\mathrm{D}))$$

$$\pi_x(A) = A(x) \qquad \pi_y(A) = A(y)$$

$$\pi_x(\neg C) = \neg\pi_x(C) \qquad \pi_y(\neg C) = \neg\pi_y(C)$$

$$\pi_x(C \sqcap D) = \pi_x(C) \wedge \pi_x(\mathrm{D}) \qquad \pi_y(C \sqcap D) = \pi_y(C) \wedge \pi_y(\mathrm{D})$$

$$\pi_x(C \sqcup D) = \pi_x(C) \vee \pi_x(\mathrm{D}) \qquad \pi_y(C \sqcup D) = \pi_y(C) \vee \pi_y(\mathrm{D})$$

$$\pi_x(\forall r.C) = \forall y.(r(x,y) \rightarrow \pi_y(C)) \qquad \pi_y(\forall r.C) = \forall x.(r(y,x) \rightarrow \pi_x(C))$$

$$\pi_x(\exists r.C) = \exists y.(r(x,y) \wedge \pi_y(C)) \qquad \pi_y(\exists r.C) = \exists x.(r(y,x) \wedge \pi_x(C))$$

Semantics via Translation into FOL

- translation only requires two variables

$\leadsto$ $\mathcal{ALC}$ is a fragment of FOL with two variables $\mathcal{L}_2$

$\leadsto$ satisfiability checking of sets of $\mathcal{ALC}$ axioms is decidable

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Inverse Roles

- a role can be
  - a role name r or
  - an inverse role r^-
- the semantics of inverse roles is defined as follows:

$$(r^-)^\mathcal{I} = \{(y, x) \mid (x, y) \in r^\mathcal{I}\}$$

- the extension of $\mathcal{ALC}$ by inverse roles is denoted as $\mathcal{ALCI}$
- corresponds to `owl:inverseOf`

Parts of a Knowledge Base

TBox $\mathcal{T}$	information about concepts and their taxonomic dependencies
ABox $\mathcal{A}$	information about individuals, their concepts and role connections

in more expressive DLs also:

RBox $\mathcal{R}$	information about roles and their mutual dependencies

Role Axioms

- for r, s roles, a role inclusion axiom – RIA has the form $r \sqsubseteq s$
- $r \equiv s$ is the abbreviation for $r \sqsubseteq s$ and $s \sqsubseteq r$
- an RBox (role box) or role hierarchy consists of a set of role axioms
- $r \sqsubseteq s$ holds in an interpretation $\mathcal{I}$ if $r^{\mathcal{I}} \subseteq s^{\mathcal{I}}$, written $\mathcal{I} \models r \sqsubseteq s$
- the extension of $\mathcal{ALC}$ by role hierarchies is denoted with $\mathcal{ALCH}$, if we also have inverse roles: $\mathcal{ALCHI}$
- corresponds to `owl:subPropertyOf`

RBox $\mathcal{R}$

An Example Knowledge Base

RBox $\mathcal{R}$

$$\texttt{own} \sqsubseteq \texttt{careFor}$$

TBox $\mathcal{T}$

$$\texttt{Healthy} \sqsubseteq \neg\texttt{Dead}$$

$$\texttt{Cat} \sqsubseteq \texttt{Dead} \sqcup \texttt{Alive}$$

$$\texttt{HappyCatOwner} \sqsubseteq \exists\texttt{owns.Cat} \sqcap \forall\texttt{caresFor.Healthy}$$

ABox $\mathcal{A}$

$$\texttt{HappyCatOwner}\ (\texttt{schrödinger})$$

An Example Knowledge Base

RBox $\mathcal{R}$

$$\texttt{own} \sqsubseteq \texttt{careFor}$$

"If somebody owns something, they care for it."

TBox $\mathcal{T}$

$$\texttt{Healthy} \sqsubseteq \neg\texttt{Dead}$$

"Healthy beings are not dead."

$$\texttt{Cat} \sqsubseteq \texttt{Dead} \sqcup \texttt{Alive}$$

"Every cat is dead or alive."

$$\texttt{HappyCatOwner} \sqsubseteq \exists\texttt{owns.Cat} \sqcap \forall\texttt{caresFor.Healthy}$$

"A happy cat owner owns a cat and everything he cares for is healthy."

ABox $\mathcal{A}$

$$\texttt{HappyCatOwner}\ (\texttt{schrödinger})$$

"Schrödinger is a happy cat owner. "

Role Transitivity

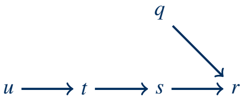
- for r a role, a transitivity axiom has the form $\text{Trans}(r)$
- $\text{Trans}(r)$ holds in an interpretation $\mathcal{I}$ if $r^{\mathcal{I}}$ is a transitive relation, i.e., $(x, y) \in r^{\mathcal{I}}$ and $(y, z) \in r^{\mathcal{I}}$ imply $(x, z) \in r^{\mathcal{I}}$, written $\mathcal{I} \models \text{Trans}(r)$
- the extension of $\mathcal{ALC}$ by transitivity axioms is denoted by $\mathcal{S}$ (after the modal logic S_4 – although it is really based on K_4)
- corresponds to `owl:TransitiveProperty`

Role Functionality

- for r a role, a functionality axiom has the form $\mathsf{Func}(r)$
- $\mathsf{Func}(r)$ holds in an interpretation $\mathcal{I}$ if $(x, y_1) \in r^{\mathcal{I}}$ and $(x, y_2) \in r^{\mathcal{I}}$ imply $y_1 = y_2$, written $\mathcal{I} \models \mathsf{Func}(r)$
- translation into FOL requires equality (=)
- the extension of $\mathcal{ALC}$ by functionality axioms is denoted by $\mathcal{ALCF}$
- corresponds to `owl:FunctionalProperty`

Simple and Non-Simple Roles

- Given a role hierarchy $\mathcal{R}$, we let $\sqsubseteq^{*}_{\mathcal{R}}$ be the smallest relation such that:
  - if $r \sqsubseteq s$ then $r \sqsubseteq^{*}_{\mathcal{R}} s$
  - $r \sqsubseteq^{*}_{\mathcal{R}} r$ for every role r (reflexive)
  - if $r \sqsubseteq^{*}_{\mathcal{R}} s$ and $s \sqsubseteq^{*}_{\mathcal{R}} t$ then $r \sqsubseteq^{*}_{\mathcal{R}} t$ (transitive)
  - if $r \sqsubseteq^{*}_{\mathcal{R}} s$ then $r^- \sqsubseteq^{*}_{\mathcal{R}} s^-$, where "$(r^-)^-$" means r
- For a role hierarchy $\mathcal{R}$, we can distinguish the roles in $\mathcal{R}$ into simple and non-simple roles:
  - a role r is non-simple w.r.t. $\mathcal{R}$, if there is a role t such that $\mathsf{Trans}(t) \in \mathcal{R}$ and $t \sqsubseteq^{*}_{\mathcal{R}} r$ holds
  - all other roles are are simple
- Example: $\mathcal{R} = \{u \sqsubseteq t, \quad t \sqsubseteq s, \quad s^- \sqsubseteq r^-, \quad q \sqsubseteq r, \quad \mathsf{Trans}(t)\}$



non-simple:

Simple and Non-Simple Roles

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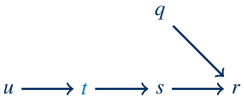

non-simple: t

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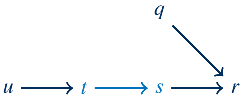

non-simple: t, s

Simple and Non-Simple Roles

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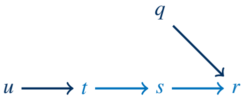

non-simple: t, s, r

Simple and Non-Simple Roles

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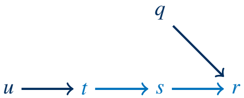

non-simple: t, s, r simple: q, u

(Unqualified) Number Restrictions

- For a simple role s and a natural number n, $\leqslant n\, s$, $\geqslant n\, s$ and $= n\, s$ are concepts
- Semantics defined by:

$$
\begin{aligned}
(\leqslant n\, s)^\mathcal{I} &= \{x \in \Delta^\mathcal{I} \mid \#\{y \in \Delta^\mathcal{I} \mid (x, y) \in s^\mathcal{I}\} \leq n\} \\
(\geqslant n\, s)^\mathcal{I} &= \{x \in \Delta^\mathcal{I} \mid \#\{y \in \Delta^\mathcal{I} \mid (x, y) \in s^\mathcal{I}\} \geq n\} \\
(= n\, s)^\mathcal{I} &= \{x \in \Delta^\mathcal{I} \mid \#\{y \in \Delta^\mathcal{I} \mid (x, y) \in s^\mathcal{I}\} = n\}
\end{aligned}
$$

- Extension of $\mathcal{ALC}$ by (unqualified) number restrictions denoted by $\mathcal{ALCN}$
- Correspond to `owl:maxCardinality`, `owl:minCardinality`, and `owl:cardinality`
- Restriction to simple roles ensures decidability, e.g., for checking knowledge base satisfiability
- Definition of TBox requires an RBox being already defined

(Unqualified) Number Restrictions in FOL

- Translation into FOL requires equality or counting quantifiers
- Translation defined as follows (likewise for π_y):

$$\pi_x(\leqslant n\, s) = \exists^{\leqslant n} y.(s(x,y))$$
$$\pi_x(\geqslant n\, s) = \exists^{\geqslant n} y.(s(x,y))$$
$$\pi_x(= n\, s) = \exists^{\leqslant n} y.(s(x,y)) \wedge \exists^{\geqslant n} y.(s(x,y))$$

- The following equivalences hold:

$$\neg(\leqslant n\, s) \mathrel{\hat{=}} \; \geqslant (n+1)\, s \qquad \neg(\geqslant n\, s) \mathrel{\hat{=}} \; \leqslant (n-1)\, s, \quad n \geq 1$$
$$\neg(\geqslant 0\, s) \mathrel{\hat{=}} \bot \qquad \geqslant 1\, s \mathrel{\hat{=}} \exists s.\top$$
$$\leqslant 0\, s \mathrel{\hat{=}} \forall s.\bot \qquad \top \sqsubseteq \; \leqslant 1 s \mathrel{\hat{=}} \mathsf{Func}(s)$$

Nominals or Closed Classes

- A nominal for an individual a is a concept of the form $\{a\}$;
  it denotes a class with a single element (denoted by a)
- Closed class with multiple elements expressed as union of nominals;
  $\{a_1\} \sqcup \ldots \sqcup \{a_n\}$ can also be abbreviated as $\{a_1, \ldots, a_n\}$
- Semantics defined as follows (the special case $n = 1$ covers nominals):

$$\text{DL: } (\{a_1, \ldots, a_n\})^{\mathcal{I}} = \{a_1^{\mathcal{I}}, \ldots, a_n^{\mathcal{I}}\}$$

$$\text{FOL: } \pi_x(\{a_1, \ldots, a_n\}) = (x = a_1 \vee \ldots \vee x = a_n)$$

- Extension of $\mathcal{ALC}$ by nominals denoted as $\mathcal{ALCO}$
- Corresponds to `owl:oneOf`

Nominals for Encoding Further OWL Constructors

- `owl:hasValue` “forces” role to a certain individual

```
<owl:Class rdf:ID="Woman">
  <owl:equivalentClass>
    <owl:Restriction>
      <owl:onProperty rdf:resource="#hasGender"/>
      <owl:hasValue rdf:resource="#female"/>
    </owl:Restriction>
  </owl:equivalentClass>
</owl:Class>
```

- In description logic:

$$\text{Woman} \equiv \exists \text{hasGender}.\{\text{female}\}$$

Further Kinds of ABox Assertions

ABox assertions can have one of the following forms:

- $C(a)$ (concept assertion)
- $r(a, b)$ (role assertion)
- $\neg r(a, b)$ (negative role assertion)
- $a \approx b$ (equality assertion)
- $a \not\approx b$ (inequality assertion)

Further Kinds of ABox Assertions

ABox assertions can have one of the following forms:

- $C(a)$ (concept assertion)
- $r(a, b)$ (role assertion)
- $\neg r(a, b)$ (negative role assertion)
- $a \approx b$ (equality assertion)
- $a \not\approx b$ (inequality assertion)

Internalization of ABox Assertions

If nominals are supported, every knowledge base with an ABox can be transformed into an equivalent one without ABox:

$$
\begin{aligned}
C(a) &\mathrel{\hat{=}} \{a\} \sqsubseteq C \\
r(a,b) &\mathrel{\hat{=}} \{a\} \sqsubseteq \exists r.\{b\} \\
\neg r(a,b) &\mathrel{\hat{=}} \{a\} \sqsubseteq \forall r.(\neg\{b\}) \\
a \approx b &\mathrel{\hat{=}} \{a\} \equiv \{b\} \\
a \not\approx b &\mathrel{\hat{=}} \{a\} \sqsubseteq \neg\{b\}
\end{aligned}
$$

Overview Nomenclature

$\mathcal{ALC}$ Attribute Language with Complement

$\mathcal{S}$ $\mathcal{ALC}$ + role transitivity

$\mathcal{H}$ subroles

$\mathcal{O}$ nominals

$\mathcal{I}$ inverse roles

$\mathcal{N}$ (unqualified) number restrictions

(D) datatypes

$\mathcal{F}$ functional roles

OWL DL is $\mathcal{SHOIN}(\mathrm{D})$ and OWL Lite is $\mathcal{SHIF}(\mathrm{D})$

Different Terms in DLs and in OWL

OWL	**DL**
class	concept
property	role
object property	abstract role
data property	concrete role
oneOf*	nominal
ontology	knowledge base
–	TBox, RBox, ABox

*) with one individual

Example: A More Complex Knowledge Base

$$\begin{aligned}
\mathsf{Human} &\sqsubseteq \mathsf{Animal} \sqcap \mathsf{Biped} \\
\mathsf{Man} &\equiv \mathsf{Human} \sqcap \mathsf{Male} \\
\mathsf{Male} &\sqsubseteq \neg\mathsf{Female} \\
\{\mathsf{President_Obama}\} &\equiv \{\mathsf{Barack_Obama}\} \\
\{\mathsf{john}\} &\sqsubseteq \neg\{\mathsf{peter}\} \\
\mathsf{hasDaughter} &\sqsubseteq \mathsf{hasChild} \\
\mathsf{hasChild} &\equiv \mathsf{hasParent}^{-} \\
\mathsf{cost} &\equiv \mathsf{price} \\
&\mathsf{Trans}(\mathsf{ancestor}) \\
&\mathsf{Func}(\mathsf{hasMother}) \\
&\mathsf{Func}(\mathsf{hasSSN}^{-})
\end{aligned}$$

Open versus Closed World Assumption

OWA Open World Assumption

- the existence of further individuals is possible, if they are not explicitly excluded
- OWL uses the OWA

CWA Closed World Assumption

- it is assumed that the knowledge base contains all individuals and facts

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	Are all of Bill's children male?	DL answers	Prolog answers
child(bill, bob) Man(bob)	$\models^?$ ($\forall$ child.Man)(bill)		
($\leqslant 1$ child)(bill)	$\models^?$ ($\forall$ child.Man)(bill)		

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Agenda

- Motivation
- Introduction Description Logics
- The Description Logic $\mathcal{ALC}$
- Extensions of $\mathcal{ALC}$
- Inference Problems

Important Inference Problems for a Knowledge Base $\mathcal{K}$

- global consistency of the knowledge base: $\mathcal{K} \models^? \text{false}$? $\mathcal{K} \models^? \top \sqsubseteq \bot$?
  - Is the knowledge base "plausible"?
- class consistency: $\mathcal{K} \models^? C \sqsubseteq \bot$?
  - Is the class C necessarily empty?
- class inclusion (subsumption): $\mathcal{K} \models^? C \sqsubseteq D$?
  - taxonomic structure of the knowledge base
- class equivalence: $\mathcal{K} \models^? C \equiv D$?
  - Do two classes comprise the same individual sets?
- class disjointness: $\mathcal{K} \models^? C \sqcap D \sqsubseteq \bot$?
  - Are two classes disjoint?
- class membership: $\mathcal{K} \models^? C(a)$?
  - Is the individual a contained in class C?
- instance retrieval: find all x with $\mathcal{K} \models C(x)$
  - Find all (known!) members of the class C.

Decidability of OWL DL

- decidability means that there is a terminating algorithm for all the aforementioned inference problems
- OWL DL is a fragment of FOL, thus FOL inference procedures could be used in principle(Resolution, Tableaux)
  - but these are not guaranteed to terminate!
- problem: find algorithms that are guaranteed to terminate
- no “naive” solutions for this

OWL 2: Outlook

- OWL 2 extends the fragments introduced here by further constructors
- OWL 2 also defines simpler fragments (PTime for standard inferencing problems)
- diverse tools for automated inferencing
- editors support creation of ontologies / knowledge bases

Further Literature

- Description Logic Primer
  A gentle introduction to DLs and pointers to further literature
  Available at `http://arxiv.org/abs/1201.4089`