TECHNISCHE UNIVERSITÄT DRESDEN

FOUNDATIONS OF SEMANTIC WEB TECHNOLOGIES

Hypertableau I

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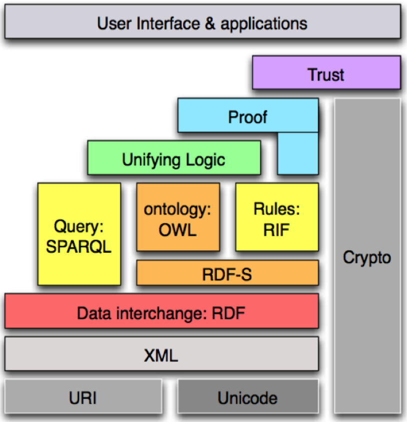
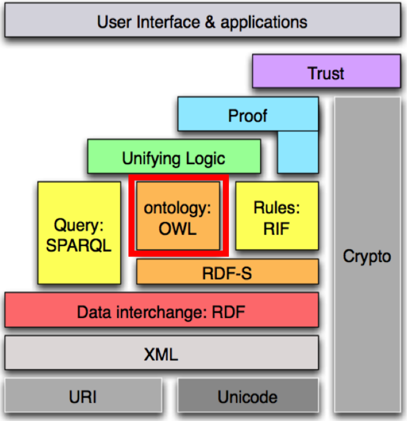
DRESDEN concept
Exzellenz aus Wissenschaft und Kultur

Content

Hypertableau



Hypertableau



Agenda

- Motivation
- Recap: Translation into FOL
- Structural Transformation
- Translation into Clauses
- Summary

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Example Standard Tableau

Let the following TBox $\mathcal{T}$ and ABox $\mathcal{A}$ be given:

$$\mathcal{T} = \{\exists r.A \sqsubseteq A\} \qquad C_{\mathcal{T}} = \forall r.(\neg A) \sqcup A$$

$$\mathcal{A} = \{\neg A(a_0), r(a_0, b_1), r(b_1, a_1), \ldots, r(a_{n-1}, b_n), r(b_n, a_n), A(a_n)\}$$

$$a_0 \xrightarrow{r} b_1 \xrightarrow{r} a_1 \xrightarrow{r} b_2 \cdots\cdots \cdots\cdots \rightarrow a_{n-1} \xrightarrow{r} b_n \xrightarrow{r} a_n$$

Assumption: we address the nodes in the tableau in alphabetic order, i.e., a's before b's

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\end{aligned}$$

Now again decisions for a_n and $b_1, \ldots, b_n$

Is that Necessarily So?

- the algorithm constructs exponentially many branches, despite dependency directed backtracking
- translation of the formula into FOL:

$$
\begin{aligned}
& \forall r.(\neg A) \sqcup A \\
= & \forall x, y. \, [r(x,y) \wedge A(y) \rightarrow A(x)] \\
= & \forall x, y. \, [\neg r(x,y) \vee \neg A(y) \vee A(x)]
\end{aligned}
$$

- note: the formula does not have real non-determinism (Horn-clause)
- hypertableau exploits this

Idea Hypertableau

- translate KB axioms into FOL
  - rewrite axioms to obtain formulae of a certain structure
- axioms are translated such that non-determinism is avoided, if possible
- the formulae thus obtained become rules for constructing a model abstraction

Simple Hypertableau Example

1. Translation into Clauses

$$
\begin{aligned}
A &\sqsubseteq A' \\
A &\sqsubseteq \exists r.B \\
D &\sqsubseteq E \sqcup F \\
F &\sqsubseteq \bot \\
\exists r.\top &\sqsubseteq C
\end{aligned}
$$

$$
\begin{aligned}
A(a) \\
(D \sqcap \neg B)(d)
\end{aligned}
$$

Simple Hypertableau Example

1. Translation into Clauses

$$A \sqsubseteq A' \qquad\qquad A(x) \rightarrow A'(x)$$

$$A \sqsubseteq \exists r.B$$

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Simple Hypertableau Example

1. Translation into Clauses

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$$D \sqsubseteq E \sqcup F$$

$$F \sqsubseteq \bot$$

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- existential quantifiers treated as in the tableau

Simple Hypertableau Example

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$$A \sqsubseteq \exists r.B \qquad A(x) \rightarrow \exists r.B(x)$$
$$D \sqsubseteq E \sqcup F \qquad D(x) \rightarrow E(x) \vee F(x)$$
$$F \sqsubseteq \bot$$
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Simple Hypertableau Example

1. Translation into Clauses

$A \sqsubseteq A'$	$A(x) \rightarrow A'(x)$
$A \sqsubseteq \exists r.B$	$A(x) \rightarrow \exists r.B(x)$
$D \sqsubseteq E \sqcup F$	$D(x) \rightarrow E(x) \vee F(x)$
$F \sqsubseteq \bot$	$F(x) \rightarrow \bot(x)$
$\exists r.\top \sqsubseteq C$	

$$A(a)$$

$$(D \sqcap \neg B)(d)$$

- existential quantifiers treated as in the tableau

Simple Hypertableau Example

1. Translation into Clauses

$$A \sqsubseteq A' \qquad A(x) \rightarrow A'(x)$$

$$A \sqsubseteq \exists r.B \qquad A(x) \rightarrow \exists r.B(x)$$

$$D \sqsubseteq E \sqcup F \qquad D(x) \rightarrow E(x) \vee F(x)$$

$$F \sqsubseteq \bot \qquad F(x) \rightarrow \bot(x)$$

$$\exists r.\top \sqsubseteq C \qquad r(x,y) \rightarrow C(x)$$

$$A(a)$$

$$(D \sqcap \neg B)(d)$$

- existential quantifiers treated as in the tableau

Simple Hypertableau Example

1. Translation into Clauses

$$
\begin{array}{rcl}
A \sqsubseteq A' & \quad & A(x) \rightarrow A'(x) \\
A \sqsubseteq \exists r.B & \quad & A(x) \rightarrow \exists r.B(x) \\
D \sqsubseteq E \sqcup F & \quad & D(x) \rightarrow E(x) \vee F(x) \\
F \sqsubseteq \bot & \quad & F(x) \rightarrow \bot(x) \\
\exists r.\top \sqsubseteq C & \quad & r(x,y) \rightarrow C(x) \\
A(a) & \quad & \rightarrow A(a) \\
(D \sqcap \neg B)(d) & &
\end{array}
$$

- existential quantifiers treated as in the tableau

Simple Hypertableau Example

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F \sqsubseteq \bot & & F(x) \rightarrow \bot(x) \\
\exists r.\top \sqsubseteq C & & r(x,y) \rightarrow C(x) \\
A(a) & & \rightarrow A(a) \\
(D \sqcap \neg B)(d) & & \rightarrow D(d) \\
 & & \rightarrow \neg B(d)
\end{array}
$$

- existential quantifiers treated as in the tableau

Simple Hypertableau Example

a d

$L(a) = \{A\}$

$L(d) = \{D, \neg B\}$

$$
\begin{aligned}
A(x) &\rightarrow A'(x) \\
A(x) &\rightarrow \exists r.B(x) \\
D(x) &\rightarrow E(x) \vee F(x) \\
F(x) &\rightarrow \bot(x) \\
r(x,y) &\rightarrow C(x) \\
&\rightarrow A(a) \\
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Simple Hypertableau Example

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$L(a) = \{A\} \cup \{A'\}$

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Simple Hypertableau Example

a $\qquad$ d

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r(x,y) &\to C(x) \\
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\end{aligned}
$$

$L(a) = \{A\} \cup \{A'\}$

$L(d) = \{D, \neg B\} \cup \{E^1\}$

Simple Hypertableau Example

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$$L(a) = \{A\} \cup \{A'\}$$
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$L(a) = \{A\} \cup \{A'\} \cup \{\exists r.B\}$

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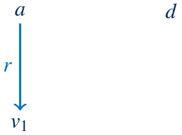
Simple Hypertableau Example



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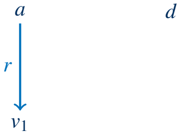
$$

$$
\begin{aligned}
L(a) &= \{A\} \cup \{A'\} \cup \{\exists r.B\} \\
L(d) &= \{D, \neg B\} \cup \{E^1\} \\
L(v_1) &= \{B\}
\end{aligned}
$$

Simple Hypertableau Example



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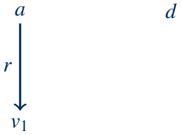
&\rightarrow D(d) \\
&\rightarrow \neg B(d)
\end{aligned}
$$



$$L(a) = \{A\} \cup \{A'\} \cup \{\exists r.B\} \cup \{C\}$$

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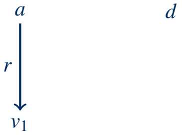
Simple Hypertableau Example



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&\rightarrow D(d) \\
&\rightarrow \neg B(d)
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$$

$$
\begin{aligned}
L(a) &= \{A\} \cup \{A'\} \cup \{\exists r.B\} \cup \{C\} \\
L(d) &= \{D, \neg B\} \cup \{E^1\} \\
L(v_1) &= \{B\}
\end{aligned}
$$

- no more rules applicable $\rightsquigarrow$ satisfiability has been shown
- the only thing left from the tableau rules: an analogue of the $\exists$-rule

Agenda

- Motivation
- Recap: Translation into FOL
- Structural Transformation
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Translation into FOL

translation of TBox axioms into FOL via the mapping π with C, D complex classes, r a role and A an atomic class:

$$\pi(C \sqsubseteq D) = \forall x.(\pi_x(C) \rightarrow \pi_x(D)) \qquad \pi(C \equiv D) = \forall x.(\pi_x(C) \leftrightarrow \pi_x(D))$$

$$\begin{aligned}
\pi_x(A) &= A(x) & \pi_y(A) &= A(y)\\
\pi_x(\neg C) &= \neg\pi_x(C) & \pi_y(\neg C) &= \neg\pi_y(C)\\
\pi_x(C \sqcap D) &= \pi_x(C) \wedge \pi_x(D) & \pi_y(C \sqcap D) &= \pi_y(C) \wedge \pi_y(D)\\
\pi_x(C \sqcup D) &= \pi_x(C) \vee \pi_x(D) & \pi_y(C \sqcup D) &= \pi_y(C) \vee \pi_y(D)\\
\pi_x(\forall r.C) &= \forall y.(r(x,y) \rightarrow \pi_y(C)) & \pi_y(\forall r.C) &= \forall x.(r(y,x) \rightarrow \pi_x(C))\\
\pi_x(\exists r.C) &= \exists y.(r(x,y) \wedge \pi_y(C)) & \pi_y(\exists r.C) &= \exists x.(r(y,x) \wedge \pi_x(C))
\end{aligned}$$

Motivation Normal Form

- the given translation creates rather complex formulae for complex axioms

$$\pi(C \sqsubseteq \exists r.(\forall s.(D \sqcup \exists r.D)))$$

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- the given translation creates rather complex formulae for complex axioms

$$\pi(C \sqsubseteq \exists r.(\forall s.(D \sqcup \exists r.D)))$$
$$\forall x.\,[\pi_x(C) \rightarrow \pi_x(\exists r.(\forall s.(D \sqcup \exists r.D)))]$$

Motivation Normal Form

- the given translation creates rather complex formulae for complex axioms

$$\pi(C \sqsubseteq \exists r.(\forall s.(D \sqcup \exists r.D)))$$
$$\forall x.\,[\pi_x(C) \rightarrow \pi_x(\exists r.(\forall s.(D \sqcup \exists r.D)))]$$
$$\forall x.\,[C(x) \rightarrow \exists y.(r(x,y) \wedge \pi_y(\forall s.(D \sqcup \exists r.D)))]$$

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$$\forall x.\,[\pi_x(C) \rightarrow \pi_x(\exists r.(\forall s.(D \sqcup \exists r.D)))]$$
$$\forall x.\,[C(x) \rightarrow \exists y.(r(x,y) \wedge \pi_y(\forall s.(D \sqcup \exists r.D)))]$$
$$\forall x.\,[C(x) \rightarrow \exists y.(r(x,y) \wedge \forall x.(s(y,x) \rightarrow \pi_x(D \sqcup \exists r.D)))]$$

Motivation Normal Form

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$$\pi(C \sqsubseteq \exists r.(\forall s.(D \sqcup \exists r.D)))$$
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$$\forall x.\,[C(x) \rightarrow \exists y.(r(x,y) \wedge \pi_y(\forall s.(D \sqcup \exists r.D)))]$$
$$\forall x.\,[C(x) \rightarrow \exists y.(r(x,y) \wedge \forall x.(s(y,x) \rightarrow \pi_x(D \sqcup \exists r.D)))]$$
$$\forall x.\,[C(x) \rightarrow \exists y.(r(x,y) \wedge \forall x.(s(y,x) \rightarrow \pi_x(D) \vee \pi_x(\exists r.D)))]$$

Motivation Normal Form

- the given translation creates rather complex formulae for complex axioms

$$
\begin{aligned}
&\pi(C \sqsubseteq \exists r.(\forall s.(D \sqcup \exists r.D))) \\
&\forall x.\, [\pi_x(C) \to \pi_x(\exists r.(\forall s.(D \sqcup \exists r.D)))] \\
&\forall x.\, [C(x) \to \exists y.(r(x,y) \land \pi_y(\forall s.(D \sqcup \exists r.D)))] \\
&\forall x.\, [C(x) \to \exists y.(r(x,y) \land \forall x.(s(y,x) \to \pi_x(D \sqcup \exists r.D)))] \\
&\forall x.\, [C(x) \to \exists y.(r(x,y) \land \forall x.(s(y,x) \to \pi_x(D) \lor \pi_x(\exists r.D)))] \\
&\forall x.\, [C(x) \to \exists y.(r(x,y) \land \forall x.(s(y,x) \to D(x) \lor \exists y.(r(x,y) \land \pi_y(D))))]
\end{aligned}
$$

Motivation Normal Form

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&\forall x.\,[C(x) \to \exists y.(r(x,y) \wedge \forall x.(s(y,x) \to \pi_x(D \sqcup \exists r.D)))]\\
&\forall x.\,[C(x) \to \exists y.(r(x,y) \wedge \forall x.(s(y,x) \to \pi_x(D) \vee \pi_x(\exists r.D)))]\\
&\forall x.\,[C(x) \to \exists y.(r(x,y) \wedge \forall x.(s(y,x) \to D(x) \vee \exists y.(r(x,y) \wedge \pi_y(D))))]\\
&\forall x.\,[C(x) \to \exists y.(r(x,y) \wedge \forall x.(s(y,x) \to D(x) \vee \exists y.(r(x,y) \wedge D(y))))]
\end{aligned}
$$

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Structural Transformation

- Structural Transformation introduces new concepts for complex subconcepts

$$C \sqsubseteq \exists r.(\forall s.(D \sqcup \exists r.D))$$

Structural Transformation

- Structural Transformation introduces new concepts for complex subconcepts

$$
\begin{aligned}
& C \sqsubseteq \exists r.(\forall s.(D \sqcup \exists r.D)) \\
\rightsquigarrow\ & C \sqsubseteq \exists r.Q_1 \\
& Q_1 \equiv \forall s.(D \sqcup \exists r.D)
\end{aligned}
$$

Structural Transformation

- Structural Transformation introduces new concepts for complex subconcepts

$$
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& C \sqsubseteq \exists r.(\forall s.(D \sqcup \exists r.D)) \\
\rightsquigarrow\ & C \sqsubseteq \exists r.Q_1 \\
& Q_1 \equiv \forall s.(D \sqcup \exists r.D) \\
\rightsquigarrow\ & Q_1 \equiv \forall s.Q_2 \\
& Q_2 \equiv D \sqcup \exists r.D
\end{aligned}
$$

Structural Transformation

- Structural Transformation introduces new concepts for complex subconcepts

$$
\begin{aligned}
& C \sqsubseteq \exists r.(\forall s.(D \sqcup \exists r.D)) \\
\rightsquigarrow\ & C \sqsubseteq \exists r.Q_1 \\
& Q_1 \equiv \forall s.(D \sqcup \exists r.D) \\
\rightsquigarrow\ & Q_1 \equiv \forall s.Q_2 \\
& Q_2 \equiv D \sqcup \exists r.D \\
\rightsquigarrow\ & Q_2 \equiv D \sqcup Q_3 \\
\rightsquigarrow\ & Q_3 \equiv \exists r.D
\end{aligned}
$$

Optimization of Structural Transformation

- do we have to introduce equivalence axioms?

$$A \sqsubseteq \forall r.(\forall r.B)$$

Optimization of Structural Transformation

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\end{aligned}
$$

Optimization of Structural Transformation

- do we have to introduce equivalence axioms?

$$A \sqsubseteq \forall r.(\forall r.B)$$

$$\rightsquigarrow \quad A \sqsubseteq \forall r.Q$$

$$\rightsquigarrow \quad Q \sqsubseteq \forall r.B \qquad \checkmark$$

Optimization of Structural Transformation

- do we have to introduce equivalence axioms?

$$A \sqsubseteq \forall r.(\forall r.B) \qquad\qquad \exists r.(A \sqcap B) \sqsubseteq C$$

$$\rightsquigarrow \; A \sqsubseteq \forall r.Q$$

$$\rightsquigarrow \; Q \sqsubseteq \forall r.B \qquad \checkmark$$

Optimization of Structural Transformation

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$$A \sqsubseteq \forall r.(\forall r.B) \qquad\qquad \exists r.(A \sqcap B) \sqsubseteq C$$

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$$\rightsquigarrow Q \sqsubseteq \forall r.B \quad \checkmark \qquad\qquad Q \sqsubseteq A \sqcap B$$

Optimization of Structural Transformation

- do we have to introduce equivalence axioms?

$$
\begin{array}{lll}
& A \sqsubseteq \forall r.(\forall r.B) & \\
\rightsquigarrow & A \sqsubseteq \forall r.Q & \\
\rightsquigarrow & Q \sqsubseteq \forall r.B & \checkmark
\end{array}
\qquad
\begin{array}{ll}
& \exists r.(A \sqcap B) \sqsubseteq C \\
\rightsquigarrow & \exists r.Q \sqsubseteq C \\
& Q \sqsubseteq A \sqcap B \\
\rightsquigarrow & Q \sqsubseteq A \\
\rightsquigarrow & Q \sqsubseteq B
\end{array}
$$

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Optimization of Structural Transformation

- do we have to introduce equivalence axioms?

$$A \sqsubseteq \forall r.(\forall r.B)$$
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$$\rightsquigarrow \exists r.Q \sqsubseteq C$$
$$Q \sqsubseteq A \sqcap B$$
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Well. Is that correct? Let $\mathcal{A} = \{r(a,b), A(b), B(b), \neg C(a)\}$

Optimization of Structural Transformation

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Well. Is that correct? Let $\mathcal{A} = \{r(a,b), A(b), B(b), \neg C(a)\}$

- In the context of a TBox with the original axiom, the ABox was contradictory.

Optimization of Structural Transformation

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Well. Is that correct? Let $\mathcal{A} = \{r(a,b), A(b), B(b), \neg C(a)\}$

- In the context of a TBox with the original axiom, the ABox was contradictory.
- In the context of a TBox with the rewritten axiom, the ABox is satisfiable

Polarity for Optimized Transformation

- we have to take care if subexpressions occur “positively” or “negatively”
- $A \sqsubseteq B$ is just $\neg A \sqcup B$
- thus, A occurs negatively in the axiom and B positively
- when replacing a negatively occurring concept, we have to use $\sqsupseteq$

Polarity for Optimized Transformation

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$$\exists r.(A \sqcap B) \sqsubseteq C$$

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$$Q \sqsupseteq A \sqcap B$$

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$$C_{\mathcal{T}} = (\forall r.(\neg Q) \sqcup C) \sqcap (\neg A \sqcup \neg B \sqcup Q)$$

Tableau for Unsatisfiability

$$C_{\mathcal{T}} = (\forall r.(\neg Q) \sqcup C) \sqcap (\neg A \sqcup \neg B \sqcup Q)$$
$$\mathcal{A} = \{r(a,b), A(b), B(b), \neg C(a)\}$$

$$L(a) = \{\neg C\}$$
$$L(b) = \{A, B\}$$

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$$L(a) = \{\neg C\} \cup \{C_{\mathcal{T}}, \forall r.(\neg Q) \sqcup C, \neg A \sqcup \neg B \sqcup Q, \forall r.(\neg Q), \neg A\}$$
$$L(b) = \{A, B\}$$

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$$L(b) = \{A, B\} \cup \{\neg Q\} \cup \{C_{\mathcal{T}}, \forall r.(\neg Q) \sqcup C, \neg A \sqcup \neg B \sqcup Q, \forall r.(\neg Q)\}$$

Tableau for Unsatisfiability

$$C_{\mathcal{T}} = (\forall r.(\neg Q) \sqcup C) \sqcap (\neg A \sqcup \neg B \sqcup Q)$$
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Tableau for Unsatisfiability

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$$\cup \xcancel{\{\neg A\}} \cup \xcancel{\{\neg B\}} \cup \xcancel{\{Q\}}$$

no further choice options
$\rightsquigarrow$ the KB is unsatisfiable

Optimized Structural Transformation

- we now want to define structural transformation formally
- we want to introduce just one new concept name per subexpression
- goal: rewrite TBox into an equisatisfiable TBox containing only “simple” axioms

Polarity of Concepts

We define the polarity of a concept C inside a formula as follows:

- C occurs in C positively,
- C occurs positively (negatively) in $\neg D$ if C occurs negatively (positively) in D,
- C occurs positively (negatively) in $D \sqcap E$ or $D \sqcup E$, if C occurs positively (negatively) in D or E,
- C occurs positively (negatively) in $\exists r.D$ or $\forall r.D$, if C occurs positively (negatively) in D,
- C occurs positively (negatively) in $D \sqsubseteq E$, if C occurs positively (negatively) in E or negatively (positively) in D.

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- C occurs positively (negatively) in $D \sqsubseteq E$, if C occurs positively (negatively) in E or negatively (positively) in D.

A concept occurs positively (negatively) in an ($\mathcal{ALC}$) TBox $\mathcal{T}$, if C occurs positively (negatively) in an axiom in $\mathcal{T}$.

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- C occurs positively (negatively) in $D \sqsubseteq E$, if C occurs positively (negatively) in E or negatively (positively) in D.

A concept occurs positively (negatively) in an ($\mathcal{ALC}$) TBox $\mathcal{T}$, if C occurs positively (negatively) in an axiom in $\mathcal{T}$.
$\leadsto$ a concept may occur both positively and negatively in an axiom

Optimized Transformation with Polarity

Let $\mathcal{T}$ be an $\mathcal{ALC}$ TBox. For every concept (sub-)expression C in $\mathcal{T}$, we introduce a fresh atomic concept A_C and define the function st(C) as follows:

$$\begin{aligned} \text{st}(A) &= A & \text{st}(\neg C) &= \neg A_C & \text{st}(\exists r.C) &= \exists r.A_C \\ \text{st}(\top) &= \top & \text{st}(C \sqcap D) &= A_C \sqcap A_D & \text{st}(\forall r.C) &= \forall r.A_C \\ \text{st}(\bot) &= \bot & \text{st}(C \sqcup D) &= A_C \sqcup A_D \end{aligned}$$

The result of the structural transformation of a TBox $\mathcal{T}$ is a TBox $\mathcal{T}'$ with the following axioms:

- $A_C \sqsubseteq$ st(C) for every concept C occurring positively in $\mathcal{T}$,
- st(C) $\sqsubseteq A_C$ for every concept C occurring negatively in $\mathcal{T}$,
- $A_C \sqsubseteq A_D$ for every GCI $C \sqsubseteq D \in \mathcal{T}$.

Simplification of Axioms

we can now use known equivalences to simplify the axioms further:

$$\{C \sqsubseteq D \sqcap E\} \equiv \{C \sqsubseteq D, C \sqsubseteq E\}$$
$$\{C \sqcup D \sqsubseteq E\} \equiv \{C \sqsubseteq E, D \sqsubseteq E\}$$

Result of the Structural Transformation

by virtue of structural transformation and the known equivalences, we can rewrite an $\mathcal{ALC}$ KB into an equisatisfiable one that contains only axioms of the following shape (A and B being atomic):

$$
\begin{aligned}
A_1 \sqcap \ldots \sqcap A_n &\sqsubseteq B_1 \sqcup \ldots \sqcup B_m \\
A &\sqsubseteq \exists r.B \\
A &\sqsubseteq \forall r.B \\
\exists r.A &\sqsubseteq B \\
\forall r.A &\sqsubseteq B
\end{aligned}
$$

Example: Optimized Structural Transformation

$$\overbrace{\underbrace{\forall r.(\overbrace{A \sqcap D}^{C_1})}_{C_3} \sqcap A \sqcap \underbrace{\exists s.(\overbrace{A \sqcap D}^{C_1})}_{C_4}}^{C_7} \sqsubseteq \underbrace{B \sqcup (\overbrace{\underbrace{\exists r.(\overbrace{C \sqcap E}^{C_2})}_{C_5} \sqcap F}^{C_8}) \sqcup \underbrace{\forall r.A}_{C_6}}_{C_9}$$

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$$A \sqcap D \sqsubseteq C_1$$

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$$A \sqcap D \sqsubseteq C_1$$
$$C_2 \sqsubseteq C \sqcap E$$

Example: Optimized Structural Transformation

$$\overbrace{\underbrace{\forall r.(\overbrace{A \sqcap D}^{C_1})}_{C_3} \sqcap A \sqcap \underbrace{\exists s.(\overbrace{A \sqcap D}^{C_1})}_{C_4}}^{C_7} \sqsubseteq \underbrace{B \sqcup (\overbrace{\underbrace{\exists r.(\overbrace{C \sqcap E}^{C_2})}_{C_5}}^{C_8} \sqcap F) \sqcup \underbrace{\forall r.A}_{C_6}}_{C_9}$$

$$\begin{aligned} A \sqcap D &\sqsubseteq C_1 \\ C_2 &\sqsubseteq C \sqcap E \\ \forall r.C_1 &\sqsubseteq C_3 \end{aligned}$$

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$$\begin{aligned} A \sqcap D &\sqsubseteq C_1 \\ C_2 &\sqsubseteq C \sqcap E \\ \forall r.C_1 &\sqsubseteq C_3 \\ \exists s.C_1 &\sqsubseteq C_4 \end{aligned}$$

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$$C_6 \sqsubseteq \forall r.A$$

Example: Optimized Structural Transformation

$$\overbrace{\underbrace{\forall r.(\overbrace{A \sqcap D}^{C_1})}_{C_3} \sqcap A \sqcap \underbrace{\exists s.(\overbrace{A \sqcap D}^{C_1})}_{C_4}}^{C_7} \sqsubseteq \underbrace{B \sqcup (\overbrace{\underbrace{\exists r.(\overbrace{C \sqcap E}^{C_2})}_{C_5} \sqcap F}^{C_8}) \sqcup \underbrace{\forall r.A}_{C_6}}_{C_9}$$

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$$\begin{aligned} C_6 &\sqsubseteq \forall r.A \\ C_3 \sqcap A \sqcap C_4 &\sqsubseteq C_7 \end{aligned}$$

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Example: Optimized Structural Transformation

we still can apply the simplification rules:

$$\overbrace{\underbrace{\forall r.(\overbrace{A \sqcap D}^{C_1})}_{C_3} \sqcap A \sqcap \underbrace{\exists s.(\overbrace{A \sqcap D}^{C_1})}_{C_4}}^{C_7} \sqsubseteq \underbrace{B \sqcup (\overbrace{\underbrace{\exists r.(\overbrace{C \sqcap E}^{C_2})}_{C_5} \sqcap F}^{C_8}) \sqcup \underbrace{\forall r.A}_{C_6}}_{C_9}$$

$$\begin{aligned} A \sqcap D &\sqsubseteq C_1 \\ C_2 &\sqsubseteq C \\ \forall r.C_1 &\sqsubseteq C_3 \\ \exists s.C_1 &\sqsubseteq C_4 \\ C_5 &\sqsubseteq \exists r.C_3 \\ C_2 &\sqsubseteq E \end{aligned} \qquad \begin{aligned} C_6 &\sqsubseteq \forall r.A \\ C_3 \sqcap A \sqcap C_4 &\sqsubseteq C_7 \\ C_8 &\sqsubseteq C_5 \\ C_9 &\sqsubseteq B \sqcup C_8 \sqcup C_6 \\ C_7 &\sqsubseteq C_9 \\ C_8 &\sqsubseteq F \end{aligned}$$

Agenda

- Motivation
- Recap: Translation into FOL
- Structural Transformation
- Translation into Clauses
- Summary

Translation into Clauses

a TBox with simplified axioms can now be translated into clauses (written as rules):

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$$A_1(x) \wedge \ldots \wedge A_n(x) \rightarrow B_1(x) \vee \ldots \vee B_m(x)$$

If $m = 0$, the rule head contains $\bot(x)$.

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$$A_1 \sqcap \ldots \sqcap A_n \sqsubseteq B_1 \sqcup \ldots \sqcup B_m$$
$$A_1(x) \land \ldots \land A_n(x) \to B_1(x) \lor \ldots \lor B_m(x)$$

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$$A \sqsubseteq \exists r.B$$

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$$A \sqsubseteq \exists r.B \qquad\qquad A(x) \rightarrow (\exists r.B)(x)$$

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$$A \sqsubseteq \exists r.B \qquad A(x) \rightarrow (\exists r.B)(x)$$
$$A \sqsubseteq \forall r.B$$

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$A \sqsubseteq \exists r.B$	$A(x) \rightarrow (\exists r.B)(x)$
$A \sqsubseteq \forall r.B$	$A(x) \wedge r(x, y) \rightarrow B(y)$

Translation into Clauses

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$$A \sqsubseteq \exists r.B \qquad A(x) \to (\exists r.B)(x)$$
$$A \sqsubseteq \forall r.B \qquad A(x) \land r(x,y) \to B(y)$$
$$\exists r.A \sqsubseteq B$$

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If $m = 0$, the rule head contains $\bot(x)$.

$$
\begin{aligned}
A &\sqsubseteq \exists r.B & A(x) &\rightarrow (\exists r.B)(x) \\
A &\sqsubseteq \forall r.B & A(x) \wedge r(x, y) &\rightarrow B(y) \\
\exists r.A &\sqsubseteq B & r(x, y) \wedge A(y) &\rightarrow B(x)
\end{aligned}
$$

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$A \sqsubseteq \exists r.B$	$A(x) \to (\exists r.B)(x)$
$A \sqsubseteq \forall r.B$	$A(x) \land r(x,y) \to B(y)$
$\exists r.A \sqsubseteq B$	$r(x,y) \land A(y) \to B(x)$
$\forall r.A \sqsubseteq B$	

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$$
\begin{array}{rcl}
A \sqsubseteq \exists r.B & & A(x) \rightarrow (\exists r.B)(x) \\
A \sqsubseteq \forall r.B & & A(x) \wedge r(x,y) \rightarrow B(y) \\
\exists r.A \sqsubseteq B & & r(x,y) \wedge A(y) \rightarrow B(x) \\
\forall r.A \sqsubseteq B & & \rightarrow (\exists r.\neg A)(x) \vee B(x)
\end{array}
$$

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Summary

- axioms can be simplified via structural transformation
- simplified axioms can be expressed as rules
- existential quantification in rule heads allowed
- often these rules allow us to avoid nondeterminism