TECHNISCHE UNIVERSITÄT DRESDEN

FOUNDATIONS OF SEMANTIC WEB TECHNOLOGIES

Semantics of RDF(S)

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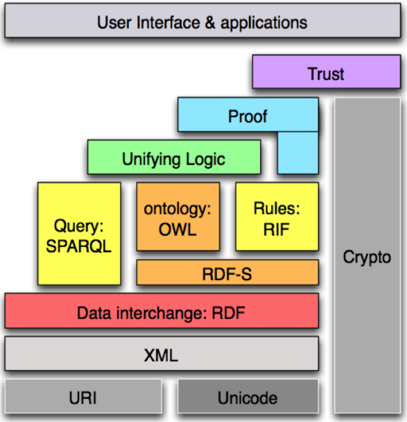
Dresden, 25 April 2014

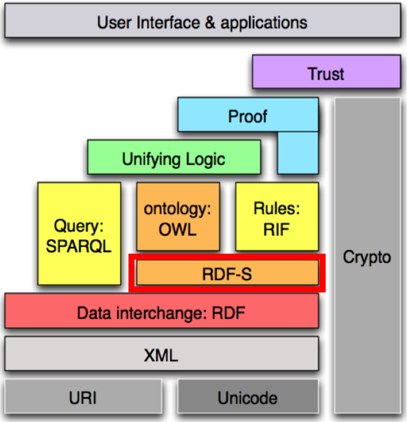
DRESDEN concept Exzellenz aus Wissenschaft und Kultur

Content

RDF Schema


User Interface & applications
Trust
Proof
Unifying Logic
Query: SPARQL
ontology: OWL
Rules: RIF
RDF-S
Crypto
Data interchange: RDF
XML
URI
Unicode

RDF Schema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Agenda

1. Motivation and Considerations
2. Simple Entailment
3. RDF Entailment
4. RDFS Entailment
5. Downsides of RDF(S)

Agenda

Why Formal Semantics?

- after introduction of RDF(S), criticism of tool developers: different tools were incompatible (despite the existing specification)
- e.g. triple stores:
  - same RDF document
  - same SPARQL query
  - different answers
- thus a model-theoretic formal semantics was defined for RDF(S)

How is RDF(S) Linked to a Logic?

- to start with: what are the sentences in RDF(S)?
  - basic elements (vocabulary V): IRIs, bnodes and literals (these are not sentences themselves)
  - every triple

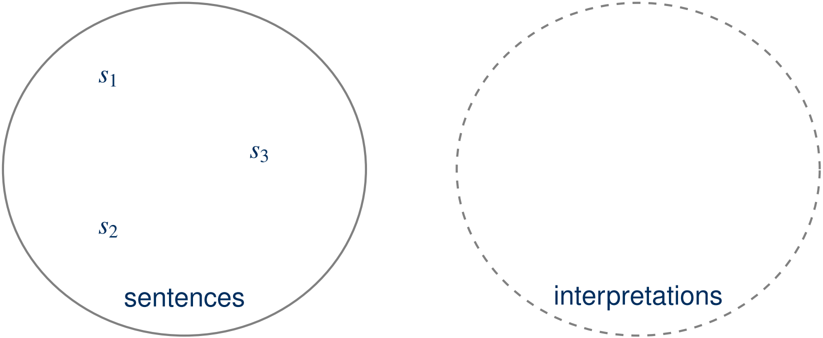
    $$(s, p, o) \in (\mathrm{IRI} \cup \mathrm{bnode}) \times \mathrm{IRI} \times (\mathrm{IRI} \cup \mathrm{bnode} \cup \mathrm{literal})$$

    is a sentence
  - every finite set of triples (denoted: graph) is a sentence

How is RDF(S) Linked to a Logic?

What is the semantics?

- consequence relation that defines when an RDF(S) graph G' logically follows from an RDF(S) graph G, i.e. $G \models G'$
- model-theoretic semantics: we define a set of interpretations and stipulate under which conditions an interpretation is a model of a graph



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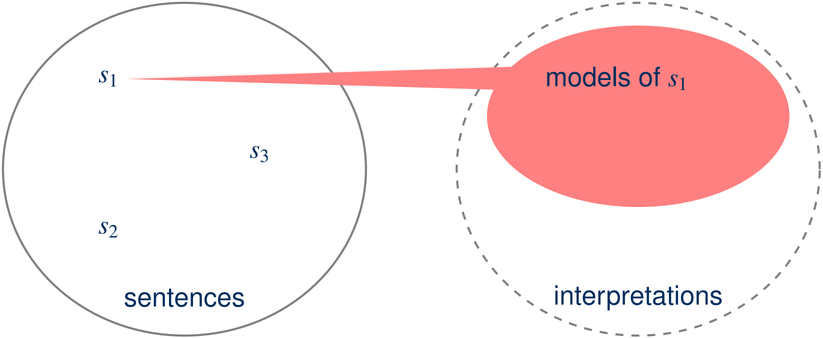

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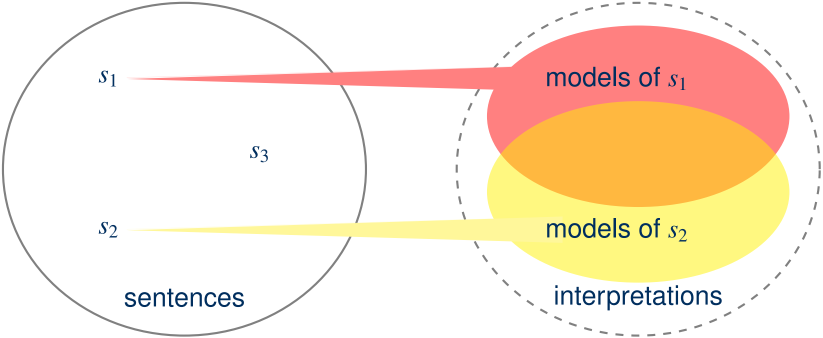

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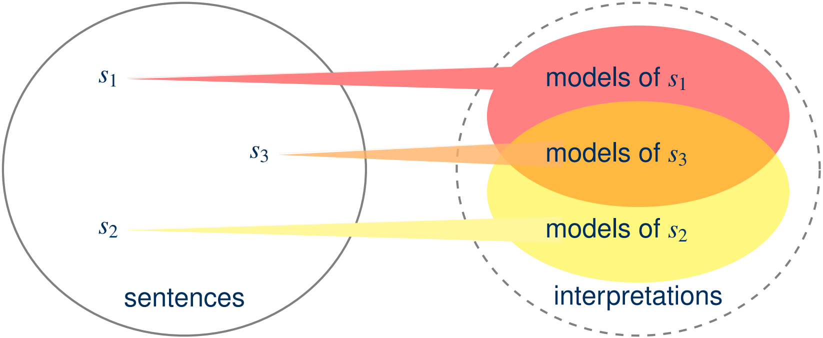

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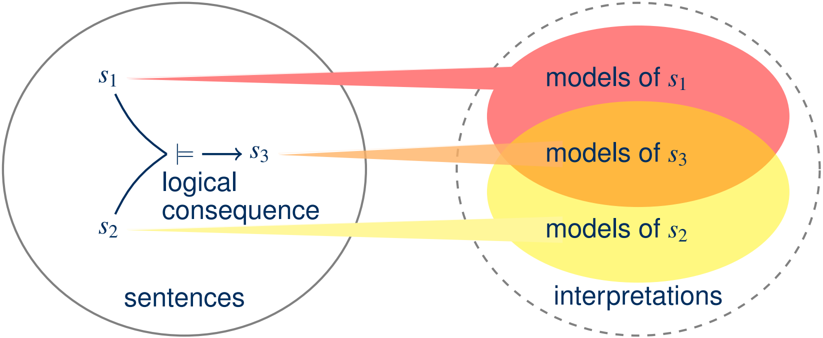

Semantics of RDF(S)

- we proceed stepwise:

simple interpretations

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RDF interpretations

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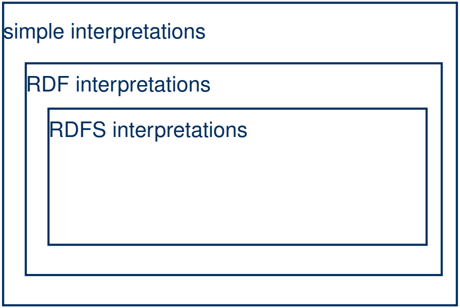

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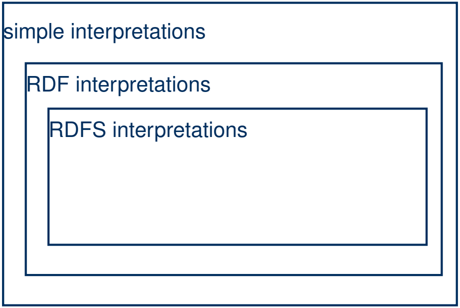


- the more we restrict the set of interpretations, the stronger the consequence relation becomes

Agenda

Semantics of the Simple Entailment

Definition (Simple Interpretation)

A simple Interpretation $\mathcal{I}$ for a vocabulary V consists of

- IR, a non-empty set of resources, also referred to as domain, with
- LV $\subseteq$ IR the set of literal values, that contains (at least) all untyped literals from V, and
- IP, the set of properties of $\mathcal{I}$;
- $\mathrm{I_S}$, a function, mapping IRIs from V to the union of the sets IR and IP, i.e., $\mathrm{I_S} : \mathrm{V} \to \mathrm{IR} \cup \mathrm{IP}$,
- $\mathrm{I_{EXT}}$, a function, mapping every property to a set of pairs from IR, i.e., $\mathrm{I_{EXT}} : \mathrm{IP} \to 2^{\mathrm{IR} \times \mathrm{IR}}$ and
- $\mathrm{I_L}$, a function mapping typed literals from V into the set IR of resources.

Semantics of the Simple Entailment

- IR is also called domain or universe of discourse of $\mathcal{I}$
- $\mathsf{I}_{\mathsf{EXT}}(p)$ is also referred to as the extension of the property p

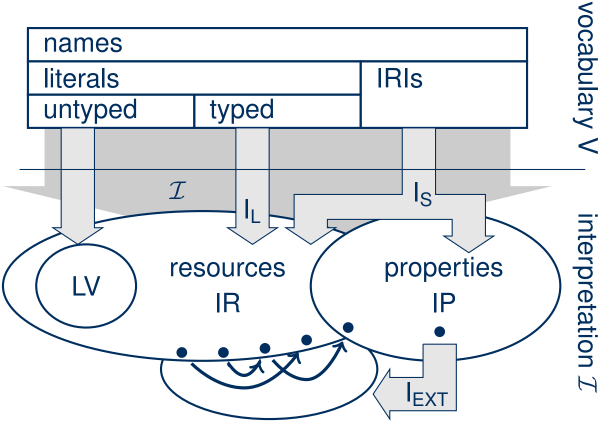
Definition (interpretation function)

based on I_L and I_S, we define $\cdot^\mathcal{I}$ as follows:

- every untyped literal `"a"` is mapped to a : $(\texttt{"a"})^\mathcal{I} = a$
- every untyped literal with language information `"a"@t` is mapped to the pair $\langle \mathsf{a}, \mathsf{t} \rangle$, that is: $(\texttt{"a"@t})^\mathcal{I} = \langle \mathsf{a}, \mathsf{t} \rangle$,
- every typed literal l is mapped to $\mathsf{I}_\mathsf{L}(l)$, that is: $l^\mathcal{I} = \mathsf{I}_\mathsf{L}(l)$ and
- every IRI i is mapped to $\mathsf{I}_\mathsf{S}(i)$, hence: $i^\mathcal{I} = \mathsf{I}_\mathsf{S}(i)$.

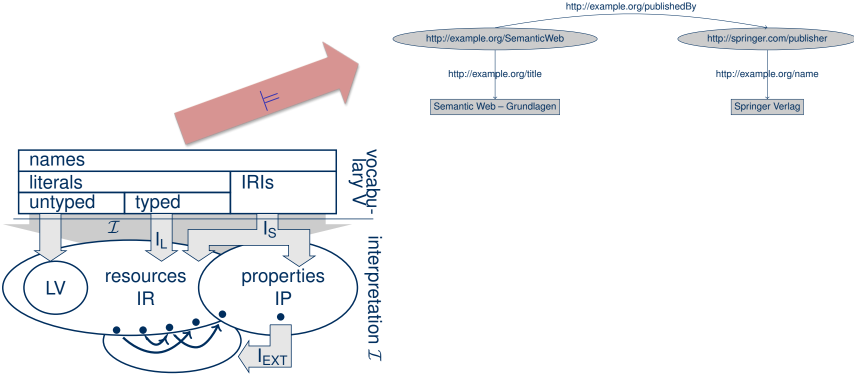
Semantics of the Simple Entailment

Interpretation (schematic):



Semantics of the Simple Entailment

- Question: When is a given interpretation a model of a graph?



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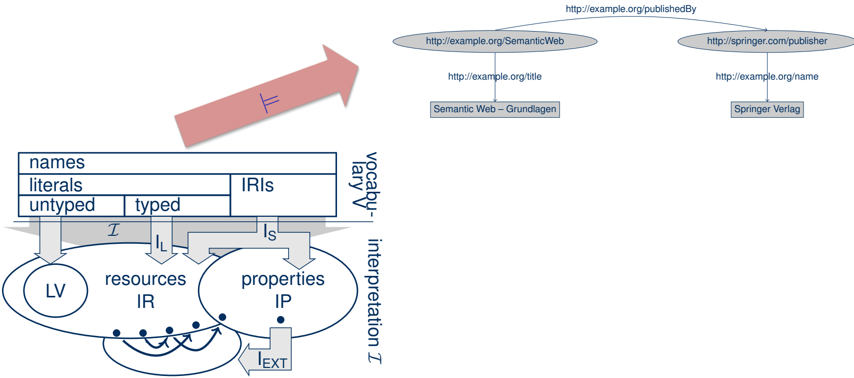
- . . . if it is a model for every triple of the graph!



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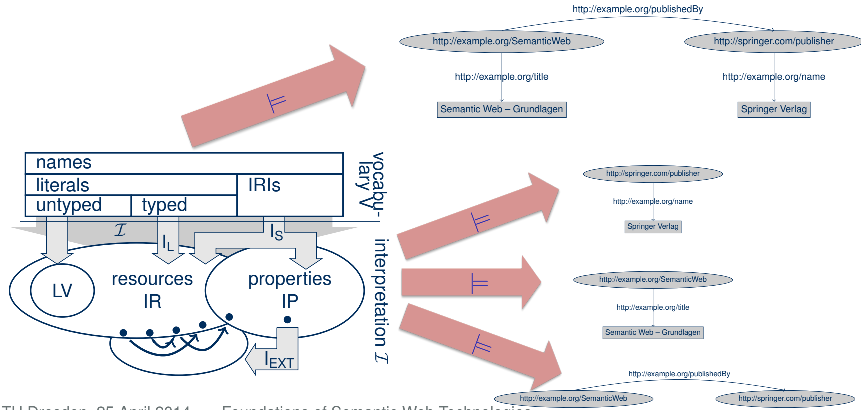

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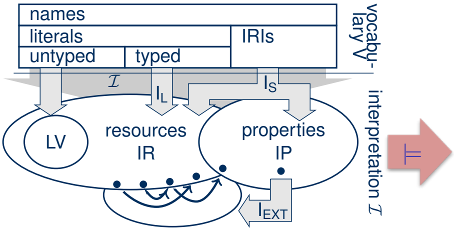
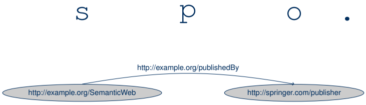
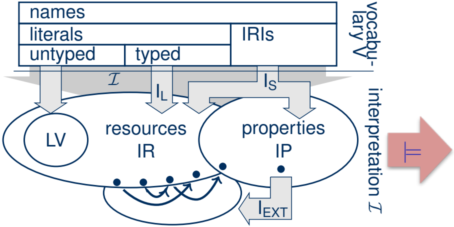
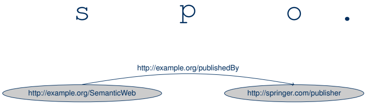




Semantics of the Simple Entailment

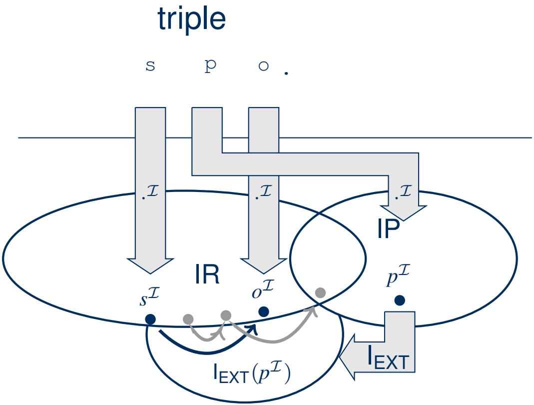
- Question: When is a given interpretation a model of a triple?
- …if all subject, predicate, and object are contained in V and additionally $\langle s^{\mathcal{I}}, o^{\mathcal{I}} \rangle \in \mathrm{I_{EXT}}(p^{\mathcal{I}})$ holds





Semantics of Simple Entailment

schematically:

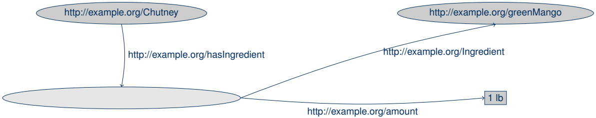


Semantics of Simple Entailment

- . . . oops, we forgot the bnodes!
- let A be a function mapping all bnodes to elements of IR
- given an interpretation $\mathcal{I}$, let $\mathcal{I} + A$ behave just like $\mathcal{I}$ on the vocabulary, and additionally for every bnode `_:label` let $(\texttt{\_:label})^{\mathcal{I}+A} = A(\texttt{\_:label})$
- now, an interpretation $\mathcal{I}$ is a model of an RDF graph G, if there exists an A such that all triples are satisfied w.r.t. $\mathcal{I} + A$

Simple Interpretations: Example

given graph G:

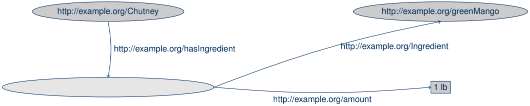


and interpretation $\mathcal{I}$:

$$
\begin{array}{rcl}
IR & = & \{c, g, h, z, l, m, \text{1 lb}\} \\
IP & = & \{h, z, m\} \\
LV & = & \{\text{1 lb}\} \\
I_{EXT} & = & h \mapsto \{\langle c, l\rangle\} \\
 & & z \mapsto \{\langle l, g\rangle\} \\
 & & m \mapsto \{\langle l, \text{1 lb}\rangle\}
\end{array}
\qquad
\begin{array}{rcll}
I_S & = & \texttt{ex:Chutney} & \mapsto c \\
 & & \texttt{ex:greenMango} & \mapsto g \\
 & & \texttt{ex:hasIngredient} & \mapsto h \\
 & & \texttt{ex:ingredient} & \mapsto z \\
 & & \texttt{ex:amount} & \mapsto m \\
I_L & & \text{is the “empty function”} &
\end{array}
$$

Is $\mathcal{I}$ a model of G?

Simple Interpretations: Example



$$
\begin{aligned}
\mathsf{IR} &= \{c, g, h, z, l, m, \text{1 lb}\} \\
\mathsf{IP} &= \{h, z, m\} \\
\mathsf{LV} &= \{\text{1 lb}\} \\
\mathsf{I}_{\mathsf{EXT}} &= h \mapsto \{\langle c, l\rangle\} \\
& \quad\; z \mapsto \{\langle l, g\rangle\} \\
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& \texttt{ex:ingredient} \mapsto z \\
& \texttt{ex:amount} \mapsto m \\
\mathsf{I}_{\mathsf{L}} \; & \text{is the “empty function”}
\end{aligned}
$$

- If we pick $A\colon \texttt{\_:id1} \mapsto l$, then we get

$$
\begin{aligned}
\langle \texttt{ex:Chutney}^{\mathcal{I}+A}, \texttt{\_:id1}^{\mathcal{I}+A}\rangle &= \langle c, l\rangle \in \mathsf{I}_{\mathsf{EXT}}(h) = \mathsf{I}_{\mathsf{EXT}}(\texttt{ex:hasIngredient}^{\mathcal{I}+A}) \\
\langle \texttt{\_:id1}^{\mathcal{I}+A}, \texttt{ex:greenMango}^{\mathcal{I}+A}\rangle &= \langle l, g\rangle \in \mathsf{I}_{\mathsf{EXT}}(z) = \mathsf{I}_{\mathsf{EXT}}(\texttt{ex:ingredient}^{\mathcal{I}+A}) \\
\langle \texttt{\_:id1}^{\mathcal{I}+A}, \texttt{"1 lb"}^{\mathcal{I}+A}\rangle &= \langle l, \texttt{1 lb}\rangle \in \mathsf{I}_{\mathsf{EXT}}(m) = \mathsf{I}_{\mathsf{EXT}}(\texttt{ex:amount}^{\mathcal{I}+A})
\end{aligned}
$$

- Therefore, $\mathcal{I}$ is a model of G.

Simple Entailment

- definition of simple interpretations fixes the notion of simple entailment for RDF graphs
- question: how can this (abstractly defined) semantics be turned something computable
- answer: deduction rules

Simple Entailment

deduction rules for simple entailment:

$$\frac{\texttt{u} \quad \texttt{a} \quad \texttt{x}\ \texttt{.}}{\texttt{u} \quad \texttt{a} \quad \texttt{\_:n}\ \texttt{.}}\ \text{se1}$$

$$\frac{\texttt{u} \quad \texttt{a} \quad \texttt{x}\ \texttt{.}}{\texttt{\_:n} \quad \texttt{a} \quad \texttt{x}\ \texttt{.}}\ \text{se2}$$

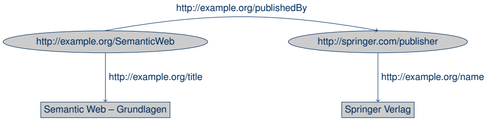
- precondition for applying this rule: the bnode has not already been associated with another IRI or literal

Simple Entailment

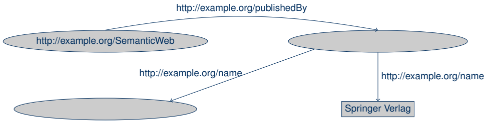
Theorem

A graph G_2 is simply entailed by a graph G_1 if G_1 can be extended to a graph G_1' by applying the rules se1 and se2 such that G_2 is contained in G_1'.

Example.: the graph



simply entails



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Agenda

RDF interpretations

RDF interpretations are specific simple interpretations, where additional conditions are imposed on the URIs of the RDF vocabulary

```
rdf:type  rdf:Property  rdf:XMLLiteral  rdf:nil
rdf:List  rdf:Statement  rdf:subject  rdf:predicate
rdf:object  rdf:first  rdf:rest  rdf:Seq  rdf:Bag
rdf:Alt  rdf:_1  rdf:_2  ...
```

inorder to realize their intended semantics.

Conditions for RDF Interpretations

An RDF interpretation for a vocabulary V is a simple interpretation for the vocabulary $V \cup V_{RDF}$ that additionally satisfies the following conditions:

1. $x \in IP$ exactly if $\langle x, \texttt{rdf:Property}^{\mathcal{I}} \rangle \in I_{EXT}(\texttt{rdf:type}^{\mathcal{I}})$.

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"For every triple predicate we can infer that it is an member of the class of all properties."

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"For every triple predicate we can infer that it is an member of the class of all properties."

$$\frac{\texttt{u a y}}{\texttt{a rdf:type rdf:Property}} \ \text{rdf1}$$

Conditions for RDF Interpretations

2. If `"s"^^rdf:XMLLiteral` is contained in V and `s` is a well-formed XML literal, then
   - $I_L(\texttt{"s"\^{}\^{}rdf:XMLLiteral})$ is the XML value of `s`;
   - $I_L(\texttt{"s"\^{}\^{}rdf:XMLLiteral}) \in LV$;
   - $\langle I_L(\texttt{"s"\^{}\^{}rdf:XMLLiteral}), \texttt{rdf:XMLLiteral}^{\mathcal{I}} \rangle \in I_{EXT}(\texttt{rdf:type}^{\mathcal{I}})$

$$\frac{\texttt{u a l}}{\texttt{l rdf:type rdf:XMLLiteral}} \quad ??? \quad \text{für } \texttt{l} \text{ a well-formed XML literal}$$

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   - $\langle I_L(\texttt{"s"\^{}\^{}rdf:XMLLiteral}), \texttt{rdf:XMLLiteral}^{\mathcal{I}}\rangle \in I_{EXT}(\texttt{rdf:type}^{\mathcal{I}})$

$$\frac{\texttt{u a l}}{\texttt{l rdf:type rdf:XMLLiteral}} \quad ??? \quad \text{für } \texttt{l} \text{ a well-formed XML literal}$$

Oops, literals must not occur in subject position!

Conditions for RDF Interpretations

2. If `"s"^^rdf:XMLLiteral` is contained in V and `s` is a well-formed XML literal, then
   - I_L(`"s"^^rdf:XMLLiteral`) is the XML value of `s`;
   - I_L(`"s"^^rdf:XMLLiteral`) $\in$ LV;
   - $\langle I_L(\texttt{"s"\^{}\^{}rdf:XMLLiteral}), \texttt{rdf:XMLLiteral}^{\mathcal{I}}\rangle \in I_{EXT}(\texttt{rdf:type}^{\mathcal{I}})$

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2. If `"s"^^rdf:XMLLiteral` is contained in V and `s` is a well-formed XML literal, then
   - $I_L($`"s"^^rdf:XMLLiteral`$)$ is the XML value of `s`;
   - $I_L($`"s"^^rdf:XMLLiteral`$) \in LV$;
   - $\langle I_L(\texttt{"s"\^{}\^{}rdf:XMLLiteral}), \texttt{rdf:XMLLiteral}^{\mathcal{I}}\rangle \in I_{EXT}(\texttt{rdf:type}^{\mathcal{I}})$

$$\frac{\texttt{u a l}}{\texttt{u a \_:n}}\ \text{lg}$$

`l` a literal, `_:n` not bound otherwise

$$\frac{\texttt{u a \_:n}}{\texttt{\_:n rdf:type rdf:XMLLiteral}}\ \text{rdf2}$$

If rule lg has assigned `_:n` to the XML Literal `l`

Conditions for RDF Interpretations

3. If `"s"^^rdf:XMLLiteral` is contained in V and `s` is an ill-formed XML literal, then
   - $I_L(\texttt{"s"\^{}\^{}rdf:XMLLiteral}) \notin LV$ and
   - $\langle I_L(\texttt{"s"\^{}\^{}rdf:XMLLiteral}), \texttt{rdf:XMLLiteral}^{\mathcal{I}}\rangle \notin I_{EXT}(\texttt{rdf:type}^{\mathcal{I}})$.

RDF Interpretations

- Note: x is a property exactly if it is linked to the resource denoted by `rdf:Property` via the `rdf:type` property (this has the direct consequence that in every RDF interpretation holds $\mathrm{IP} \subseteq \mathrm{IR}$).
- The value space of the `rdf:XMLLiteral` datatype contains for every well-formed XML string exactly one so-called XML value. The RDF specs only stipulate that this value is neither an XML string itself nor a data value of any XML Schema datatype nor a Unicode string.

RDF Interpretations

- additional requirement: every RDF interpretation must be a model of the following "axiomatic" triples:

```
rdf:type        rdf:type  rdf:Property .
rdf:subject     rdf:type  rdf:Property .
rdf:predicate   rdf:type  rdf:Property .
rdf:object      rdf:type  rdf:Property .
rdf:first       rdf:type  rdf:Property .
rdf:rest        rdf:type  rdf:Property .
rdf:value       rdf:type  rdf:Property .
rdf:_1          rdf:type  rdf:Property .
rdf:_2          rdf:type  rdf:Property .
...             rdf:type  rdf:Property .
rdf:nil         rdf:type  rdf:List .
```

$$\frac{}{\texttt{u a x}}\ \text{rdfax}$$

every axiomatic triple "u a x ." can always be derived

RDF Entailment

- Theorem: A graph G_2 is RDF-entailed by a graph G_1, if there is a graph G_1', such that
  - G_1' can be derived from G_1 via lg, rdf1, rdf2 and rdfax and
  - G_2 is simply entailed by G_1'.

- note: two-stage deduction process

Agenda

RDFS Interpretations

. . . RDFS interpretations are specific RDF interpretations, where additional constraints are imposed for the URIs of the RDFS vocabulary

```
rdfs:domain       rdfs:range            rdfs:Resource
rdfs:Literal      rdfs:Datatype         rdfs:Class
rdfs:subClassOf   rdfs:subPropertyOf    rdfs:Container
rdfs:member       rdfs:ContainerMembershipProperty
rdfs:comment      rdfs:seeAlso          rdfs:isDefinedBy
rdfs:label
```

such that the intended semantics of these URIs is realized.

RDFS Interpretations

- for the sake of easier representation, we introduce – given an interpretation $\mathcal{I}$ – a function $\mathsf{I}_{\mathsf{CEXT}}$ that maps resources to sets of resources (thus: $\mathsf{I}_{\mathsf{CEXT}} : \mathsf{IR} \to 2^{\mathsf{IR}}$) by letting $\mathsf{I}_{\mathsf{CEXT}}(y)$ contain exactly those elements x, for which $\langle x, y \rangle$ is contained in $\mathsf{I}_{\mathsf{EXT}}(\texttt{rdf:type}^{\mathcal{I}})$. We call $\mathsf{I}_{\mathsf{CEXT}}(y)$ the (class) extension of y.
- moreover, we let IC be the extension of the specific IRI `rdfs:Class`, hence: $\mathsf{IC} = \mathsf{I}_{\mathsf{CEXT}}(\texttt{rdfs:Class}^{\mathcal{I}})$.
- note: both $\mathsf{I}_{\mathsf{CEXT}}$ as well as IC are fully determined by $\cdot^{\mathcal{I}}$ and $\mathsf{I}_{\mathsf{EXT}}$.

RDFS Interpretations

An RDFS interpretation for a vocabulary V is an RDF interpretation for the vocabulary $V \cup V_{RDFS}$, that additionally satisfies the following criteria:

- $IR = I_{CEXT}(\texttt{rdfs:Resource}^{\mathcal{I}})$
  Every resource is of type `rdfs:Resource`.
- $LV = I_{CEXT}(\texttt{rdfs:Literal}^{\mathcal{I}})$
  Every untyped and every well-formed typed literal is of type `rdfs:Literal`.
- If $\langle x, y \rangle \in I_{EXT}(\texttt{rdfs:domain}^{\mathcal{I}})$ and $\langle u, v \rangle \in I_{EXT}(x)$, then $u \in I_{CEXT}(y)$.
  If the property `rdfs:domain` connects x with y and the property x connects the resources u and v, then u is of type y.

RDFS Interpretations

- If $\langle x, y\rangle \in \mathsf{I}_{\mathsf{EXT}}(\texttt{rdfs:range}^{\mathcal{I}})$ and $\langle u, v\rangle \in \mathsf{I}_{\mathsf{EXT}}(x)$, then $v \in \mathsf{I}_{\mathsf{CEXT}}(y)$.
  If the property `rdfs:range` connects x with y and the property x connects the resources u and v, then v is of type y.
- $\mathsf{I}_{\mathsf{EXT}}(\texttt{rdfs:subPropertyOf}^{\mathcal{I}})$ is reflexive and transitive on IP.
  The `rdfs:subPropertyOf` property connects every property with itself. Moreover, if `rdfs:subPropertyOf` connects a property x with a property y and additionally y with a property z, then `rdfs:subPropertyOf` also connects x directly with z.

RDFS Interpretations

- If $\langle x, y \rangle \in \mathsf{I}_{\mathsf{EXT}}(\texttt{rdfs:subPropertyOf}^{\mathcal{I}})$, then $x, y \in \mathsf{IP}$ and $\mathsf{I}_{\mathsf{EXT}}(x) \subseteq \mathsf{I}_{\mathsf{EXT}}(y)$.
  If `rdfs:subPropertyOf` connects x with y, then both x and y are properties every pair of resources contained in the extension of x is also contained in the extension of y.
- If $x \in \mathsf{IC}$, then $\langle x, \texttt{rdfs:Resource}^{\mathcal{I}} \rangle \in \mathsf{I}_{\mathsf{EXT}}(\texttt{rdfs:subClassOf}^{\mathcal{I}})$.
  If x represents a class, then it has to be a subclass of the class of all resources, i.e., the pair containing x and `rdfs:Resource` is in the extension of `rdfs:subClassOf`.

RDFS Interpretations

- If $\langle x, y\rangle \in \mathsf{I}_{\mathsf{EXT}}(\texttt{rdfs:subClassOf}^{\mathcal{I}})$, then $x, y \in$ IC and $\mathsf{I}_{\mathsf{CEXT}}(x) \subseteq \mathsf{I}_{\mathsf{CEXT}}(y)$.
  If x and y are connected via the `rdfs:subClassOf` property, then both x and y are classes and the (class) extension of x is a subset of the (class) extension of y.
- $\mathsf{I}_{\mathsf{EXT}}(\texttt{rdfs:subClassOf}^{\mathcal{I}})$ is reflexive and transitive on IC.
  The `rdfs:subClassOf` property connects every class to itself.
  Moreover, whenever this property connects a class x with a class y and a class y with a class z, then it also directly connects x with z.

RDFS Interpretations

- If $x \in \mathsf{I}_{\mathsf{CEXT}}(\texttt{rdfs:ContainerMembershipProperty}^{\mathcal{I}})$, then $\langle x, \texttt{rdfs:member}^{\mathcal{I}}\rangle \in \mathsf{I}_{\mathsf{EXT}}(\texttt{rdfs:subPropertyOf}^{\mathcal{I}})$.
  If x is a property of the type `rdfs:ContainerMembershipProperty`, then it is `rdfs:subPropertyOf`-connected with the property `rdfs:member`.
- If $x \in \mathsf{I}_{\mathsf{CEXT}}(\texttt{rdfs:Datatype}^{\mathcal{I}})$, then $\langle x, \texttt{rdfs:Literal}^{\mathcal{I}}\rangle \in \mathsf{I}_{\mathsf{EXT}}(\texttt{rdfs:subClassOf}^{\mathcal{I}})$.
  If some x is typed as element of the class `rdfs:Datatype`, then it must be a subclass of the class of all literal values (denoted by `rdfs:Literal`).
- . . . additionally we require satisfaction of the following axiomatic triples:

RDFS Interpretations

```
rdf:type            rdfs:domain    rdfs:Resource .
rdfs:domain         rdfs:domain    rdf:Property .
rdfs:range          rdfs:domain    rdf:Property .
rdfs:subPropertyOf  rdfs:domain    rdf:Property .
rdfs:subClassOf     rdfs:domain    rdfs:Class .
rdf:subject         rdfs:domain    rdf:Statement .
rdf:predicate       rdfs:domain    rdf:Statement .
rdf:object          rdfs:domain    rdf:Statement .
rdfs:member         rdfs:domain    rdfs:Resource .
rdf:first           rdfs:domain    rdf:List .
rdf:rest            rdfs:domain    rdf:List .
rdfs:seeAlso        rdfs:domain    rdfs:Resource .
rdfs:isDefinedBy    rdfs:domain    rdfs:Resource .
rdfs:comment        rdfs:domain    rdfs:Resource .
rdfs:label          rdfs:domain    rdfs:Resource .
rdf:value           rdfs:domain    rdfs:Resource .
```

RDFS Interpretations

```
rdf:type            rdfs:range          rdfs:Class .
rdfs:domain         rdfs:range          rdfs:Class .
rdfs:range          rdfs:range          rdfs:Class .
rdfs:subPropertyOf  rdfs:range          rdf:Property .
rdfs:subClassOf     rdfs:range          rdfs:Class .
rdf:subject         rdfs:range          rdfs:Resource .
rdf:predicate       rdfs:range          rdfs:Resource .
rdf:object          rdfs:range          rdfs:Resource .
rdfs:member         rdfs:range          rdfs:Resource .
rdf:first           rdfs:range          rdfs:Resource .
rdf:rest            rdfs:range          rdf:List .
rdfs:seeAlso        rdfs:range          rdfs:Resource .
rdfs:isDefinedBy    rdfs:range          rdfs:Resource .
rdfs:comment        rdfs:range          rdfs:Literal .
rdfs:label          rdfs:range          rdfs:Literal .
rdf:value           rdfs:range          rdfs:Resource .
```

RDFS Interpretations

```
rdfs:ContainerMembershipProperty
                    rdfs:subClassOf     rdf:Property .
rdf:Alt             rdfs:subClassOf     rdfs:Container .
rdf:Bag             rdfs:subClassOf     rdfs:Container .
rdf:Seq             rdfs:subClassOf     rdfs:Container .

rdfs:isDefinedBy    rdfs:subPropertyOf  rdfs:seeAlso .

rdf:XMLLiteral      rdf:type            rdfs:Datatype .
rdf:XMLLiteral      rdfs:subClassOf     rdfs:Literal .
rdfs:Datatype       rdfs:subClassOf     rdfs:Class .

rdf:_1              rdf:type
                       rdfs:ContainerMembershipProperty .
rdf:_1              rdfs:domain         rdfs:Resource .
rdf:_1              rdfs:range          rdfs:Resource .
rdf:_2              rdf:type
                       rdfs:ContainerMembershipProperty .
...
```

RDFS Entailment

Automatic inference is again realized via deduction rules:

$$\frac{}{\texttt{u a x .}}\ \text{rdfsax}$$

every axiomatic triple “u a x .” can always be derived

$$\frac{\texttt{u a \_:n .}}{\texttt{u a l .}}\ \text{gl}$$

the converse of Rule lg: `_:n` has been assigned (via Rule lg) to the untyped literal `l`

$$\frac{\texttt{u a l .}}{\texttt{\_:n rdf:type rdfs:Literal}}\ \text{rdfs1}$$

`_:n` has been assigned (via Rule lg) to the untyped literal `l`

$$\frac{\texttt{a rdfs:domain x .}\quad \texttt{u a y .}}{\texttt{u rdf:type x .}}\ \text{rdfs2}$$

implements the semantics of property domains

$$\frac{\texttt{a rdfs:range x .}\quad \texttt{u a v .}}{\texttt{v rdf:type x .}}\ \text{rdfs3}$$

implementis the semantics of property ranges

`a`, `b`	IRIs	`x`, `y`	IRI, blank node or literal		
`u`, `v`	IRI or blank node	`l`	literal	`_:n`	blank nodes

RDFS Entailment

$$\frac{\texttt{u a x .}}{\texttt{u rdf:type rdfs:Resource .}}$$ rdfs4a — the subject of every triple is a resource

$$\frac{\texttt{u a v .}}{\texttt{v rdf:type rdfs:Resource .}}$$ rdfs4b — objects that are not literals are resources as well

$$\frac{\texttt{u rdfs:subPropertyOf v . v rdfs:subPropertyOf x .}}{\texttt{u rdfs:subPropertyOf x .}}$$ rdfs5 — transitivity

$$\frac{\texttt{u rdf:type rdf:Property .}}{\texttt{u rdfs:subPropertyOf u .}}$$ rdfs6 — reflexivity

$$\frac{\texttt{a rdfs:subPropertyOf b . u a y .}}{\texttt{u b y .}}$$ rdfs7 — subproperty inferences for instances

$$\frac{\texttt{u rdf:type rdfs:Class .}}{\texttt{u rdf:subClassOf rdfs:Resource .}}$$ rdfs8 — classes contain only resources

RDFS Entailment

$$\frac{\texttt{u rdfs:subClassOf x . \quad v rdf:type u .}}{\texttt{v rdf:type x .}}\ \text{rdfs9}$$

subclassen inferences for instances

$$\frac{\texttt{u rdf:type rdfs:Class .}}{\texttt{u rdfs:subClassOf u .}}\ \text{rdfs10}$$

reflexivity

$$\frac{\texttt{u rdfs:subClassOf v . v rdfs:subClassOf x .}}{\texttt{u rdfs:subClassOf x .}}\ \text{rdfs11}$$

transitivity

$$\frac{\texttt{u rdf:type rdfs:ContainerMembershipProperty .}}{\texttt{u rdfs:subPropertyOf rdfs:member .}}\ \text{rdfs12}$$

$$\frac{\texttt{u rdf:type rdfs:Datatype .}}{\texttt{u rdfs:subClassOf rdfs:Literal .}}\ \text{rdfs10}$$

every datatype is a subclass of `rdfs:Literal`

RDFS Entailment

- important definition: XML clash

```
ex:hasSmiley    rdfs:range     rdfs:Literal .
ex:evilRemark   ex:hasSmiley   ">:->"^^rdf:XMLLiteral .
```

- occurs if a node of type `rdfs:Literal` gets assigned an ill-formed literal value

RDFS Entailment

Theorem:

A graph G_2 is RDFS entailed by G_1, if there is a graph G_1' obtained by applying the rules lg, gl, rdfax, rdf1, rdf2, rdfs1 – rdfs13 and rdfsax to G_1, such that

- G_2 is simply entailed by G_1' or
- G_1' contains an XML clash.

Agenda

What RDF(S) Cannot Do

- Certain seemingly sensible consequences are not RDFS-entailed, e.g.

```
ex:talksTo   rdfs:domain       ex:Homo .
ex:Homo      rdfs:subClassOf   ex:Primates .
```

  should imply

```
ex:talksTo   rdfs:domain   ex:Primates .
```

- possible solution: use a stronger, so-called “extensional” semantics (but this would be outside the standard)
- no possibility to express negation