Exercise 4

Foundations of Logic Programming

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International Master Program in Computational Logic — winter term 2008/2009

Date of Exercise: 08.12.2008

Exercise 4.1

Reconsider the program P from Exercise 3.4:

```
add(X,0,X).
add(X,s(Y),s(Z)) :- add(X,Y,Z).

mul(X,0,0).
mul(X,s(Y),Z) :- mul(X,Y,Z1), add(Z1,X,Z).
```

a) Give the Herbrand universe HU_F and the Herbrand base $HB_{\Pi,F}$ determined by P.

b) Give a Herbrand model of P.

c) Consider the interpretation I_6 from Slide IV/9 together with the additional definition $mul_{I_6} = \{(m, n, n) \mid m, n \in \{0, 1\}\}$. Show either $I_6 \models P$ or $I_6 \not\models P$.

d) Show that $P \not\models mul(s(0), s(0), X)$.

Exercise 4.2

Consider the following program P:

```
p(X,a).
p(X,f(Y)) :- p(X,Y), q(X,Y).

q(X,Y)    :- r(Y).
q(f(X),Y) :- q(X,f(Y)).

r(f(X)).
```

a) Give an implication tree whose root is the atom $p(f(a), f(f(a)))$ and whose nodes are ground. How many trees of this kind are there? And if the nodes are not required to be ground?

b) Show that the query $? - p(f(f(x)), f(a)), q(f(x), f(x)).$ is n-deep (cf. Slide IV/28) for $n = 7, 8, 9$.

Exercise 4.3

Is there a finite set of definite clauses S and a non-empty set $\{M_i\}_{i \in I}$ of Herbrand models of S such that $\cap_{i \in I} M_i$ is not a model of S? Justify your answer.

Exercise 4.4

Consider the Herbrand universe $HU = \{1, 2, 3\}$ together with the following program P:

```
a(3).
a(X) :- a(2).
b(X) :- c(3).
b(X) :- a(X).
c(1) :- b(3).
c(3) :- c(X).
```

a) Compute $T_P \uparrow \omega$ and show the values of $T_P \uparrow i$ for $i = 0, 1, 2, \ldots$. For which value of i do you get a fixpoint?

b) Is $T_P \uparrow \omega \cup \{c(2)\}$ also a model of P? And $T_P \uparrow \omega \cup \{a(2)\}$?

c) Let HB be the Herbrand base of P. Compute $T_P \uparrow \omega(HB)$.