Foundations of Logic Programming

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Exercise 4.1

Consider the following program:

```
p(X) :- q(X), r(X).
q(f(X)).
r(f(a)).
```

a) Give an SLD-derivation ξ for query `?- p(X)` that uses the Prolog selection rule.

b) For each derivation step of ξ, give the resultant that is associated with this step (Sl. III/18).

c) Give the resultants of every level i of ξ (Sl. III/19).

Exercise 4.2

Give a program P, a query Q and two selection rules R_1 and R_2 such that:

- every SLD-derivation of $P \cup \{Q\}$ via R_1 is infinite

- every SLD-derivation of $P \cup \{Q\}$ via R_2 is failed

Is it possible to construct P and Q such that additionally to the properties specified above there exists a successful SLD-derivation via some selection rule R_3? Justify your answer.

Exercise 4.3

Consider the following definition of a selection rule R:

"Choose the atom A of the query such that the number of elements in $var(A)$ is minimal; in case several atoms have the same minimal number of variables, choose the rightmost."

Moreover, consider the following program P:

```
add(X,0,X).
add(X,s(Y),s(Z)) :- add(X,Y,Z).

mul(X,0,0).
mul(X,s(Y),Z) :- mul(X,Y,Z1), add(Z1,X,Z).
```

and a query $Q = $ `mul(X,Y,s(0))`. Solve the following tasks:

a) Build the SLD-tree for $P \cup \{Q\}$ via R.

b) Find a selection rule R' such that there is a successful derivation which always uses the first applicable program clause in P. Build the SLD-tree for $P \cup \{Q\}$ via R'.

Exercise 4.4

Find a counterexample to the following claim: For every infinite SLD-derivation of $P \cup \{Q\}$ and every selection rule $\mathcal{R}$ there exists an infinite SLD-derivation of $P \cup \{Q\}$ via $\mathcal{R}$.

Exercise 4.5

Reconsider the program P from Slide V/10 that checks if `Xs` is a sublist of `Ys`:

```
sub(Xs,Ys) :- app(Xs,_,Zs), app(_,Zs,Ys).

app([],Ys,Ys).
app([X|Xs],Ys,[X|Zs]) :- app(Xs,Ys,Zs).
```

a) Construct the LD-Tree for $P \cup \{sub(L,[a])\}$ via operation $expand$ from Slides V/5-6.

b) Which of the notions from Slides V/7-8 apply to the query from a): universally terminates, diverges, potentially diverges, produces infinitely many answers, fails?