Exercise 6

Foundations of Logic Programming

Prof. Michael Thielscher, Sebastian Voigt

International Master Program in Computational Logic — winter term 2009/2010

Date of Exercise: 11.01.2010

Exercise 6.1 cf. Exercise 5.4

Reconsider the program P from Exercise 4.3:

```
add(X,0,X).
add(X,s(Y),s(Z)) :- add(X,Y,Z).

mul(X,0,0).
mul(X,s(Y),Z) :- mul(X,Y,Z1), add(Z1,X,Z).
```

d) Show that $P \not\models mul(s(0), s(0), X)$.

e) Show that $P \models mul(0, s(0), 0)$.

Exercise 6.2

Consider the following program P :

```
p(X,a).
p(X,f(Y)) :- p(X,Y), q(X,Y).

q(X,Y)    :- r(Y).
q(f(X),Y) :- q(X,f(Y)).

r(f(X)).
```

a) Give an implication tree whose root is the atom $p(f(a), f(f(a)))$ and whose nodes are ground. How many trees of this kind are there? And if the nodes are not required to be ground?

b) Show that the query $? - p(f(f(x)), f(a)), q(f(x), f(x))$. is n-deep (cf. Slide IV/28) for $n = 7, 8, 9$.

Exercise 6.3

Is there a program P and a non-empty set $\{M_i\}_{i \in I}$ of Herbrand models of P such that $\cap_{i \in I} M_i$ is not a model of P ? Justify your answer.

Exercise 6.4

Consider the following program P:

```
a(1).
a(2).
a(3).
b(X) :- a(X).
c(X) :- b(X).
d(X) :- d(s(X)).
e(X,X) :- a(X),b(X).
e(X,s(Y)) :- e(X,Y).
```

a) Compute $T_P \uparrow 0$, $T_P \uparrow 1$, $T_P \uparrow 2$, $T_P \uparrow 3$, and $T_P \uparrow 4$.

b) Give the least Herbrand model I of P.

c) Is $I \cup \{d(2)\}$ a Herbrand model of P? And $I \cup \{d(s(1))\}$?