Exercise 2

Foundations of Logic Programming

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Exercise 2.1

Find two terms s and t such that s is an instance of t but where s and t cannot be unified. Explain your answer.

Exercise 2.2

a) Compute the substitution composition $\theta\eta\tau$, where w, x, y, z are variables and

$$\theta = \{y/a(x,z), z/y\} \qquad \eta = \{y/x, x/f(w)\} \qquad \tau = \{w/g(a), x/z, z/b\}$$

b) Use structural induction to prove that for arbitrary terms t and substitutions δ_1, δ_2:

$$(t\delta_1)\delta_2 = t(\delta_1\delta_2)$$

Exercise 2.3

Use the Martelli-Montanari algorithm step by step to unify the following pairs of terms with variables x, y, and z. For each step indicate which rule you have used and the reduction wrt. $\prec_3$ of the termination proof for the MM-algorithm.

a) $f(g(x), g(c), y)$ and $f(g(g(y)), x, a)$

b) $f(b, x, x, y)$ and $f(b, g(y), g(g(z)), g(a))$

c) $f(x, g(z), g(z))$ and $f(h(y), y, g(h(x)))$

Give the corresponding most general unifier (mgu) or give the reason why the terms are not unifiable.

Exercise 2.4

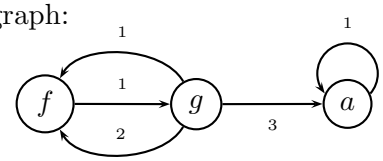
In the proof of the termination of the MM algorithm, a lexicographic ordering on triples of the form $(\mathit{uns}(E), \mathit{lfun}(E), \mathit{card}(E))$ is used. Show examples why orderings on triples of the form $(\mathit{lfun}(E), \mathit{uns}(E), \mathit{card}(E))$ or of the form $(\mathit{uns}(E), \mathit{card}(E), \mathit{lfun}(E))$ would not work.

Exercise 2.5

For a finite ranked alphabet F, give a partial mapping $t : \mathbb{N}_+^* \twoheadrightarrow F$ (cf. Slide 2b/4) which defines a non-regular tree.

Exercise 2.6

Consider the following labeled graph:



a) Give the infinite tree with root f which is represented by this graph as partial mapping $t : \mathbb{N}_+^* \twoheadrightarrow \{f, g, a\}$.

b) How many subtrees does the tree from a) have?