Exercise 4

Foundations of Logic Programming

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Exercise 4.1

Consider the following definition of a selection rule R:

"Choose the atom A of the query such that the number of elements in $var(A)$ is minimal; in case several atoms have the same minimal number of variables, choose the rightmost."

Moreover, consider the following program P:

```
add(X,0,X).
add(X,s(Y),s(Z)) :- add(X,Y,Z).

mul(X,0,0).
mul(X,s(Y),Z) :- mul(X,Y,A), add(A,X,Z).
```

and a query $Q =$ `mul(X,Y,s(0))`. Solve the following tasks:

a) Build the SLD-tree for $P \cup \{Q\}$ via R. Give the CAS for each node with the empty query $\Box$.

b) Build the SLD-tree for $P \cup \{Q\}$ via the "leftmost"-selection rule and give all CAS.

Exercise 4.2

Reconsider the program P from Exercise 4.1.

a) Give the Herbrand universe HU_F and the Herbrand base $HB_{\Pi,F}$ determined by P.

b) Give two Herbrand models of P.

c) Consider the interpretation I_6 from Slide IV/9 together with the additional definition $mul_{I_6} = \{(m,n,n) \mid m,n \in \{0,1\}\}$. Show either $I_6 \models P$ or $I_6 \not\models P$.

d) Show that $P \not\models mul(s(0), s(0), X)$.

e) Show that $P \models mul(0, s(0), 0)$.

Exercise 4.3

Consider the following program P:

```
p(X,a).
p(X,f(Y)) :- p(X,Y), q(X,Y).

q(X,Y)    :- r(Y).
q(f(X),Y) :- q(X,f(Y)).

r(f(X)).
```

a) Give an implication tree (cf. Slide 4a/22) whose root is the atom $p(f(a), f(f(a)))$ and whose nodes are ground. How many trees of this kind are there? And if the nodes are not required to be ground?

b) Show that the query $? - p(f(f(x)), f(a)), q(f(x), f(x))$. is n-deep (cf. Slide IV/28) for $n = 7, 8, 9$.

Exercise 4.4

Is there a program P and a non-empty set $\{M_i\}_{i \in I}$ of Herbrand models of P such that $\cap_{i \in I} M_i$ is not a model of P? Justify your answer.