Exercise 5

Foundations of Logic Programming

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International Master Program in Computational Logic — winter term 2010/2011

Date of Exercise: 21.12.2010

Exercise 5.1 cf. Exercise 4.3

Consider the following program P:

```
p(X,a).
p(X,f(Y)) :- p(X,Y), q(X,Y).

q(X,Y)    :- r(Y).
q(f(X),Y) :- q(X,f(Y)).

r(f(X)).
```

a) Give an implication tree (cf. Slide 4a/22) whose root is the atom $p(f(a), f(f(a)))$ and whose nodes are ground. How many trees of this kind are there? And if the nodes are not required to be ground?

b) Show that the query $? - p(f(f(x)), f(a)), q(f(x), f(x))$. is n-deep (cf. Slide IV/28) for $n = 7, 8, 9$.

Exercise 5.2 cf. Exercise 4.4

Is there a program P and a non-empty set $\{M_i\}_{i \in I}$ of Herbrand models of P such that $\cap_{i \in I} M_i$ is not a model of P? Justify your answer.

Exercise 5.3

Consider the following program P:

```
a(1).
a(2).
a(3).
b(X) :- a(X).
c(X) :- b(X).
d(X) :- d(s(X)).
e(X,X) :- a(X),b(X).
e(X,s(Y)) :- e(X,Y).
```

a) Compute $T_P \uparrow 0$, $T_P \uparrow 1$, $T_P \uparrow 2$, $T_P \uparrow 3$, and $T_P \uparrow 4$.

b) Give the least Herbrand model I of P.

c) Is $I \cup \{d(2)\}$ a Herbrand model of P? And $I \cup \{d(s(1))\}$?

Exercise 5.4

Reconsider the program P from Slide V/10 that checks if `Xs` is a sublist of `Ys`:

```
sub(Xs,Ys) :- app(Xs,_,Zs), app(_,Zs,Ys).

app([],Ys,Ys).
app([X|Xs],Ys,[X|Zs]) :- app(Xs,Ys,Zs).
```

a) Construct the LD-Tree for $P \cup \{\texttt{sub(L,[a])}\}$ via operation *expand* from Slides V/5-6.

b) Which of the notions from Slides V/7-8 apply to the query from a): universally terminates, diverges, potentially diverges, produces infinitely many answers, fails?

Exercise 5.5

Write a predicate `most_frequent(List, Item)` in Pure Prolog that finds the most frequently occurring item `Item` in a list `List`. If there are two or more items in the list that occur equally often (i.e., with the same frequency), return only the one whose first occurrence is closest to the beginning of the list.

Examples:
`?-most_frequent([1,2,3,1,2], Item).` returns `Item = 1`.
`?-most_frequent([1,2,3,1,2,3,2], Item).` returns `Item = 2`.