Exercise 3.1:
Apply the Domain Reduction Rules from Slide III/5 to the following CSPs until you reach a successful, failed or stabilising CSP. At each step give the rule you have used.

a) \( \langle x = y, y = z, x \neq w, w \neq z; x \in \{a, b, c\}, y \in \{a, c, d\}, z \in \{c, d, e\}, w = c \rangle \)

b) \( \langle x \neq w, w < y, w < z, y < z; x \in [4..8], y \in [2..6], z \in [3..6], w \in [4..9] \rangle \)

Exercise 3.2:
Take the following set of linear equations:

\[
\begin{align*}
a + b + c &= 0 \\
4a + 2b + c &= 1 \\
9a + 3b + c &= 3
\end{align*}
\]

a) Apply Gauss-Jordan Elimination to compute a most general unifier (mgu) for this set of equations.

b) Apply Gaussian Elimination to compute an mgu for this set of equations.

Exercise 3.3:
Apply the Hyper-Arc Consistency rule (cf. Slides IV/10) to the following CSP until it is closed under this rule. Indicate the chosen constraint at each step.

\( \langle x < y, x \neq z, y - x = z; x \in [1..4], y \in [1..4], z \in [2..3] \rangle \)
Exercise 3.4:
Construct an Eclipse-Prolog program which solves the following Einstein Puzzle:

There are four different persons: Marc, Joey, Sandra and Ellen. Each person likes exactly one of the sports hiking, volleyball, basketball or tennis and exactly one of the drinks tea, water, coffee or beer. The favorite sport and drink of each person differs from those of the respective other persons. Moreover you have the following clues:

1. Joey drinks beer.
2. Marc likes neither tea nor volleyball.
3. Either Sandra goes hiking or Joey plays basketball.
4. Ellen plays basketball if Sandra likes tea.
5. The water drinker plays tennis or volleyball.

Use the constraint solving library ic, and use numbers 1, 2, 3, 4 to encode the names of the persons and variables Tea, Coffee, Hiking, etc. which range over these numbers.