



# SEMINAR ABSTRACT ARGUMENTATION

## Introduction to Formal Argumentation

\* slides adapted from Stefan Woltran's lecture on Abstract Argumentation

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Dresden, 1st November 2013



# Roadmap for the Lecture

- Introduction
- Abstract Argumentation Frameworks
- Complexity
- Implementation Techniques
- Extensions of Abstract Argumentation Frameworks
- Students' Topics

# Introduction

## Argumentation:

... the study of processes “concerned with how assertions are **proposed**, **discussed**, and **resolved** in the context of issues upon which several **diverging opinions** may be held”.

[Bench-Capon and Dunne, Argumentation in AI, AIJ 171:619-641, 2007]

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## Formal Models of Argumentation are concerned with

- representation of an argument
- representation of the relationship between arguments
- solving conflicts between the arguments (“acceptability”)

# Introduction (ctd.)

## Increasingly important area

- “Argumentation” as keyword at all major AI conferences
- dedicated conference: [COMMA](#); several workshops
- specialized journal: [Argument and Computation](#) (Taylor & Francis)
- two text books:
  - Besnard, Hunter: *Elements of Argumentation*. MIT Press, 2008
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## Applications

- PARMENIDES-system for E-Democracy: facilitates structured arguments over a proposed course of action [Atkinson et al.; 2006]
- IMPACT project: argumentation toolbox for supporting open, inclusive and transparent deliberations about public policy
- Decision support systems, etc.

# The Overall Process

## Steps

- Starting point:  
knowledge-base
- Form arguments
- Identify conflicts
- Abstract from  
internal structure
- Resolve conflicts
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$$\Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\}$$



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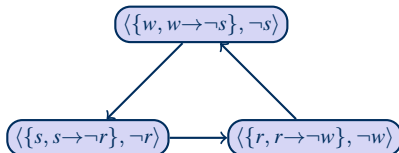
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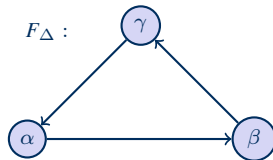
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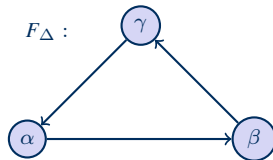
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$$\begin{aligned} \text{pref}(F_{\Delta}) &= \{\emptyset\} \\ \text{stage}(F_{\Delta}) &= \{\{\alpha\}, \{\beta\}, \{\gamma\}\} \end{aligned}$$

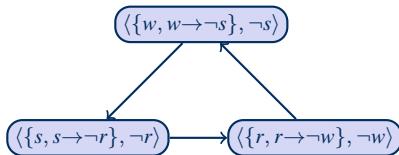
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- Starting point: knowledge-base
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$$Cn_{pref}(F_{\Delta}) = Cn(\top)$$

$$Cn_{stage}(F_{\Delta}) = Cn(\neg r \vee \neg w \vee \neg s)$$

# The Overall Process (ctd.)

## Some Remarks

- Main idea dates back to Dung [1995]; has then been refined by several authors (Prakken, Gordon, Caminada, etc.)
- Separation between logical (forming arguments) and nonmonotonic reasoning (“[abstract argumentation frameworks](#)”)
- Abstraction allows to compare several KR formalisms on a conceptual level (“calculus of conflict”)

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## Main Challenge

- [All Steps](#) in the argumentation process are, in general, [intractable](#).
- This calls for:
  - careful complexity analysis (identification of tractable fragments)
  - re-use of established tools for implementations (reduction method)

# Approaches to Form Arguments

## Classical Arguments [Besnard & Hunter, 2001]

- Given is a KB (a set of propositions)  $\Delta$
- argument is a pair  $(\Phi, \alpha)$ , such that  $\Phi \subseteq \Delta$  is consistent,  $\Phi \models \alpha$  and for no  $\Psi \subset \Phi$ ,  $\Psi \models \alpha$
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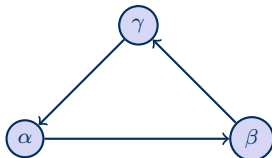


## Other Approaches

- Arguments are trees of statements
- claims are obtained via strict and defeasible rules
- different notions of conflict: rebuttal, undercut, etc.

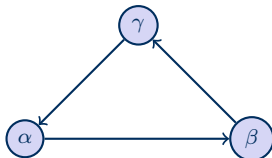
# Dung's Abstract Argumentation Frameworks

## Example



# Dung's Abstract Argumentation Frameworks

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## Main Properties

- Abstract from the concrete content of arguments but only consider the relation between them
- Semantics select subsets of arguments respecting certain criteria
- Simple, yet powerful, formalism
- Most active research area in the field of argumentation.
  - “plethora of semantics”

# Dung's Abstract Argumentation Frameworks

## Definition

An **argumentation framework** (AF) is a pair  $(A, R)$  where

- $A$  is a set of arguments
- $R \subseteq A \times A$  is a relation representing the conflicts (“attacks”)

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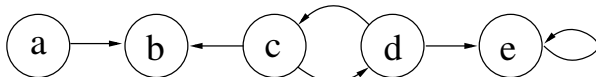
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## Example

$F = (\{a, b, c, d, e\}, \{(a, b), (c, b), (c, d), (d, c), (d, e), (e, e)\})$



# Basic Properties

## Conflict-Free Sets

Given an AF  $F = (A, R)$ .

A set  $S \subseteq A$  is **conflict-free** in  $F$ , if, for each  $a, b \in S$ ,  $(a, b) \notin R$ .

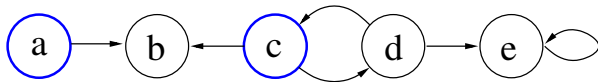
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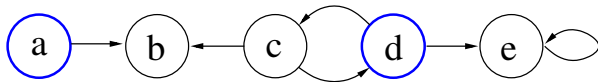
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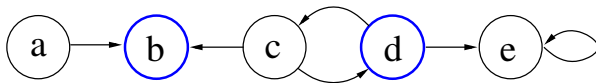
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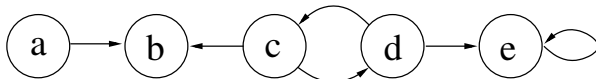
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# Basic Properties (ctd.)

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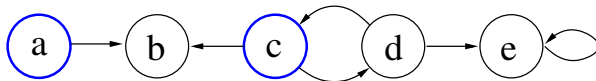
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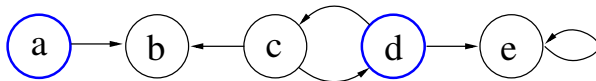
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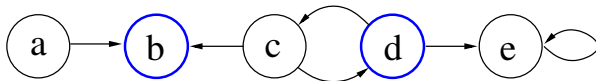
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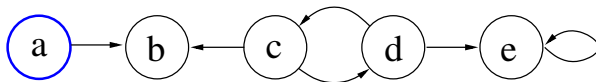
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$$\text{adm}(F) = \{\{a, c\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \emptyset\}$$



# Basic Properties (ctd.)

## Dung's Fundamental Lemma

Let  $S$  be admissible in an AF  $F$  and  $a, a'$  arguments in  $F$  defended by  $S$  in  $F$ .  
Then,

- 1  $S' = S \cup \{a\}$  is admissible in  $F$
- 2  $a'$  is defended by  $S'$  in  $F$

# Semantics

## Naive Extensions

Given an AF  $F = (A, R)$ . A set  $S \subseteq A$  is a **naive extension** of  $F$ , if

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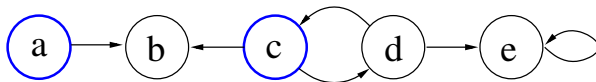
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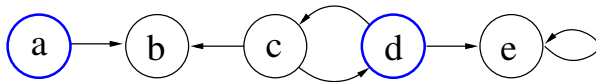
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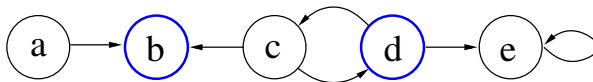
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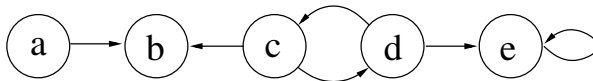
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# Semantics (ctd.)

## Grounded Extension [Dung, 1995]

Given an AF  $F = (A, R)$ . The unique **grounded extension** of  $F$  is defined as the outcome  $S$  of the following “algorithm”:

- 1 put each argument  $a \in A$  which is not attacked in  $F$  into  $S$ ; if no such argument exists, return  $S$ ;
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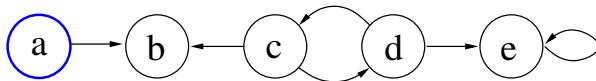
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$$\text{ground}(F) = \{\{a\}\}$$



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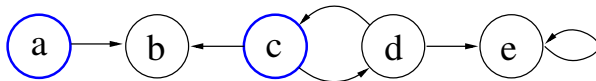
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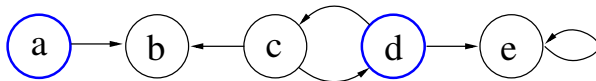
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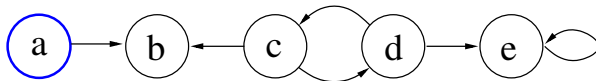
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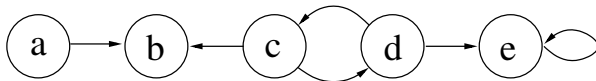
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## Remark

Since there exists exactly one grounded extension for each AF  $F$ , we often write  $ground(F) = S$  instead of  $ground(F) = \{S\}$ .

# Semantics (ctd.)

## Preferred Extensions [Dung, 1995]

Given an AF  $F = (A, R)$ . A set  $S \subseteq A$  is a **preferred extension** of  $F$ , if

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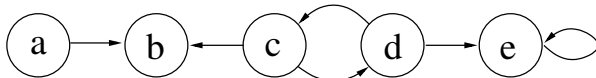
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### Example



$$\text{pref}(F) = \{\{a, c\}, \{a, d\}, \{a\}, \{e\}, \{d\}, \emptyset\}$$

# Semantics (ctd.)

## Stable Extensions [Dung, 1995]

Given an AF  $F = (A, R)$ . A set  $S \subseteq A$  is a **stable extension** of  $F$ , if

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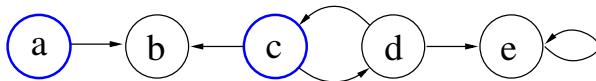
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### Example



$$\text{stable}(F) = \{\{a, e\}\}$$

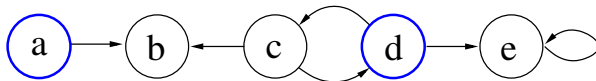
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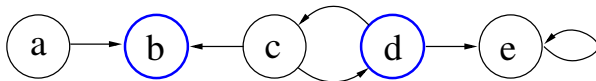
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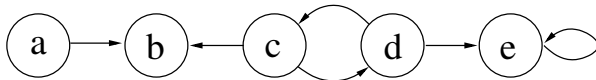
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### Example



$$\text{stable}(F) = \{\{a, e\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \emptyset, \}$$

# Semantics (ctd.)

## Some Relations

For any AF  $F$  the following relations hold:

- 1 Each stable extension of  $F$  is admissible in  $F$
- 2 Each stable extension of  $F$  is also a preferred one
- 3 Each preferred extension of  $F$  is also a complete one

# Semantics (ctd.)

## Semi-Stable Extensions [Caminada, 2006]

Given an AF  $F = (A, R)$ . A set  $S \subseteq A$  is a **semi-stable extension** of  $F$ , if

- $S$  is admissible in  $F$
- for each  $T \subseteq A$  admissible in  $F$ ,  $S^+ \not\subseteq T^+$ 
  - for  $S \subseteq A$ , define  $S^+ = S \cup \{a \mid \exists b \in S \text{ with } (b, a) \in R\}$



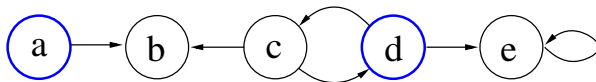
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### Example



$$\text{semi}(F) = \{\{a, c\}, \{a, d\}, \{a\}, \{c\}, \{d\}, \emptyset\}$$

# Semantics (ctd.)

## Stage Extensions [Verheij, 1996]

Given an AF  $F = (A, R)$ . A set  $S \subseteq A$  is a **stage extension** of  $F$ , if

- $S$  is conflict-free in  $F$
- for each  $T \subseteq A$  conflict-free in  $F$ ,  $S^+ \not\subseteq T^+$ 
  - recall  $S^+ = S \cup \{a \mid \exists b \in S \text{ with } (b, a) \in R\}$

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## Ideal Extension [Dung, Mancarella & Toni 2007]

Given an AF  $F = (A, R)$ . A set  $S \subseteq A$  is an **ideal extension** of  $F$ , if

- $S$  is admissible in  $F$  and contained in each preferred extension of  $F$
- there is no  $T \supset S$  admissible in  $F$  and contained in each of  $\text{pref}(F)$

# Semantics (ctd.)

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## Eager Extension [Caminada, 2007]

Given an AF  $F = (A, R)$ . A set  $S \subseteq A$  is an **eager extension** of  $F$ , if

- $S$  is admissible in  $F$  and contained in each semi-stable extension of  $F$
- there is no  $T \supset S$  admissible in  $F$  and contained in each of  $\text{semi}(F)$

## Properties of Ideal Extensions

For any AF  $F$  the following observations hold:

- 1 there exists exactly one ideal extension of  $F$
- 2 the ideal extension of  $F$  is also a complete one

The same results hold for the eager extension and similar variants [Dvořák et al., 2011].

## Resolution-based grounded Extensions [Baroni, Giacomin 2008]

A **resolution**  $\beta$  of an AF  $F = (A, R)$  contains exactly one of the attacks  $(a, b)$ ,  $(b, a)$  for each pair  $a, b \in A$  with  $\{(a, b), (b, a)\} \subseteq R$ .

A set  $S \subseteq A$  is a **resolution-based grounded extension** of  $F$ , if

- there exists a resolution  $\beta$  such that  $ground((A, R \setminus \beta)) = S$
- and there is no resolution  $\beta'$  such that  $ground((A, R \setminus \beta')) \subset S$

# Semantics (ctd.)

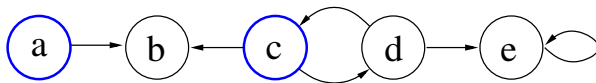
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### Example



$$ground^*(F) = \{\{a, c\},$$

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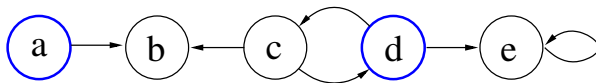
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### Example



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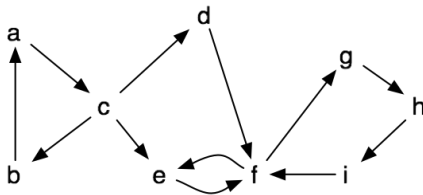


# cf2 Semantics [Baroni, Giacomin & Guida 2005]

## Definition (Separation)

An AF  $F = (A, R)$  is called **separated** if for each  $(a, b) \in R$ , there exists a path from  $b$  to  $a$ . We define  $[[F]] = \bigcup_{C \in \text{SCCs}(F)} F|_C$  and call  $[[F]]$  the **separation** of  $F$ .

## Example

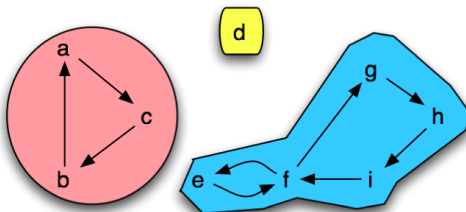


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## Example



## cf2 Semantics (ctd.)

### Definition (Reachability)

Let  $F = (A, R)$  be an AF,  $B$  a set of arguments, and  $a, b \in A$ . We say that  $b$  is **reachable** in  $F$  from  $a$  **modulo**  $B$ , in symbols  $a \Rightarrow_F^B b$ , if there exists a path from  $a$  to  $b$  in  $F|_B$ .

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### Definition ( $\Delta_{F,S}$ )

For an AF  $F = (A, R)$ ,  $D \subseteq A$ , and a set  $S$  of arguments,

$$\Delta_{F,S}(D) = \{a \in A \mid \exists b \in S : b \neq a, (b, a) \in R, a \not\Rightarrow_F^{A \setminus D} b\}.$$

By  $\Delta_{F,S}$ , we denote the lfp of  $\Delta_{F,S}(\emptyset)$ .

## cf2 Semantics (ctd.)

### cf2 Extensions [G & Woltran 2010]

Given an AF  $F = (A, R)$ . A set  $S \subseteq A$  is a **cf2-extension** of  $F$ , if

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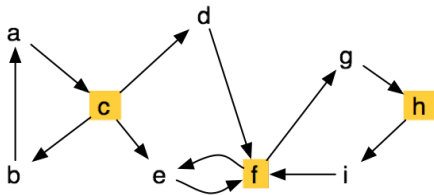
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### Example

$S = \{c, f, h\}$ ,  $S \in \text{cf}(F)$ .



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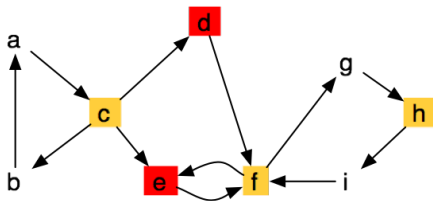
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### Example

$S = \{c, f, h\}$ ,  $S \in \text{cf}(F)$ ,  $\Delta_{F,S}(\emptyset) = \{d, e\}$ .



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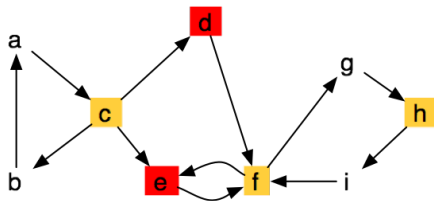
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### Example

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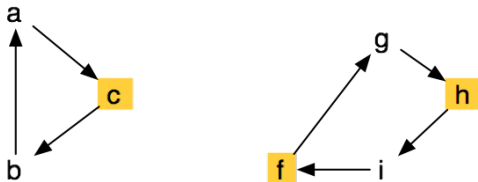
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### Example

$S = \{c, f, h\}$ ,  $S \in \text{cf}(F)$ ,  $\Delta_{F,S} = \{d, e\}$ ,  $S \in \text{naive}([F - \Delta_{F,S}])$ .



# Relations between Semantics

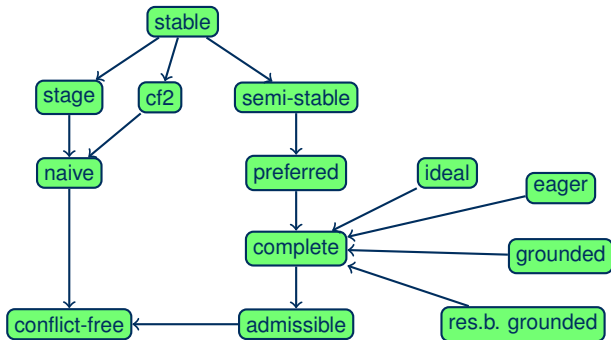
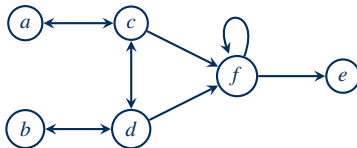


Figure: An arrow from semantics  $\sigma$  to semantics  $\tau$  encodes that each  $\sigma$ -extension is also a  $\tau$ -extension.

# Characteristics of Argumentation Semantics

## Example



$pref(F) = \{\{a, d, e\}, \{b, c, e\}, \{a, b\}\}$

$naive(F) = \{\{a, d, e\}, \{b, c, e\}, \{a, b, e\}\}$

## Natural Questions

- How to change the AF if we want  $\{a, b, e\}$  instead of  $\{a, b\}$  in  $pref(F)$ ?
- How to change the AF if we want  $\{a, b, d\}$  instead of  $\{a, b\}$  in  $pref(F)$ ?
- Can we have equivalent AFs without argument  $f$ ?

# Some Properties . . .

## Theorem

For any AFs  $F$  and  $G$ , we have

- $adm(F) = adm(G) \implies \sigma(F) = \sigma(G)$ , for  $\sigma \in \{pref, ideal\}$ ;
- $comp(F) = comp(G) \implies \vartheta(F) = \vartheta(G)$ , for  $\vartheta \in \{pref, ideal, ground\}$ ;
- no other such relation between the different semantics (*adm, pref, ideal, semi, eager, ground, comp, stable*) in terms of standard equivalence holds.

# Strong Equivalence [Oikarinen & Woltran 2011, G & Woltran 2011]

## Definition

Two AFs  $F$  and  $G$  are strongly equivalent wrt. a semantics  $\sigma \in \{\text{stable}, \text{adm}, \text{pref}, \text{ideal}, \text{semi}, \text{comp}, \text{ground}, \text{stage}\}$ , in symbols  $F \equiv_s^\sigma G$ , iff  $\sigma(F \cup H) = \sigma(G \cup H)$ , for each AF  $H$ .

- Idea: Find “ $\sigma$ -kernels” of AFs, such that the  $\sigma$ -kernels of  $F$  and  $G$  coincide iff  $F \equiv_s^\sigma G$ .
  - Verification of strong equivalence then reduces to checking syntactical equivalence

# Strong Equivalence for Stable Semantics

## Kernel for stable semantics

For AF  $F = (A, R)$ , we define *stable*-kernel of  $F$  as  $F^\kappa = (A, R^\kappa)$  with

$$R^\kappa = R \setminus \{(a, b) \mid a \neq b, (a, a) \in R\}.$$

## Theorem

For any AFs  $F$  and  $G$ :  $F^\kappa = G^\kappa$  iff  $F \equiv_s^{stable} G$  iff  $F \equiv_s^{stage} G$ .

# Exercises

- 1 Give an AF  $F$  such that  $stable(F) = \emptyset$  and  $semi(F) \neq \{\emptyset\}$ .
- 2 Show that the following statement holds for any AF  $F$ .  
If  $stable(F) \neq \emptyset$  then  $stable(F) = semi(F) = stage(F)$ .
- 3 Select three different semantics  $\sigma, \sigma', \sigma''$  out of  $\{pref, ideal, semi, eager, ground, stable\}$  of your choice and provide three pairs of AFs such that
  - $\sigma(F_1) = \sigma(G_1)$  but  $\sigma'(F_1) \neq \sigma'(G_1)$
  - $\sigma'(F_2) = \sigma'(G_2)$  but  $\sigma''(F_2) \neq \sigma''(G_2)$
  - $\sigma''(F_3) = \sigma''(G_3)$  but  $\sigma(F_3) \neq \sigma(G_3)$



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