SEMINAR ABSTRACT ARGUMENTATION

Introduction to Formal Argumentation
* slides adapted from Stefan Woltran’s lecture on Abstract Argumentation

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Dresden, 1st November 2013
Roadmap for the Lecture

- Introduction
- Abstract Argumentation Frameworks
- Complexity
- Implementation Techniques
- Extensions of Abstract Argumentation Frameworks
- Students’ Topics
Introduction

Argumentation:

...the study of processes “concerned with how assertions are proposed, discussed, and resolved in the context of issues upon which several diverging opinions may be held”.

[Bench-Capon and Dunne, Argumentation in AI, AIJ 171:619-641, 2007]
**Introduction**

**Argumentation:**

...the study of processes “concerned with how assertions are proposed, discussed, and resolved in the context of issues upon which several diverging opinions may be held”.

[Bench-Capon and Dunne, Argumentation in AI, AIJ 171:619-641, 2007]

**Formal Models of Argumentation are concerned with**

- representation of an argument
- representation of the relationship between arguments
- solving conflicts between the arguments (“acceptability”)
Increasingly important area

- “Argumentation” as keyword at all major AI conferences
- dedicated conference: COMMA; several workshops
- specialized journal: Argument and Computation (Taylor & Francis)
- two text books:
Increasingly important area

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Applications

- PARMENIDES-system for E-Democracy: facilitates structured arguments over a proposed course of action [Atkinson et al.; 2006]
- IMPACT project: argumentation toolbox for supporting open, inclusive and transparent deliberations about public policy
- Decision support systems, etc.
### The Overall Process

<table>
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<td>Form arguments</td>
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<td>Identify conflicts</td>
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<td>Abstract from internal structure</td>
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<td>Draw conclusions</td>
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The Overall Process

Steps

- Starting point: knowledge-base
- Form arguments
- Identify conflicts
- Abstract from internal structure
- Resolve conflicts
- Draw conclusions

Example

$$\Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\}$$
The Overall Process

Steps

- Starting point: knowledge-base
- **Form arguments**
- Identify conflicts
- Abstract from internal structure
- Resolve conflicts
- Draw conclusions

Example

\[ \Delta = \{ s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s \} \]

\[\langle \{ w, w \rightarrow \neg s \}, \neg s \rangle\]

\[\langle \{ s, s \rightarrow \neg r \}, \neg r \rangle\]  \[\langle \{ r, r \rightarrow \neg w \}, \neg w \rangle\]
The Overall Process

Steps
- Starting point: knowledge-base
- Form arguments
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Example

\[ \Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\} \]

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\{\{w, w \rightarrow \neg s\}, \neg s\} \\
\{\{s, s \rightarrow \neg r\}, \neg r\} \\
\{\{r, r \rightarrow \neg w\}, \neg w\}
\]
The Overall Process

Steps

• Starting point: knowledge-base
• Form arguments
• Identify conflicts
• Abstract from internal structure
• Resolve conflicts
• Draw conclusions

Example

\[ \Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\} \]

Diagram:

\[ F_{\Delta} : \]

\[ \alpha \rightarrow \beta \rightarrow \gamma \]
The Overall Process

Steps

- Starting point: knowledge-base
- Form arguments
- Identify conflicts
- Abstract from internal structure
- Resolve conflicts
- Draw conclusions

Example

\[ \Delta = \{ s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s \} \]

\[ F_\Delta : \]

\[ \text{pref}(F_\Delta) = \{ \emptyset \} \]

\[ \text{stage}(F_\Delta) = \{ \{ \alpha \}, \{ \beta \}, \{ \gamma \} \} \]
The Overall Process

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Example

\[ \Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\} \]

\[
\begin{align*}
\langle \{w, w\rightarrow \neg s\}, \neg s \rangle \\
\langle \{s, s\rightarrow \neg r\}, \neg r \rangle \\
\langle \{r, r\rightarrow \neg w\}, \neg w \rangle 
\end{align*}
\]

\[
Cn_{pref}(F_\Delta) = Cn(\top)
\]

\[
Cn_{stage}(F_\Delta) = Cn(\neg r \lor \neg w \lor \neg s)
\]
Some Remarks

- Main idea dates back to Dung [1995]; has then been refined by several authors (Prakken, Gordon, Caminada, etc.)
- Separation between logical (forming arguments) and nonmonotonic reasoning ("abstract argumentation frameworks")
- Abstraction allows to compare several KR formalisms on a conceptual level ("calculus of conflict")
Some Remarks

- Main idea dates back to Dung [1995]; has then been refined by several authors (Prakken, Gordon, Caminada, etc.)
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- Abstraction allows to compare several KR formalisms on a conceptual level ("calculus of conflict")

Main Challenge

- **All Steps** in the argumentation process are, in general, intractable.
- This calls for:
  - careful complexity analysis (identification of tractable fragments)
  - re-use of established tools for implementations (reduction method)
Approaches to Form Arguments

Classical Arguments [Besnard & Hunter, 2001]

- Given is a KB (a set of propositions) $\Delta$
- argument is a pair $(\Phi, \alpha)$, such that $\Phi \subseteq \Delta$ is consistent, $\Phi \models \alpha$ and for no $\Psi \subset \Phi$, $\Psi \models \alpha$
- conflicts between arguments $(\Phi, \alpha)$ and $(\Phi', \alpha')$ arise if $\Phi$ and $\alpha'$ are contradicting.
Approaches to Form Arguments

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Example

$$\langle \{s, s \rightarrow \neg r\}, \neg r \rangle \rightarrow \langle \{r, r \rightarrow \neg w\}, \neg w \rangle$$
Classical Arguments [Besnard & Hunter, 2001]

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Example

Other Approaches

- Arguments are trees of statements
- claims are obtained via strict and defeasible rules
- different notions of conflict: rebuttal, undercut, etc.
Dung’s Abstract Argumentation Frameworks

Example

\[ \alpha \rightarrow \gamma \rightarrow \beta \]
Dung’s Abstract Argumentation Frameworks

Example

Main Properties

- Abstract from the concrete content of arguments but only consider the relation between them
- Semantics select subsets of arguments respecting certain criteria
- Simple, yet powerful, formalism
- Most active research area in the field of argumentation.
  - “plethora of semantics”
Dung’s Abstract Argumentation Frameworks

**Definition**

An **argumentation framework** (AF) is a pair \((A, R)\) where

- \(A\) is a set of arguments
- \(R \subseteq A \times A\) is a relation representing the conflicts (“attacks”)

**Example**

\[ F = (\{a, b, c, d, e\}, \{(a, b), (c, b), (c, d), (d, c), (d, e), (e, e)\}) \]
Dung’s Abstract Argumentation Frameworks

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## Conflict-Free Sets

Given an AF \( F = (A, R) \).

A set \( S \subseteq A \) is conflict-free in \( F \), if, for each \( a, b \in S \), \( (a, b) \notin R \).
Conflict-Free Sets

Given an AF $F = (A, R)$. A set $S \subseteq A$ is conflict-free in $F$, if, for each $a, b \in S$, $(a, b) \notin R$.

Example

cf($F$) = $\{\{a, c\}\}$,
Basic Properties

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Example

$$\text{cf}(F) = \{\{a, c\}, \{a, d\}\},$$
Basic Properties

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Example

$c_f(F) = \{\{a, c\}, \{a, d\}, \{b, d\}\}.$
Basic Properties

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Given an AF $F = (A, R)$.
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Example

$cf(F) = \{\{a, c\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \emptyset\}$
Basic Properties (ctd.)

**Admissible Sets [Dung, 1995]**

Given an AF $F = (A, R)$. A set $S \subseteq A$ is admissible in $F$, if

- $S$ is conflict-free in $F$
- each $a \in S$ is **defended** by $S$ in $F$
  - $a \in A$ is defended by $S$ in $F$, if for each $b \in A$ with $(b, a) \in R$, there exists a $c \in S$, such that $(c, b) \in R$. 

Example:

```
b c d e a
adm(F) = \{\{a, c\}\}
```
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$$adm(F) = \{\{a, c\}, \{a, d\},$$
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Example

\[
\text{adm}(F) = \{\{a, c\}, \{a, d\}, \{b, d\}\},
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Basic Properties (ctd.)

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Example

![Diagram](image)

$adm(F) = \{\{a, c\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \emptyset\}$
Basic Properties (ctd.)

Dung’s Fundamental Lemma

Let $S$ be admissible in an AF $F$ and $a, a'$ arguments in $F$ defended by $S$ in $F$. Then,

1. $S' = S \cup \{a\}$ is admissible in $F$
2. $a'$ is defended by $S'$ in $F$
Naive Extensions

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a naive extension of $F$, if

- $S$ is conflict-free in $F$
- for each $T \subseteq A$ conflict-free in $F$, $S \not\subset T$
Naive Extensions

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Example

$naive(F) = \{\{a, c\}\}$
Semantics

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Grounded Extension [Dung, 1995]

Given an AF $F = (A, R)$. The unique grounded extension of $F$ is defined as the outcome $S$ of the following “algorithm”:

1. put each argument $a \in A$ which is not attacked in $F$ into $S$; if no such argument exists, return $S$;

2. remove from $F$ all (new) arguments in $S$ and all arguments attacked by them (together with all adjacent attacks); and continue with Step 1.
Semantics (ctd.)

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Example

```
ground(F) = \{ \{a\} \}
```

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Complete Extension [Dung, 1995]

Given an AF \((A, R)\). A set \(S \subseteq A\) is complete in \(F\), if

- \(S\) is admissible in \(F\)
- each \(a \in A\) defended by \(S\) in \(F\) is contained in \(S\)
  
  Recall: \(a \in A\) is defended by \(S\) in \(F\), if for each \(b \in A\) with \((b, a) \in R\), there exists a \(c \in S\), such that \((c, b) \in R\).
Semantics (ctd.)

**Complete Extension [Dung, 1995]**

Given an AF $(A, R)$. A set $S \subseteq A$ is **complete** in $F$, if

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**Example**

```
\begin{array}{cccc}
a & \rightarrow & b & \rightarrow c \\
\rightarrow & & \rightarrow & \rightarrow \\
\end{array}
```

$comp(F) = \{\{a, c\}\}$,
Complete Extension [Dung, 1995]

Given an AF \((A, R)\). A set \(S \subseteq A\) is complete in \(F\), if

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  - Recall: \(a \in A\) is defended by \(S\) in \(F\), if for each \(b \in A\) with \((b, a) \in R\), there exists a \(c \in S\), such that \((c, b) \in R\).

Example

\[ comp(F) = \{\{a, c\}, \{a, d\}\}, \]

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Example

\[ \text{comp}(F) = \{ \{a, c\}, \{a, d\}, \{a\} \}, \]
Semantics (ctd.)

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Given an AF \((A, R)\). A set \(S \subseteq A\) is complete in \(F\), if

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Example

\[
\text{comp}(F) = \{\{a, c\}, \{a, d\}, \{a\}, \{e\}, \{d\}, \emptyset\}
\]
Properties of the Grounded Extension

For any AF $F$, the grounded extension of $F$ is the subset-minimal complete extension of $F$. 

Remark: Since there exists exactly one grounded extension for each AF $F$, we often write $\text{ground}(F) = \{S\}$ instead of $\text{ground}(F) = \{\}$. 

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Semantics (ctd.)

### Properties of the Grounded Extension

For any AF $F$, the grounded extension of $F$ is the subset-minimal complete extension of $F$.

### Remark

Since there exists exactly one grounded extension for each AF $F$, we often write $\text{ground}(F) = S$ instead of $\text{ground}(F) = \{S\}$.
Preferred Extensions [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a preferred extension of $F$, if

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Example

$\text{pref}(F) = \{\{a, c\}, \{a, d\}, \{a\}, \{c\}, \{d\}, \emptyset\}$
Semantics (ctd.)

Stable Extensions [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a stable extension of $F$, if

- $S$ is conflict-free in $F$
- for each $a \in A \setminus S$, there exists a $b \in S$, such that $(b, a) \in R$
Semantics (ctd.)

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Example

$stable(F) = \{\{a, e\}\}$
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Example

\[
\text{stable}(F) = \{ \{a, e\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{e\}, \{d\}, \emptyset \}
\]
Some Relations

For any AF $F$ the following relations hold:

1. Each stable extension of $F$ is admissible in $F$
2. Each stable extension of $F$ is also a preferred one
3. Each preferred extension of $F$ is also a complete one
Semi-Stable Extensions [Caminada, 2006]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a semi-stable extension of $F$, if

- $S$ is admissible in $F$
- for each $T \subseteq A$ admissible in $F$, $S^+ \not\subset T^+$
- for $S \subseteq A$, define $S^+ = S \cup \{a \mid \exists b \in S \text{ with } (b, a) \in R\}$
Semi-Stable Extensions [Caminada, 2006]

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- for \( S \subseteq A \), define \( S^+ = S \cup \{a \mid \exists b \in S \text{ with } (b, a) \in R\} \)

Example

\[
\text{semi}(F) = \{\{a, e\}, \{a, d\}, \{a\}, \{c\}, \{d\}, \emptyset\}
\]
Stage Extensions [Verheij, 1996]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a stage extension of $F$, if

- $S$ is conflict-free in $F$
- for each $T \subseteq A$ conflict-free in $F$, $S^+ \not\subseteq T^+$
  - recall $S^+ = S \cup \{a \mid \exists b \in S \text{ with } (b, a) \in R\}$
Semantics (ctd.)

### Stage Extensions [Verheij, 1996]
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### Ideal Extension [Dung, Mancarella & Toni 2007]
Given an AF $F = (A, R)$. A set $S \subseteq A$ is an ideal extension of $F$, if
- $S$ is admissible in $F$ and contained in each preferred extension of $F$
- there is no $T \supset S$ admissible in $F$ and contained in each of $\text{pref}(F)$
Stage Extensions [Verheij, 1996]
Given an AF $F = (A, R)$. A set $S \subseteq A$ is a stage extension of $F$, if

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Eager Extension [Caminada, 2007]
Given an AF $F = (A, R)$. A set $S \subseteq A$ is an eager extension of $F$, if

- $S$ is admissible in $F$ and contained in each semi-stable extension of $F$
- there is no $T \supset S$ admissible in $F$ and contained in each of $\text{semi}(F)$
Semantics (ctd.)

Properties of Ideal Extensions

For any AF $F$ the following observations hold:

1. there exists exactly one ideal extension of $F$
2. the ideal extension of $F$ is also a complete one

The same results hold for the eager extension and similar variants [Dvořák et al., 2011].
Resolution-based grounded Extensions
[Baroni, Giacomin 2008]

A resolution $\beta$ of an AF $F = (A, R)$ contains exactly one of the attacks $(a, b)$, $(b, a)$ for each pair $a, b \in A$ with $\{(a, b), (b, a)\} \subseteq R$.

A set $S \subseteq A$ is a resolution-based grounded extension of $F$, if

- there exists a resolution $\beta$ such that $\text{ground}((A, R \setminus \beta)) = S$
- and there is no resolution $\beta'$ such that $\text{ground}((A, R \setminus \beta')) \subset S$
Resolution-based grounded Extensions
[Baroni,Giacomin 2008]

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- and there is no resolution $\beta'$ such that $\text{ground}((A, R \setminus \beta')) \subset S$

Example

$\text{ground}^*(F) = \{ \{a, c\} \}$,
Resolution-based grounded Extensions
[Baroni,Giacomin 2008]

A resolution $\beta$ of an AF $F = (A, R)$ contains exactly one of the attacks $(a, b)$, $(b, a)$ for each pair $a, b \in A$ with $\{(a, b), (b, a)\} \subseteq R$.

A set $S \subseteq A$ is a resolution-based grounded extension of $F$, if
- there exists a resolution $\beta$ such that $\text{ground}((A, R \setminus \beta)) = S$
- and there is no resolution $\beta'$ such that $\text{ground}((A, R \setminus \beta')) \subset S$

Example

$\text{ground}^*(F) = \{\{a, c\}, \{a, d\}\}$
Definition (Separation)

An AF $F = (A, R)$ is called separated if for each $(a, b) \in R$, there exists a path from $b$ to $a$. We define $[[F]] = \bigcup_{C \in SCCs(F)} F|_C$ and call $[[F]]$ the separation of $F$.

Example
Definition (Separation)

An AF $F = (A, R)$ is called separated if for each $(a, b) \in R$, there exists a path from $b$ to $a$. We define $[[F]] = \bigcup_{C \in SCCs(F)} F|_C$ and call $[[F]]$ the separation of $F$.
Definition (Reachability)

Let $F = (A, R)$ be an AF, $B$ a set of arguments, and $a, b \in A$. We say that $b$ is reachable in $F$ from $a$ modulo $B$, in symbols $a \Rightarrow^B_F b$, if there exists a path from $a$ to $b$ in $F|_B$. 
Definition (Reachability)

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Definition ($\Delta_{F,S}$)

For an AF $F = (A, R)$, $D \subseteq A$, and a set $S$ of arguments,

$$\Delta_{F,S}(D) = \{a \in A \mid \exists b \in S : b \neq a, (b, a) \in R, a \not\Rightarrow^A_D b\}.$$  

By $\Delta_{F,S}$, we denote the lfp of $\Delta_{F,S}(\emptyset)$.
Given an AF $F = (A, R)$. A set $S \subseteq A$ is a cf2-extension of $F$, if

- $S$ is conflict-free in $F$
- and $S \in naive([F - \Delta_{F,s}])$. 

Example $S = \{c, f, h\}$, $S \in cf2(F)$. 

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Semantics (ctd.)

**cf2 Extensions [G & Woltran 2010]**

Given an AF \( F = (A, R) \). A set \( S \subseteq A \) is a cf2-extension of \( F \), if

- \( S \) is conflict-free in \( F \)
- and \( S \in naive([F - \Delta_{F,S}]) \).

**Example**

\( S = \{c,f,h\}, S \in cf(F) \).
cf2 Semantics (ctd.)

**cf2 Extensions [G & Woltran 2010]**

Given an AF \( F = (A, R) \). A set \( S \subseteq A \) is a cf2-extension of \( F \), if

- \( S \) is conflict-free in \( F \)
- and \( S \in naive([[F - \Delta_F], S]]) \).

**Example**

\( S = \{c, f, h\}, \ S \in cf(F), \ \Delta_{F, S}(\emptyset) = \{d, e\} \).
cf2 Semantics (ctd.)

cf2 Extensions [G & Woltran 2010]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a cf2-extension of $F$, if

- $S$ is conflict-free in $F$
- and $S \in \text{naive}([[F - \Delta_{F,S}]]).$

Example

$S = \{c, f, h\}$, $S \in \text{cf}(F)$, $\Delta_{F,S}(\{d, e\}) = \{d, e\}$. 

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cf2 Extensions [G & Woltran 2010]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a cf2-extension of $F$, if

- $S$ is conflict-free in $F$
- and $S \in naive([[F - \Delta_{F,S}]]).

Example

$S = \{c, f, h\}$, $S \in cf(F)$, $\Delta_{F,S} = \{d, e\}$, $S \in naive([[F - \Delta_{F,S}]]).$
Relations between Semantics

Figure: An arrow from semantics $\sigma$ to semantics $\tau$ encodes that each $\sigma$-extension is also a $\tau$-extension.
Characteristics of Argumentation Semantics

Example

\[ \text{pref}(F) = \{\{a, d, e\}, \{b, c, e\}, \{a, b\}\} \]
\[ \text{naive}(F) = \{\{a, d, e\}, \{b, c, e\}, \{a, b, e\}\} \]

Natural Questions

- How to change the AF if we want \(\{a, b, e\}\) instead of \(\{a, b\}\) in \(\text{pref}(F)\)?
- How to change the AF if we want \(\{a, b, d\}\) instead of \(\{a, b\}\) in \(\text{pref}(F)\)?
- Can we have equivalent AFs without argument \(f\)?
Some Properties . . .

**Theorem**

For any AFs $F$ and $G$, we have

- $\text{adm}(F) = \text{adm}(G) \implies \sigma(F) = \sigma(G)$, for $\sigma \in \{\text{pref}, \text{ideal}\}$;
- $\text{comp}(F) = \text{comp}(G) \implies \vartheta(F) = \vartheta(G)$, for $\vartheta \in \{\text{pref}, \text{ideal}, \text{ground}\}$;
- no other such relation between the different semantics ($\text{adm}$, $\text{pref}$, $\text{ideal}$, $\text{semi}$, $\text{eager}$, $\text{ground}$, $\text{comp}$, $\text{stable}$) in terms of standard equivalence holds.
Strong Equivalence [Oikarinen & Woltran 2011, G & Woltran 2011]

**Definition**

Two AFs $F$ and $G$ are strongly equivalent wrt. a semantics $\sigma \in \{\text{stable}, \text{adm}, \text{pref}, \text{ideal}, \text{semi}, \text{comp}, \text{ground}, \text{stage}\}$, in symbols $F \equiv^\sigma_s G$, iff $\sigma(F \cup H) = \sigma(G \cup H)$, for each AF $H$.

- Idea: Find “$\sigma$-kernels” of AFs, such that the $\sigma$-kernels of $F$ and $G$ coincide iff $F \equiv^\sigma_s G$.
- Verification of strong equivalence then reduces to checking syntactical equivalence
For any AFs $F$ and $G$: $F^\kappa = G^\kappa$ iff $F \equiv_s^{\text{stable}} G$ iff $F \equiv_s^{\text{stage}} G$. 
Exercises

1. Give an AF $F$ such that $\text{stable}(F) = \emptyset$ and $\text{semi}(F) \neq \{\emptyset\}$.

2. Show that the following statement holds for any AF $F$.
   If $\text{stable}(F) \neq \emptyset$ then $\text{stable}(F) = \text{semi}(F) = \text{stage}(F)$.

3. Select three different semantics $\sigma, \sigma', \sigma''$ out of \{pref, ideal, semi, eager, ground, stable\} of your choice and provide three pairs of AFs such that
   - $\sigma(F_1) = \sigma(G_1)$ but $\sigma'(F_1) \neq \sigma'(G_1)$
   - $\sigma'(F_2) = \sigma'(G_2)$ but $\sigma''(F_2) \neq \sigma''(G_2)$
   - $\sigma''(F_3) = \sigma''(G_3)$ but $\sigma(F_3) \neq \sigma(G_3)$
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