TECHNISCHE UNIVERSITÄT DRESDEN

Artificial Intelligence, Computational Logic

SEMINAR ABSTRACT ARGUMENTATION

Computational Complexity of Abstract Argumentation

*slides adapted from Stefan Woltran's lecture on Abstract Argumentation

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Dresden, 8th November 2013

DRESDEN concept
Exzellenz aus Wissenschaft und Kultur

Outline

Computational Complexity Theory

. . . studies the asymptotic worst case complexity of problems.

That is Computational Complexity Theory addresses the

- costs to solve a problem (and not the runtime of an algorithm).
- computational costs, i.e. how hard is it for a computer to solve the problem.
- asymptotic behavior, i.e. how the costs scale with the size of the problem instances.
- worst case behavior, i.e. the costs to solve the hardest instances of a problem.

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To show that a problem A is at most as hard as a problem B, (denoted as $A \leq^R B$) one gives a reduction R from A to B satisfying:

- If I is an instance of A then $R(I)$ is an instance of B.
- $A(I) = B(R(I))$, e.g. I is a yes instance of A iff $R(I)$ is a yes instance of B
- R can be efficiently computed (i.e. in polynomial time).

Complexity classes: Class of problems that can be solved within some computational limits.

A problem A is a complete problem for a complexity class, if A is in the class and each problem B in the class can be reduced to A.
So the complete problems w.r.t. a complexity class form a equivalence class w.r.t. reducibility.

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the class Σ_2^p:

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Relating Complexity classes

The following relations hold for the introduced complexity classes:

$$\mathsf{L} \subseteq \mathsf{P} \subseteq \begin{matrix}\mathsf{NP}\\ \text{co-NP}\end{matrix} \subseteq \Theta_2^p \subseteq \begin{matrix}\Sigma_2^p\\ \Pi_2^p\end{matrix}$$

We consider problems in L and P to be computationally tractable and problems complete one of the classes NP, co-NP, Θ_2^p, Σ_2^p, Π_2^p as computationally intractable.

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  If a formalism has too low complexity, there are a lot of interesting problems of higher complexity that can not be expressed within the formalism (Knowledge-Representation View).
- For applying the Translation-Approach, i.e. encode my problem in other formalisms. The target formalism must be at least of the same complexity. To avoid unnecessary computational effort one would prefer a (fragment of a) formalism providing a complexity close to the original problem (Practitioners View).

Decision Problems on AFs

Credulous Acceptance

Cred$_\sigma$: Given AF $F = (A, R)$ and $a \in A$; is a contained in at least one σ-extension of F?

Skeptical Acceptance

Skept$_\sigma$: Given AF $F = (A, R)$ and $a \in A$; is a contained in every σ-extension of F?

If no extension exists then all arguments are skeptically accepted and no argument is credulously accepted[1].

[1] This is only relevant for stable semantics.

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Hence we are also interested in the following problem:

Skeptically and Credulously accepted

Skept'_σ: Given AF $F = (A, R)$ and $a \in A$; is a contained in every and at least one σ-extension of F?

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Further Decision Problems

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Does there exist an extension?

Exists$_\sigma$: Given AF $F = (A, R)$; Does there exist a σ-extension for F?

Does there exist a nonempty extensions?

Exists$_\sigma^{\neg\emptyset}$: Does there exist a non-empty σ-extension for F?

Computationally Easy Problems

Given an Argumentation Framework $F = (A, R)$, an argument $a \in A$ and a set $E \subseteq A$.

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  Compute the grounded extension $ground(F)$.

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Intractable problems in Abstract Argumentation

Most problems in Abstract Argumentation are computationally intractable, i.e. at least NP-hard. To show intractability for a specific reasoning problem we follow the schema given below:

Goal: Show that a reasoning problem is NP-hard.

Method: Reducing the NP-hard SAT problem to the reasoning problem.

- Consider an arbitrary CNF formula φ
- Give a reduction that maps φ to an Argumentation Framework F_φ containing an argument φ.
- Show that φ is satisfiable iff the argument φ is accepted.

Canonical Reduction

Definition

For $\varphi = \bigwedge_{i=1}^{m} l_{i1} \vee l_{i2} \vee l_{i3}$ over atoms Z, build $F_\varphi = (A_\varphi, R_\varphi)$ with

$$
\begin{aligned}
A_\varphi &= Z \cup \bar{Z} \cup \{C_1, \ldots, C_m\} \cup \{\varphi\} \\
R_\varphi &= \{(z, \bar{z}), (\bar{z}, z) \mid z \in Z\} \cup \{(C_i, \varphi) \mid i \in \{1, \ldots, m\}\} \cup \\
&\quad \{(z, C_i) \mid i \in \{1, \ldots, m\}, z \in \{l_{i1}, l_{i2}, l_{i3}\}\} \cup \\
&\quad \{(\bar{z}, C_i) \mid i \in \{1, \ldots, m\}, \neg z \in \{l_{i1}, l_{i2}, l_{i3}\}\}
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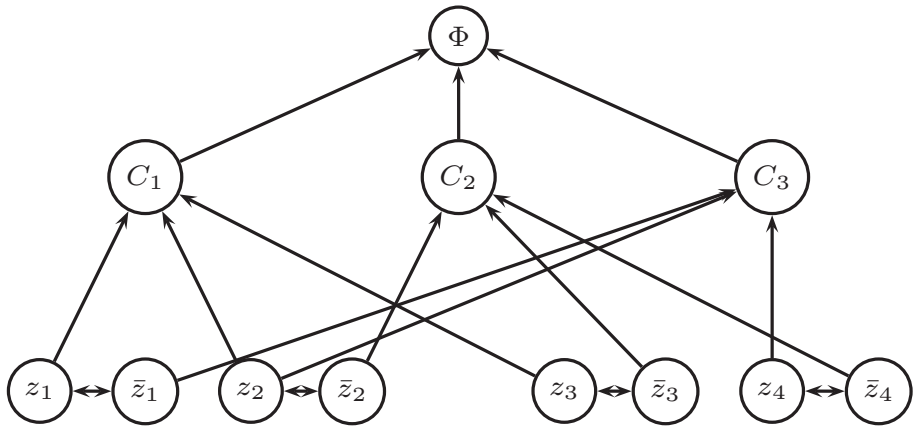
Example

Let $\Phi = (z_1 \vee z_2 \vee z_3) \wedge (\neg z_2 \vee \neg z_3 \vee \neg z_4) \wedge (\neg z_1 \vee z_2 \vee z_4)$.



Canonical Reduction: CNF $\Rightarrow$ AF (ctd.)

Theorem

The following statements are equivalent:

1. φ is satisfiable
2. F_φ has an admissible set containing φ
3. F_φ has a complete extension containing φ
4. F_φ has a preferred extension containing φ
5. F_φ has a stable extension containing φ

Proof.

$(1) \Leftrightarrow (5)$: blackboard $\square$

Complexity Results

Theorem

1. Cred_{stable} is NP-complete
2. Cred_{adm} is NP-complete
3. Cred_{comp} is NP-complete
4. Cred_{pref} is NP-complete

Proof.

(1) The hardness is immediate by the last theorem.
For the NP-membership we use the following guess & check algorithm:

- Guess a set $E \subseteq A$
- verify that E is stable
  - for each $a, b \in E$ check $(a, b) \notin R$
  - for each $a \in A \setminus E$ check if there exists $b \in E$ with $(b, a) \in R$

As this algorithm is in polynomial time we obtain NP-membership. □

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Theorem

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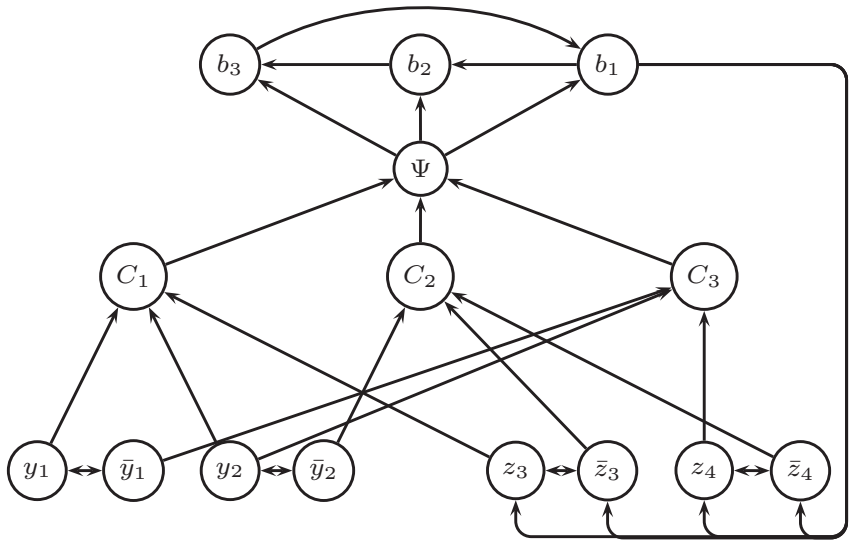
Skept$_{pref}$ is actually harder than co-NP.

Reduction for Skept$_{pref}$ (sketched)

Idea

Use a reduction from quantified Boolean formulae (QBFs) to AFs.

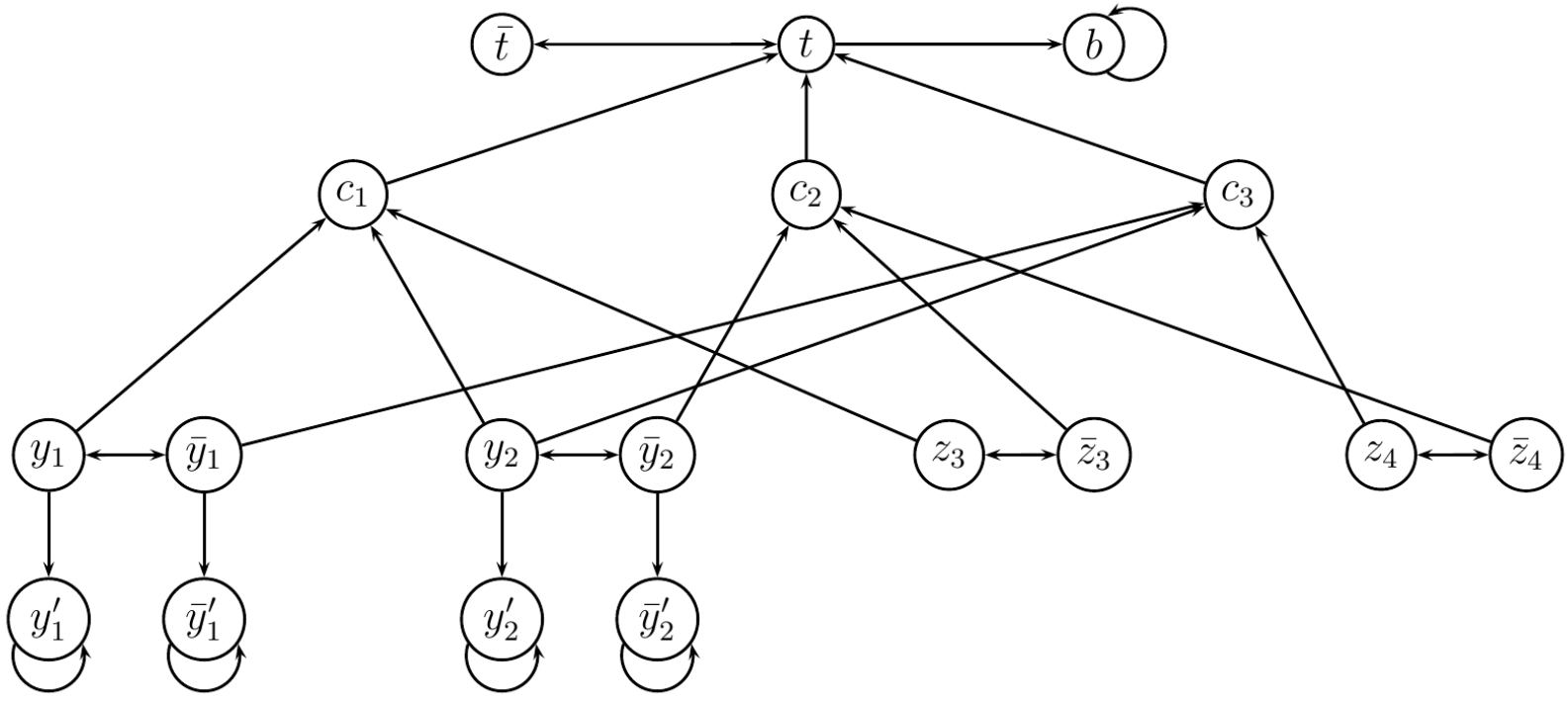
For $\forall y_1 y_2 \exists z_3 z_4 (y_1 \vee y_2 \vee z_3) \wedge (\neg y_2 \vee \neg z_3 \vee \neg z_4) \wedge (\neg y_1 \vee y_2 \vee z_4)$ use



Note that Ψ is contained in each preferred extension iff the QBF is true.

Reduction for Semi-Stable / Stage Semantics (sketched)

For $\forall y_1, y_2\ \exists z_3, z_4\ (y_1 \vee y_2 \vee z_3) \wedge (\neg y_2 \vee \neg z_3 \vee \neg z_4) \wedge (\neg y_1 \vee \neg y_2 \vee z_4)$ use



t is contained in all semi-stable extensions iff the QBF is true.

Complexity Results (Summary)

Complexity for decision problems in AFs.

σ	Cred_σ	Skept_σ
ground	P-c	P-c
naive	in L	in L
stable	NP-c	co-NP-c
adm	NP-c	trivial
comp	NP-c	P-c
pref	NP-c	Π_2^p-c

σ	Cred_σ	Skept_σ
semi	Σ_2^p-c	Π_2^p-c
stage	Σ_2^p-c	Π_2^p-c
ideal	in Θ_2^p	in Θ_2^p
eager	Π_2^p-c	Π_2^p-c
ground*	NP-c	co-NP-c
cf2	NP-c	co-NP-c

see [Baroni et al.2011, Coste-Marquis et al.2005, Dimopoulos and Torres1996, Dung1995, Dunne2008, Dunne and Bench-Capon2002, Dunne and Bench-Capon2004, Dunne and Caminada2008, Dvořák et al.2011, Dvořák and Woltran2010a, Dvořák and Woltran2010b]

Tractable Fragments

Complexity results show that many argumentation problems are computationally intractable! In order to obtain tractable fragments, imposing restrictions on the graph structure seems natural:

[2] Selfattacks, i.e. attacks $(a, a) \in R$, are prohibited

[3] Problems w.r.t. stage semantics remain hard.

Tractable Fragments

Complexity results show that many argumentation problems are computationally intractable! In order to obtain tractable fragments, imposing restrictions on the graph structure seems natural:

- acyclic AFs . . . problems become tractable (folklore)
- symmetric AFs [Coste-Marquis et al. 05][2]
- bipartite AFs [Dunne 07]
- AFs without even-length cycles [Dunne and Bench-Capon 02][3]

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[3] Problems w.r.t. stage semantics remain hard.

Fixed-Parameter Tractability (FPT) – Motivation

Some Observations

- For intractable problems, computational costs often depend primarily on some problem parameters rather than on the mere size of the instances.
- Many hard problems become tractable if some problem parameter is fixed or bounded by a fixed constant.
- Typical parameters for graphs: tree-width and clique-width.
  - Meta-theorems allow for rather easy proofs of FPT results w.r.t. these parameters
  - Dedicated dynamic algorithms required for practical realization!

  (Recall: AFs are naturally represented as graphs)

Fixed-Parameter Tractability (FPT)

FPT is one branch in the area of Parameterized Complexity

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Goal:

- Find a parameter k and algorithm such that each problem instance I can be decided in time $f(k) \cdot |I|^{O(1)}$ or
- prove that an given parameter does not provide such an algorithm.

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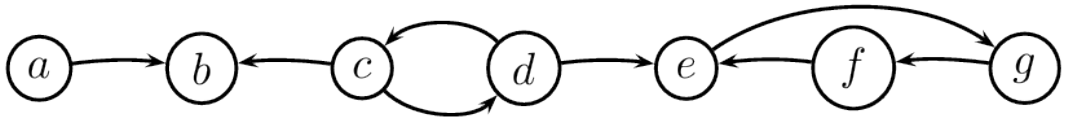
Here we briefly address:

- Definition and comparison: tree-width and clique-width
- FPT results for abstract argumentation via meta-theorems [Dunne, 2007; Dvořák et al., 2012]

Tree-Width

Argumentation Framework

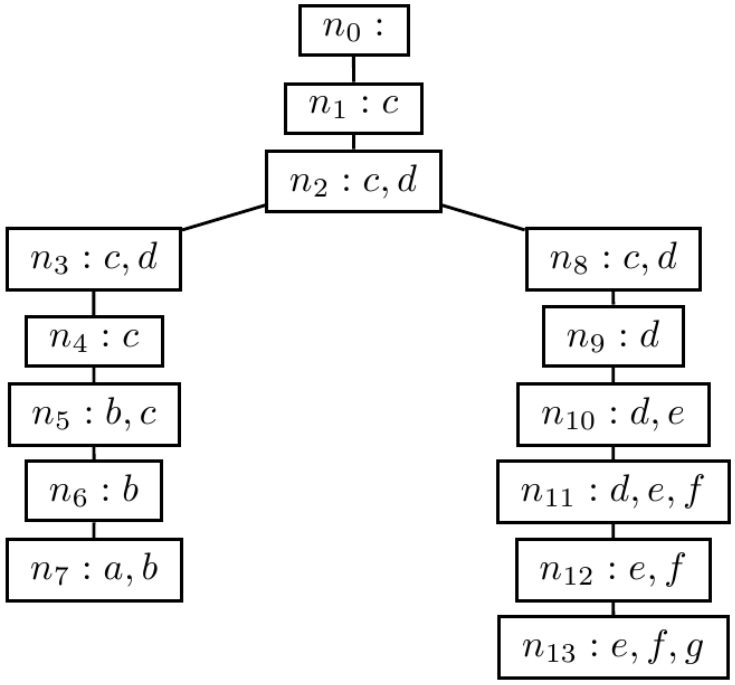


Properties

For an AF $F = (A, R)$:

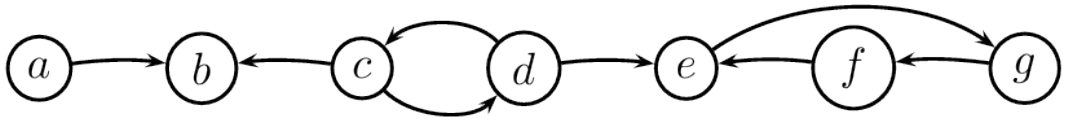
1. Each argument $a \in A$ and each attack $(b, c) \in R$ is contained in at least one bag
2. Bags containing the same argument are connected

Tree-Decomposition:



Tree-Width

Argumentation Framework

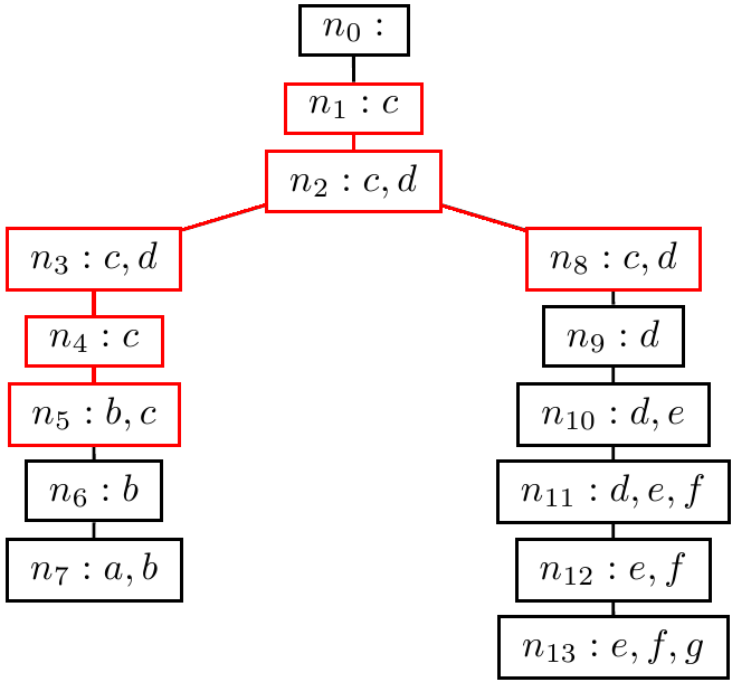


Properties

For an AF $F = (A, R)$:

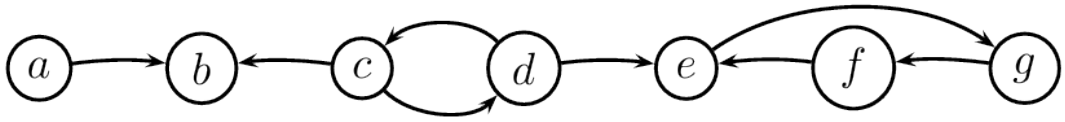
1. Each argument $a \in A$ and each attack $(b, c) \in R$ is contained in at least one bag
2. Bags containing the same argument are connected

Tree-Decomposition:



Tree-Width

Argumentation Framework

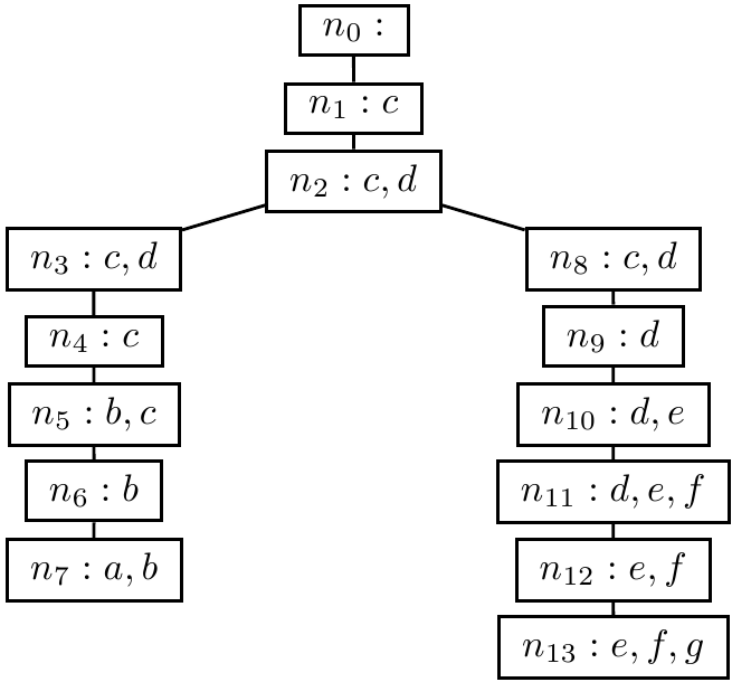


Definition

width of a tree-decomposition is the size off the largest bag -1

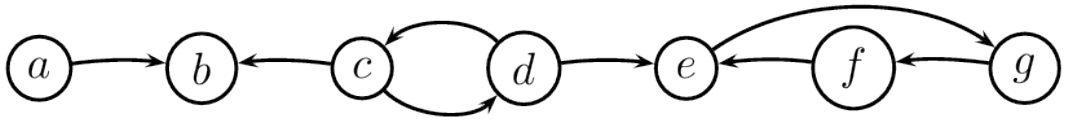
tree-width of an AF is the minimum width over all possible tree-decompositions

Tree-Decomposition:



Tree-Width

Argumentation Framework



Definition

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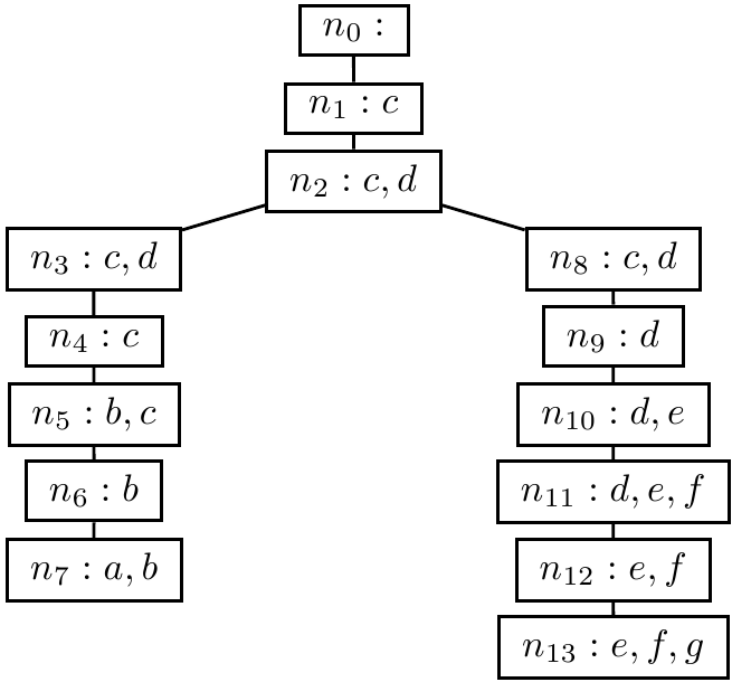
Example

AF has tree-width 2

Tree-Decomposition:



Tree-Width (ctd)

DYNPARTIX System

- Implements dynamic algorithms on tree-decompositions for several argumentation semanitcs following [Dvořák et al., 2012].
- Available under

  http://www.dbai.tuwien.ac.at/research/project/argumentation/dynpartix/

Clique-Width

Operations

We allow the following operations on labeled AFs:

- Node introduction (denote by $i(v)$);
- Disjoint union (denoted by $\oplus$);
- Relabeling: changing all labels i to j (denoted by $\rho_{i \to j}$);
- Edge insertion: connecting all vertices labeled by i with all vertices labeled by j (denoted by $\eta_{i,j}$); already existing edges are not doubled.

Note: $\rho_{i \to j}$ and $\eta_{i,j}$ treat equally labeled arguments in the same way.

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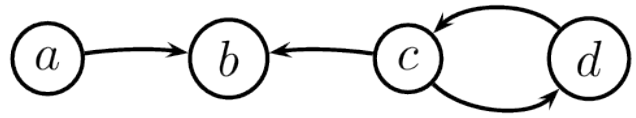
Definition

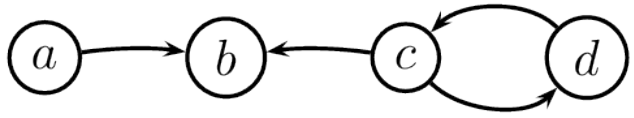
A term composed of $i(v)$, $\oplus$, $\rho_{i \to j}$, and $\eta_{i,j}$ using labels from $\{1, \ldots, k\}$ is called k-expression.
The clique-width of an AF F is the smallest integer k such that F can be defined by a k-expression.

Clique-Width – Example

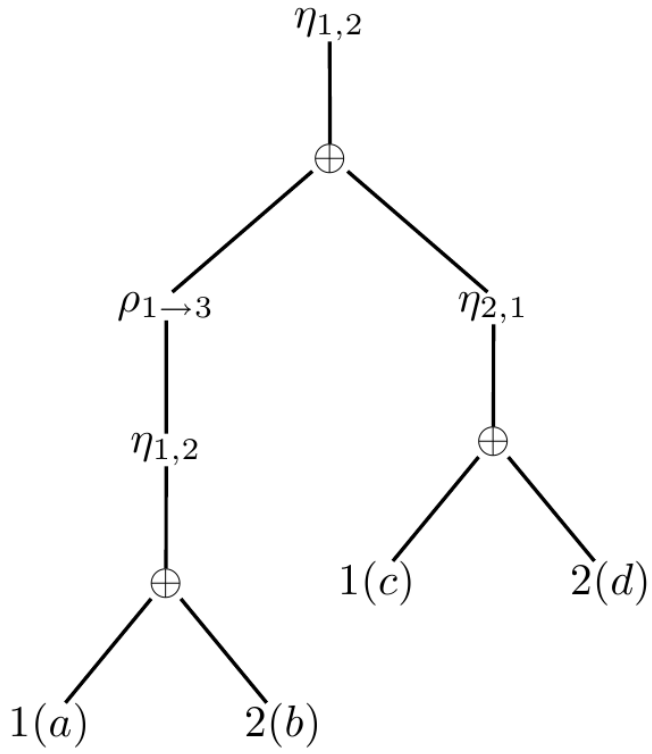
k-expression for the following AF?



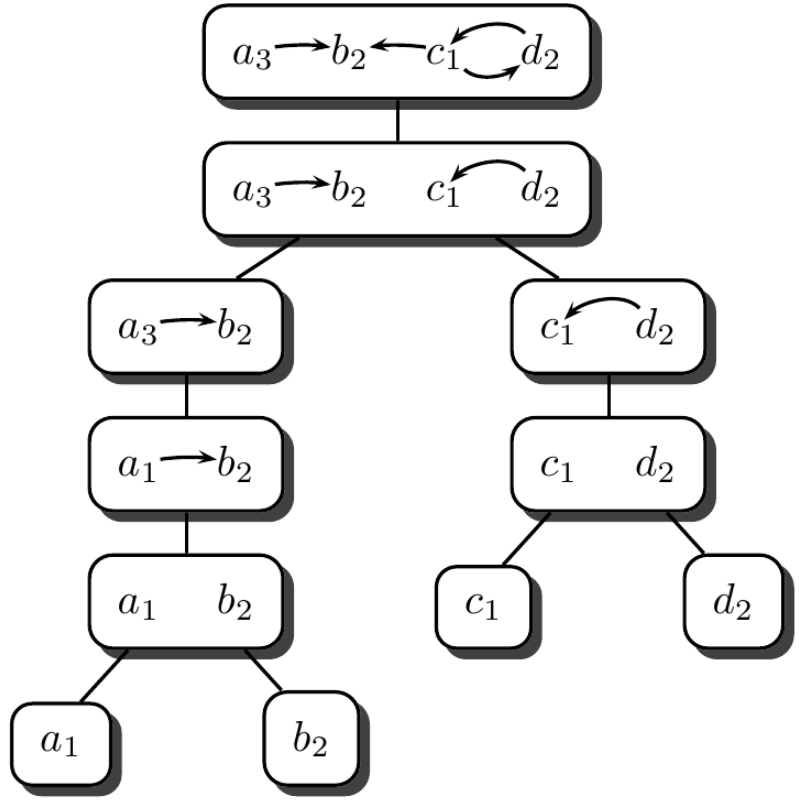
Clique-Width – Example



Parse Tree



associated AFs



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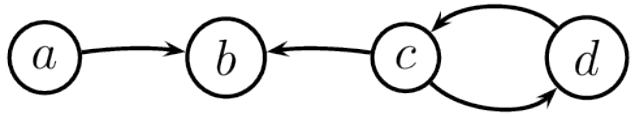

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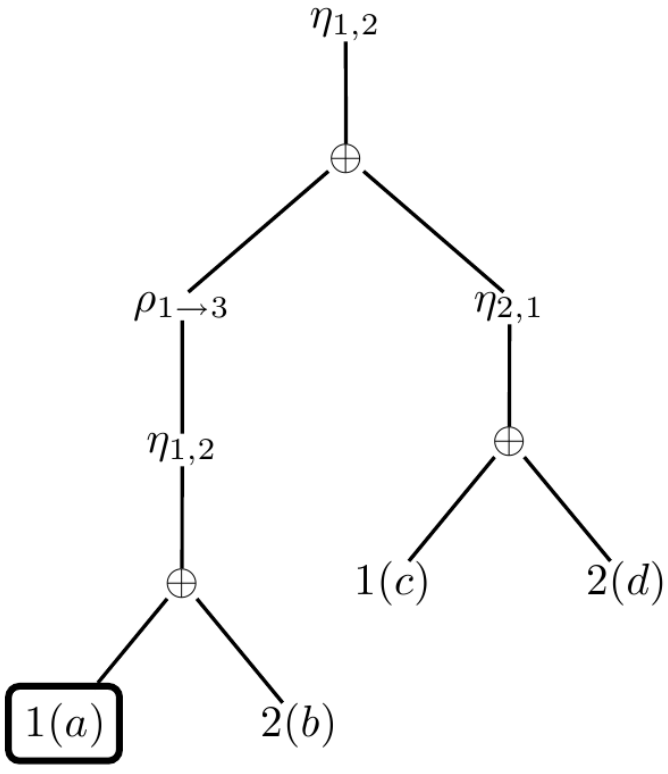

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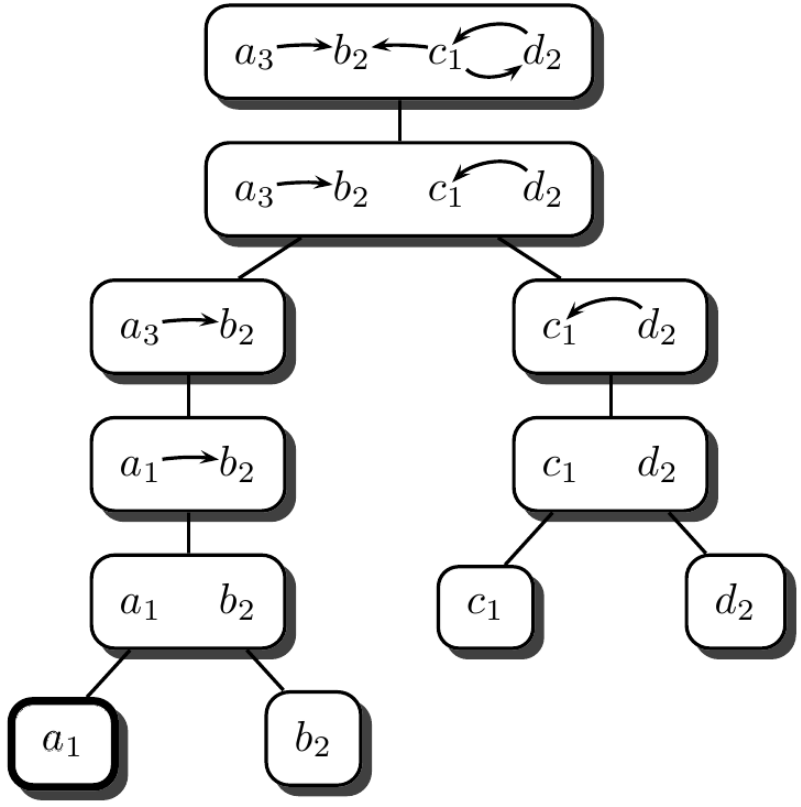

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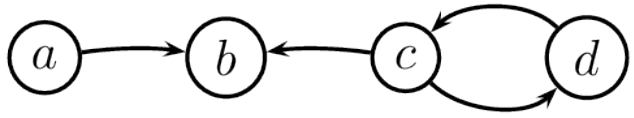

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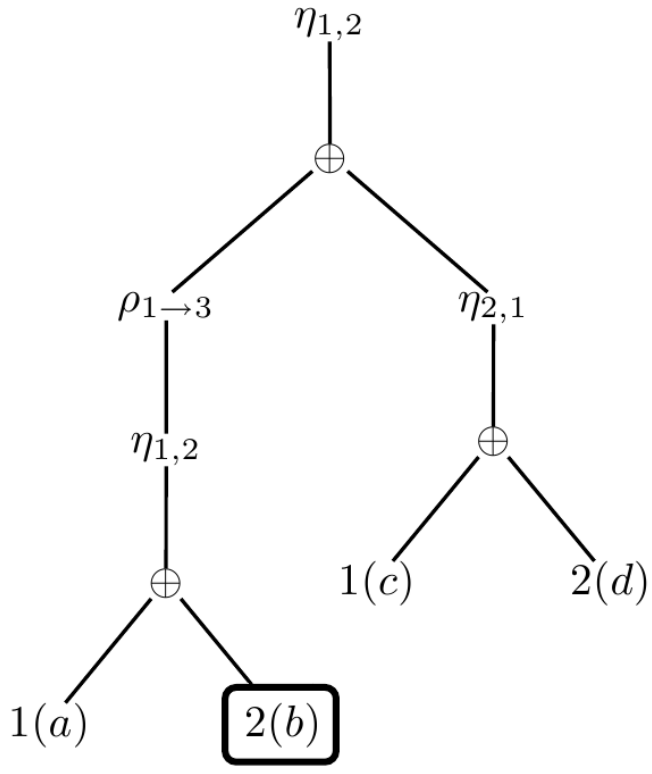

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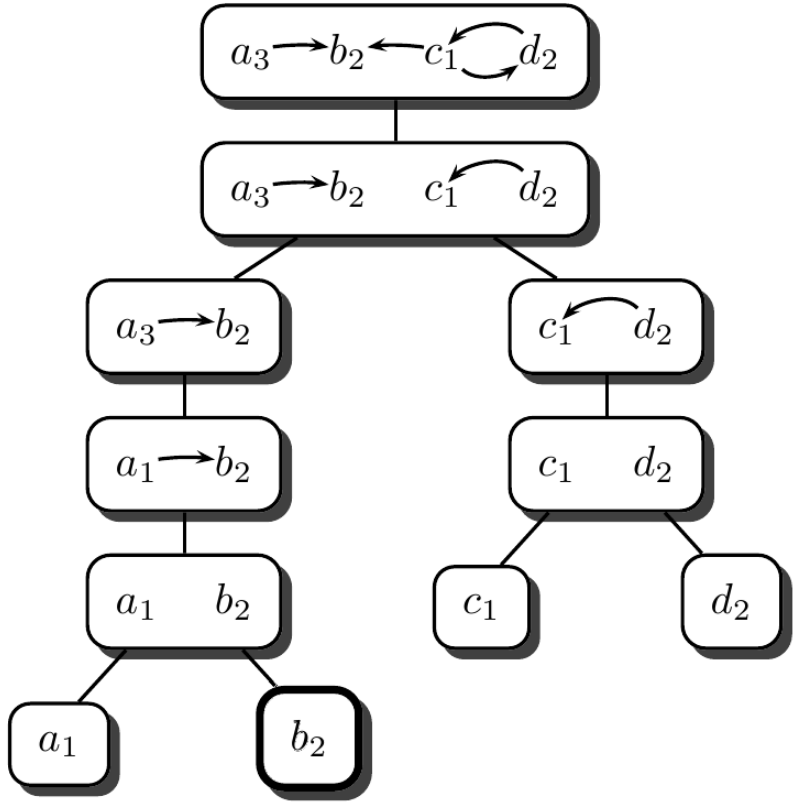

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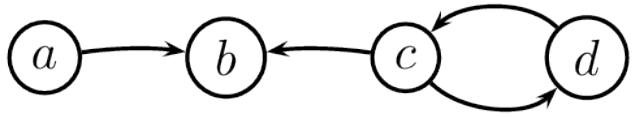

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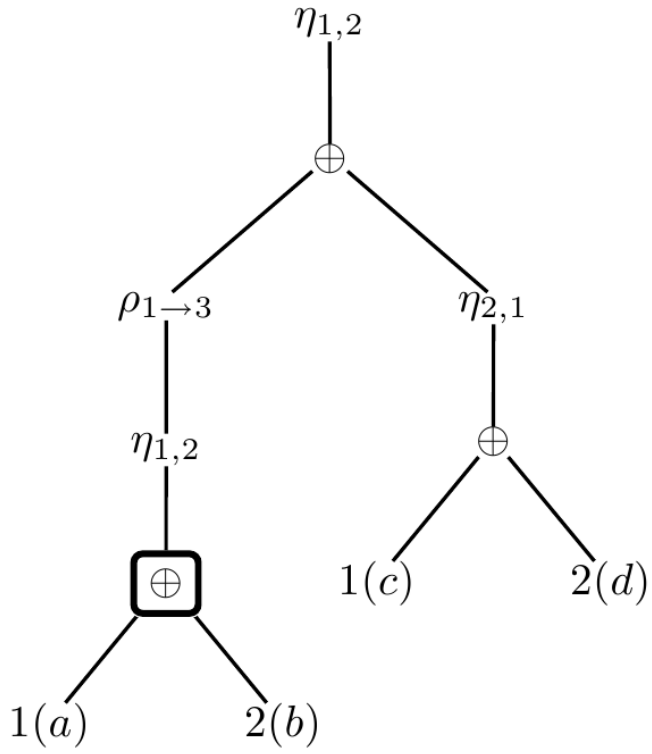

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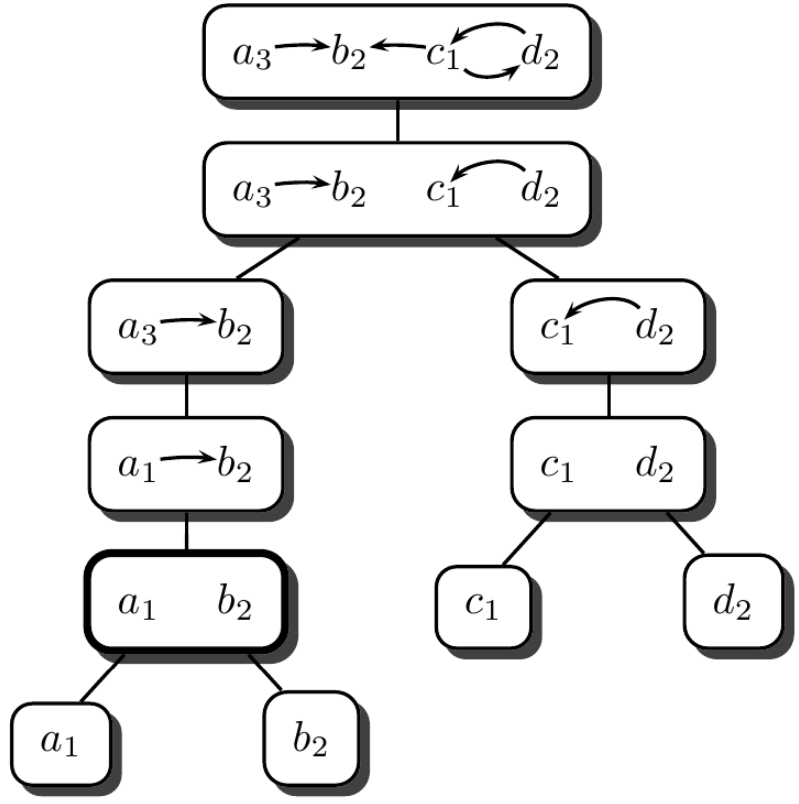

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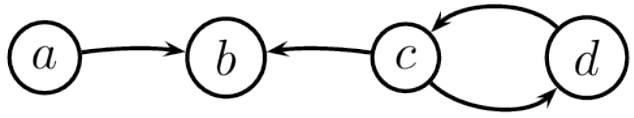

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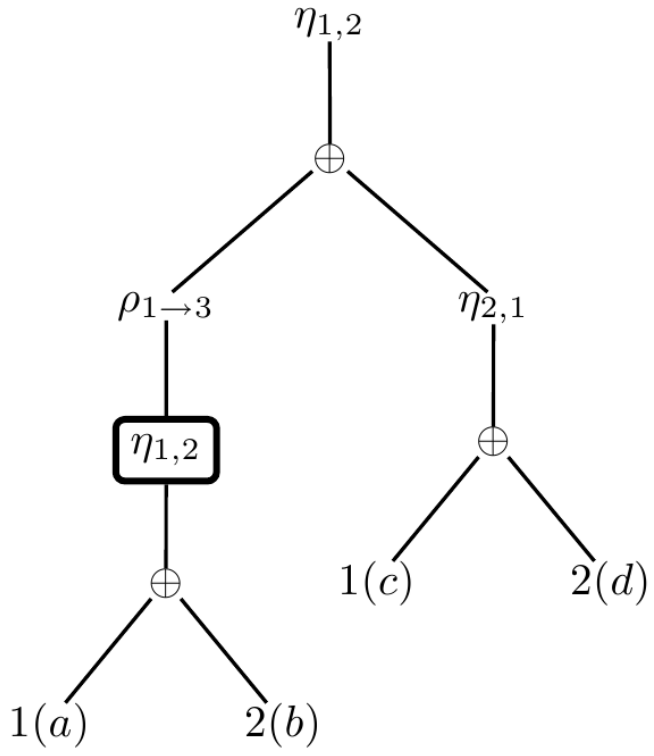

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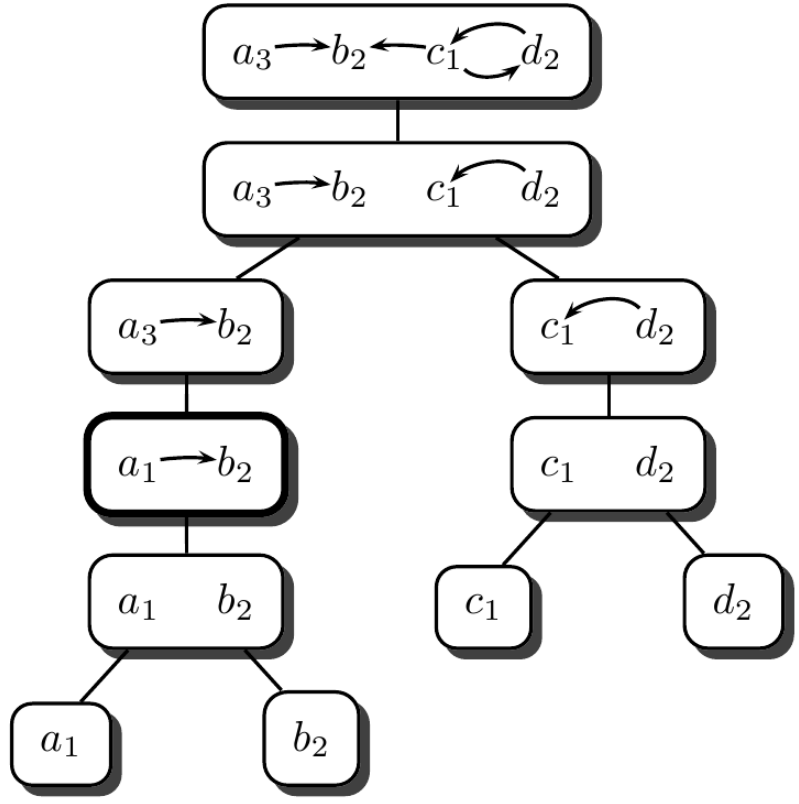

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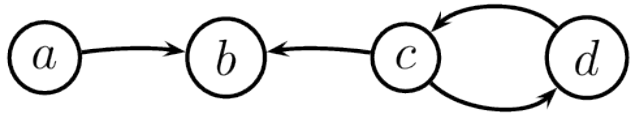

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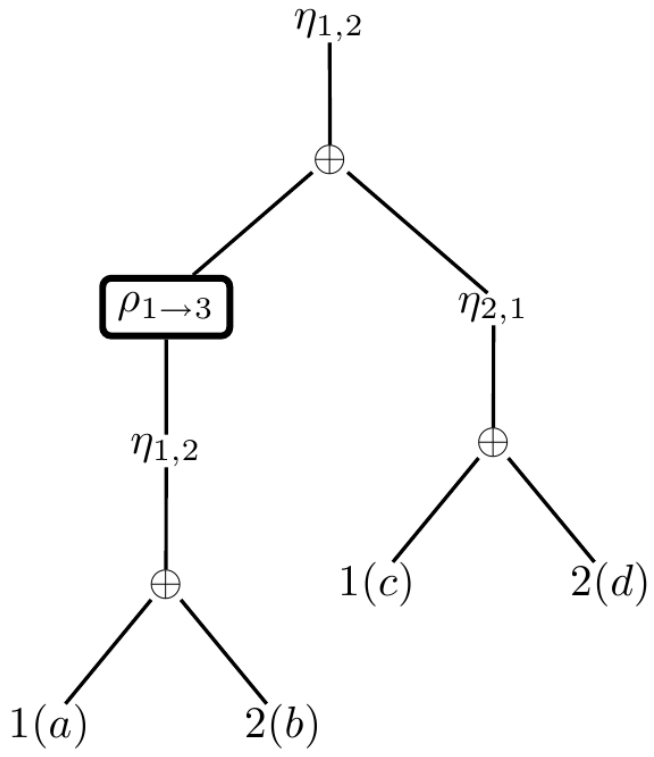

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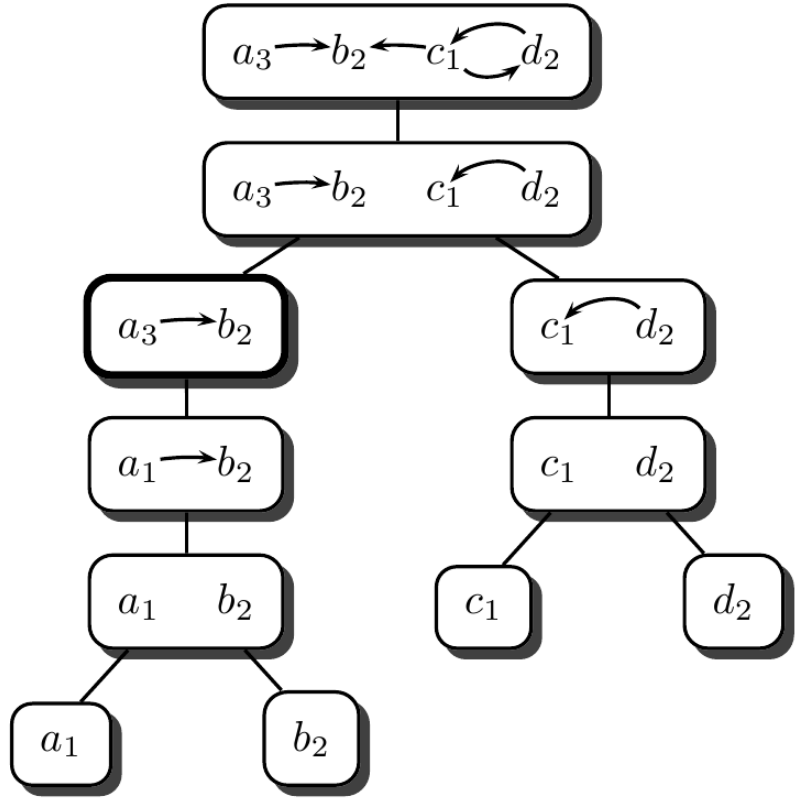

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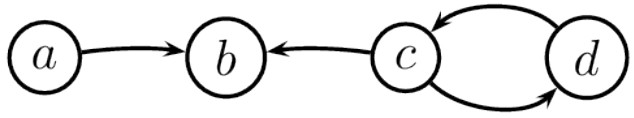

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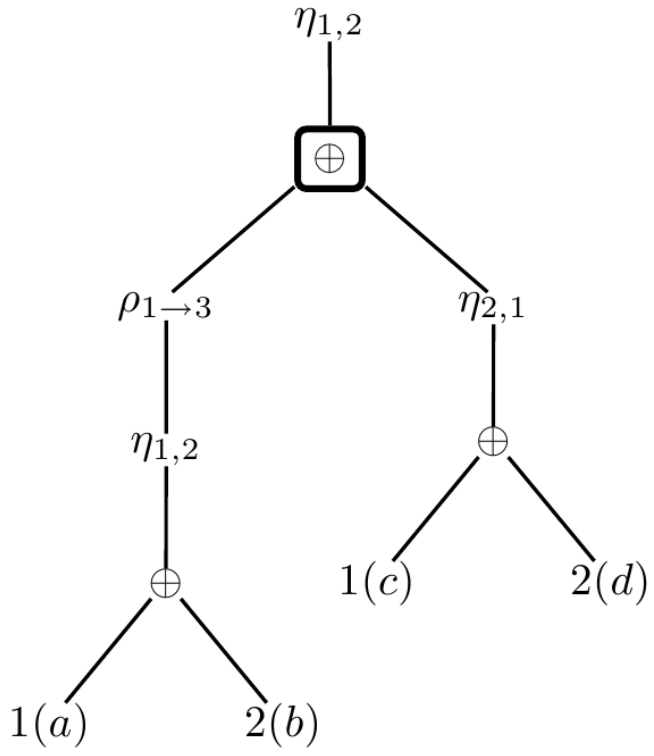

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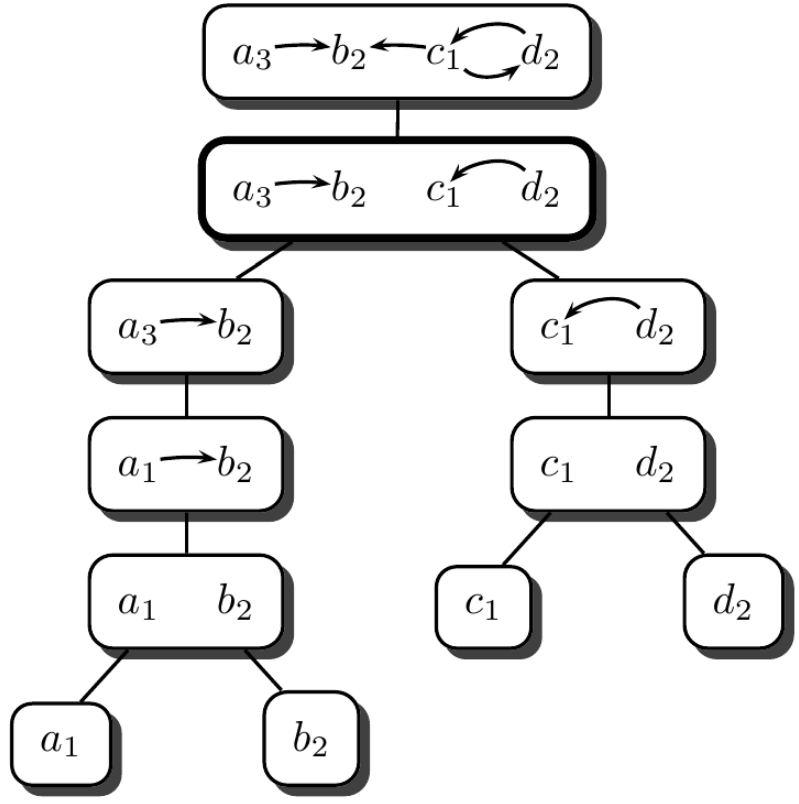

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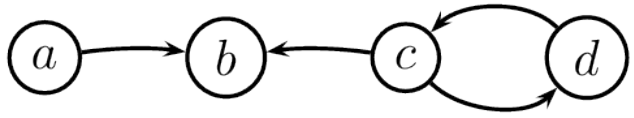

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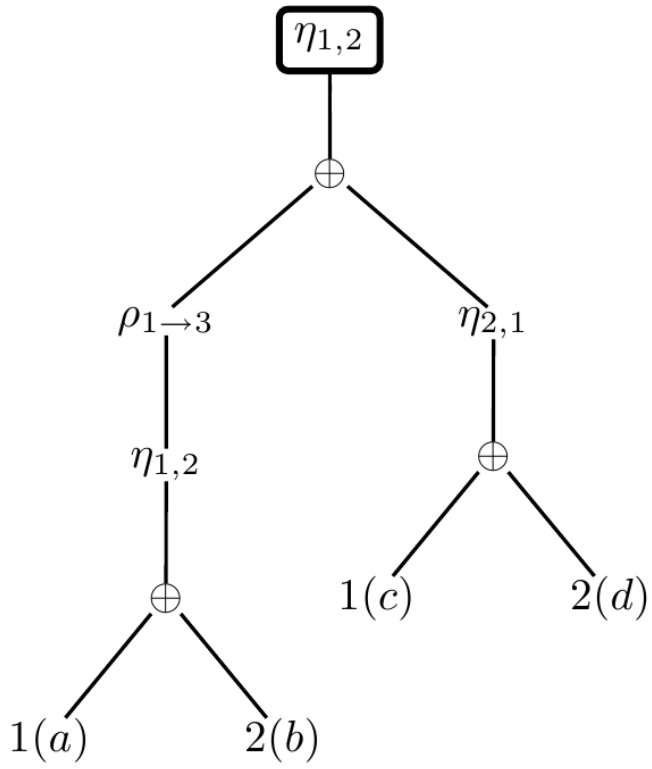

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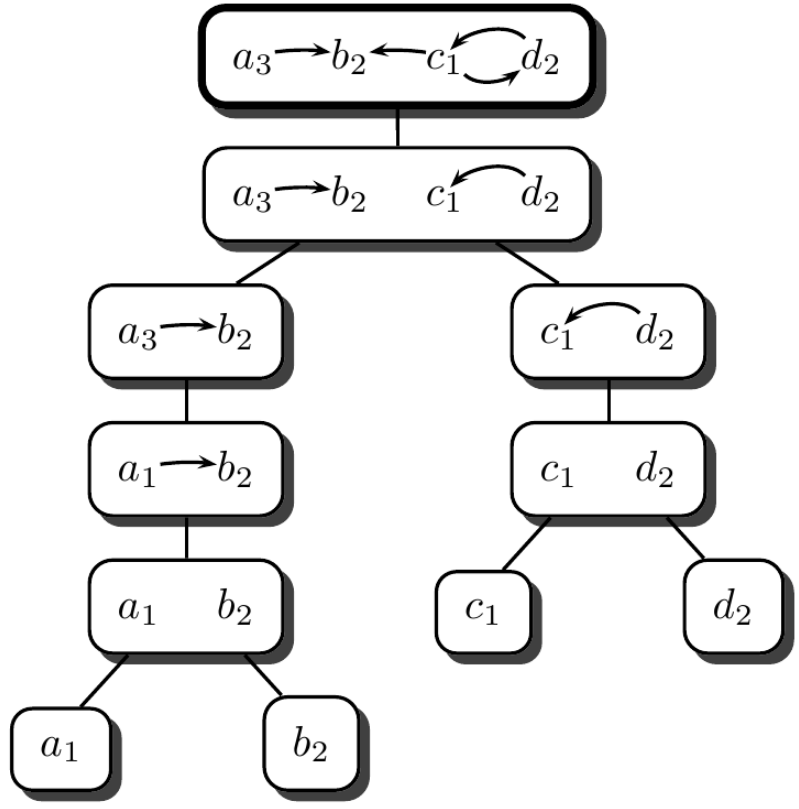
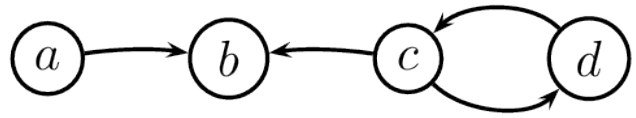


Clique-Width – Example

k-expression for the following AF?



Result:

we found a k-expression describing the AF with $k = 3$:

$$\eta_{1,2}(\rho_{1\to 3}(\eta_{1,2}(1(a) \oplus 2(b))) \oplus \eta_{2,1}(1(c) \oplus 2(d)))$$

Clique-Width – Example

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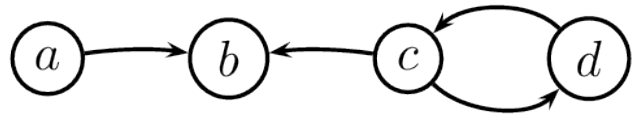

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In fact, above AF has clique-width 3.

Tree-Width vs. Clique-Width

Some Observations

- Class of AFs with bounded clique-width subsumes and significantly extends the class of AFs with bounded tree-width
- There are both, sparse (e.g. tree-like AFs) and dense AFs (e.g. clique-like AFs) that possess small clique-width
- Clique-width allows a direct handle for the orientation of attacks
- However, finding a tree-decomposition of width $\leq k$ (for given k) is much easier than finding a k-expression

Meta-Theorems

[Courcelle 1987; Courcelle, Makowsky & Rotics, 2000]

Any graph problem that can be expressed in MSO_2 (resp. MSO_1) can be solved in linear time for graphs of tree-width (resp. clique-width) bounded by some constant k.

Meta-Theorems

[Courcelle 1987; Courcelle, Makowsky & Rotics, 2000]

Any graph problem that can be expressed in MSO_2 (resp. MSO_1) can be solved in linear time for graphs of tree-width (resp. clique-width) bounded by some constant k.

Example: Existence of a stable extension [Dunne 2007]

$$\begin{aligned} SE(A,R) \quad = \quad \exists U \quad & \forall x,y\big(\langle x,y\rangle \in R \rightarrow (\neg x \in U \vee \neg y \in U)\big) \wedge \\ & \forall x\big((x \in A \wedge \neg x \in U) \rightarrow \exists z(z \in U \wedge \langle z,y\rangle \in R)\big) \end{aligned}$$

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Corollary

Cred_σ and Skept_σ when restricted to graphs of bounded clique-width (and thus of bounded tree-width) can be decided in linear time.

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But the meta-theorems do not lead to efficient algorithms

Exercises

1. Show that the following problems can be decided in polynomial time Skept$_{adm}$, Skept$_{comp}$, Skept$_{naive}$, Exists$^{\neg\emptyset}_{comp}$.
2. Show that the problems Cred$_{adm}$, Cred$_{comp}$, Cred$_{pref}$ are NP-complete.
3. Give MSO encodings of the following problems: Cred$_{adm}$, Cred$_{pref}$, Skept$_{pref}$.


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