



# SEMINAR ABSTRACT ARGUMENTATION

## Generalizations of Argumentation Frameworks

\* slides adapted from Stefan Woltran's lecture on Abstract Argumentation

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# Motivation

## Observations

For many scenarios, limitations of abstract AFs become apparent

- “positive” (support) links between arguments
- “joint attacks”
- making attacks also subject of evaluation
- weights, priorities, etc.

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## In the literature

- BAFs: Bipolar Argumentation Frameworks (Attack and Support) [1]
- EAFs: Extended Argumentation Frameworks (Attack on Attacks) [6]
- AFRAs: Argumentation Frameworks with Recursive Attacks [2]

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## In the lecture

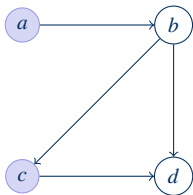
- ADFs: Abstract Dialectical Frameworks [3]

# ADFs

## Basic Idea

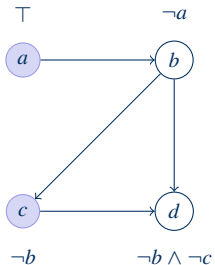
Abstract Dialectical Framework  
=  
Dependency Graph + Acceptance Conditions

# ADFs - Basic idea



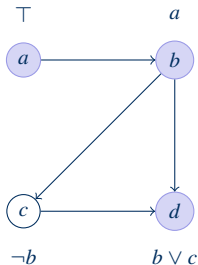
An Argumentation Framework

# ADFs - Basic idea (ctd.)



An Argumentation Framework  
with explicit acceptance conditions

# ADFs - Basic idea (ctd.)



A Dialectical Framework  
with flexible acceptance conditions



# ADFs - The Formal Framework

- Like AFs, use graph to describe dependencies among nodes.
- Unlike AFs, allow individual acceptance condition for each node.
- Assigns **t**(rue) or **f**(alse) depending on status of parents.

## Definition

An **abstract dialectical framework** (ADF) is a tuple  $D = (S, L, C)$  where

- $S$  is a set of statements (positions, nodes),
- $L \subseteq S \times S$  is a set of links,
- $C = \{C_s\}_{s \in S}$  is a set of total functions  $C_s : 2^{par(s)} \rightarrow \{\mathbf{t}, \mathbf{f}\}$ , one for each statement  $s$ .  $C_s$  is called acceptance condition of  $s$ .

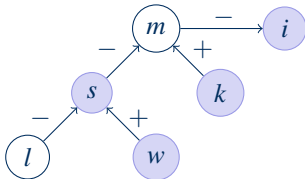
Propositional formula representing  $C_s$  denoted  $F_s$ . In the remainder:  $(S, C)$

# Example

Person innocent, unless she is a murderer.

A killer is a murderer, unless she acted in self-defense.

Evidence for self-defense needed, e.g. witness not known to be a liar.



Propositionally:

$$w : \top, k : \top, l : \perp, s : w \wedge \neg l, m : k \wedge \neg s, i : \neg m$$

# Argumentation frameworks: a special case

- AFs have attacking links only and a single type of nodes.
- Can easily be captured as ADFs.
- $\mathcal{A} = (AR, attacks)$ . Associated ADF  $D_{\mathcal{A}} = (AR, C)$
- $C_s$  as propositional formula:  
 $F_s = \neg r_1 \wedge \dots \wedge \neg r_n$ , where  $r_i$  are the attackers of  $s$ .

# ADF Semantics

- AF semantics specify for an AF = (A,R) subsets of A:  $S \subseteq A$
- We begin with a basic semantics of ADF using interpretations  $v : S \rightarrow \{\mathbf{t}, \mathbf{f}\}$

## Definition

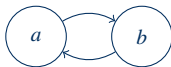
Let  $D = (S, C)$  be an ADF. An interpretation  $v$  is a **model** of  $D$  if for all  $s \in S$ :  $v(s) = v(C_s)$ .

Less formally: a node is accepted (resp. true) iff its acceptance condition says so.

Notation:  $v(\varphi)$  is the evaluation of  $\varphi$  under  $v$ , i.e.  $v(\varphi) = \begin{cases} \mathbf{t} & \text{if } v \models \varphi \\ \mathbf{f} & \text{if } v \not\models \varphi \end{cases}$

# Example

Consider  $D = (S, C)$  with  $S = \{a, b\}$ :



- For  $C_a = \neg b$ ,  $C_b = \neg a$  (AF): two models,  $v_1, v_2$
- For  $C_a = b$ ,  $C_b = a$  (mutual support): two models,  $v_3, v_4$
- For  $C_a = b$  and  $C_b = \neg a$  ( $a$  attacks  $b$ ,  $b$  supports  $a$ ): no model.

	$a$	$b$
$v_1$	<b>t</b>	<b>f</b>
$v_2$	<b>f</b>	<b>t</b>
$v_3$	<b>f</b>	<b>f</b>
$v_4$	<b>t</b>	<b>t</b>

When  $C$  is represented as set of propositional formulas, then models are just propositional models of  $\{s \equiv C_s \mid s \in S\}$ .

# A Short Excursion to Labeling of AFs

- Classical interpretations are not suited for remaining semantics of ADFs
- Extensions of AFs inherently assign to every argument two values: *in* or *out*
- Equivalently one can use **labelings** [5], which assign three values: *in* (**t**), *out* (**f**) and *undecided* (**u**)

## Definition

Given an AF  $F = (A, R)$ , a function  $\mathcal{L} : A \rightarrow \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$  is a **complete labeling** if the following conditions hold:

- $\mathcal{L}(a) = \mathbf{t}$  iff for each  $b$  with  $(b, a) \in R$ ,  $\mathcal{L}(b) = \mathbf{f}$
- $\mathcal{L}(b) = \mathbf{f}$  iff there exists  $a$  with  $(b, a) \in R$ ,  $\mathcal{L}(a) = \mathbf{t}$

# Example Labeling

## Example

Given the following AF



Then its complete labelings are given by

	<i>a</i>	<i>b</i>	<i>c</i>
$\mathcal{L}_1$	<b>u</b>	<b>u</b>	<b>u</b>
$\mathcal{L}_2$	<b>t</b>	<b>f</b>	<b>u</b>
$\mathcal{L}_3$	<b>f</b>	<b>t</b>	<b>u</b>

# Characteristic Function of AF Semantics

- Characteristic function of AFs gives easy definition of semantics via fixed points and is based on defense

## Definition

Given an AF  $F = (A, R)$ . The characteristic function  $\mathcal{F}_F : 2^A \rightarrow 2^A$  of  $F$  is defined as

$$\mathcal{F}_F(E) = \{x \in A \mid x \text{ is defended by } E\}$$

- For an AF  $F = (A, R)$  we have a conflict-free set  $E \subseteq A$  is
  - admissible if  $E \subseteq \mathcal{F}_F(E)$
  - grounded if  $E$  is lfp of  $\mathcal{F}_F$
  - complete if  $E = \mathcal{F}_F(E)$
  - preferred if  $E$  is  $\subseteq$ -maximal admissible
- Our goal now: define char. function for ADFs with three-valued interpretations
- For three-values, what does " $\subseteq$ " mean? How to compare?



# Information Ordering

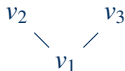
- In ADFs three-valued interpretations  $v : S \rightarrow \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$  are well-suited for defining semantics
- We can define an information ordering:  $\mathbf{u} <_i \mathbf{t}$  and  $\mathbf{u} <_i \mathbf{f}$

## Information Ordering



## Example

	<i>a</i>	<i>b</i>	<i>c</i>
$v_1$	<b>u</b>	<b>u</b>	<b>u</b>
$v_2$	<b>t</b>	<b>f</b>	<b>u</b>
$v_3$	<b>f</b>	<b>t</b>	<b>u</b>



# A Characteristic Function for ADFs

- Our goal: define a characteristic function for ADFs [7] like for AFs
- Intuitively, **u** means a not yet decided value
- Let  $[v]_2$  be the set of  $\{v' \mid v \leq_i v', v' \text{ two-valued}\}$
- Special case: if  $v$  is two-valued then  $[v]_2 = v$

## Example

	<i>a</i>
$v_1$	<b>u</b>
$v_2$	<b>t</b>
$v_3$	<b>f</b>

$[v_1]_2 = \{v_2, v_3\}$ ,  $[v_2]_2 = v_2$  and  $[v_3]_2 = v_3$

# A Characteristic Function for ADFs (contd)

- $[v]_2$  denotes the set of interpretations that refine  $v$ , i.e. set  $\mathbf{u}$  to true or false
- Given  $v$  and a boolean formula  $C_s$  for a statement  $s$ , we might have different outcomes for each  $v_1, v_2 \in [v]_2$
- E.g.  $v_1(C_s) \neq v_2(C_s)$ , hence how to update the status of  $s$  given  $v$ ?
- Idea: compute a “consensus”
- The set  $\{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$  forms a meet-semilattice w.r.t.  $<_i$ , i.e. take as consensus the meet ( $\sqcap$ )

## Information Ordering



# A Characteristic Function for ADFs (contd)

- For the characteristic function for ADFs we now take the consensus of  $[v]_2$  applied to  $C_s$ :

## Definition

$\Gamma_D(v)$  is given by

$$s \mapsto \prod_{w \in [v]_2} w(C_s)$$

## Example

Let  $C_a = \neg a$  and  $v(a) = \mathbf{u}$ , then  $[v]_2 = \{v_2, v_3\}$

	$a$
$v$	$\mathbf{u}$
$v_2$	$\mathbf{t}$
$v_3$	$\mathbf{f}$

$v_2(C_a) = \mathbf{f}$        $v_3(C_a) = \mathbf{t}$

$\mathbf{u}$

the result is  $\prod_{w \in [v]_2} w(C_a) = \mathbf{u}$

# A Characteristic Function for ADFs (contd)

## Example

Let  $C_a = \top$  and  $v(a) = \mathbf{u}$ , then  $[v]_2 = \{v_2, v_3\}$

	$a$	
$v$	$\mathbf{u}$	$v_2(C_a) = \mathbf{t} = v_3(C_a)$
$v_2$	$\mathbf{t}$	
$v_3$	$\mathbf{f}$	

the result is  $\prod_{w \in [v]_2} w(C_a) = \mathbf{t}$

# A Characteristic Function for ADFs (contd)

## Example

Let  $C_a = a \vee b$  and  $v(a) = \mathbf{t}$ ,  $v(b) = \mathbf{u}$ , then  $[v]_2 = \{v_2, v_3\}$

	$a$	$b$	
$v$	$\mathbf{t}$	$\mathbf{u}$	$v_2(C_a) = \mathbf{t} = v_3(C_a)$
$v_2$	$\mathbf{t}$	$\mathbf{t}$	
$v_3$	$\mathbf{t}$	$\mathbf{f}$	

the result is  $\prod_{w \in [v]_2} w(C_a) = \mathbf{t}$

- Here  $v$  incorporates already information:  $v(a) = \mathbf{t}$

# ADF Semantics

- Using the concept of consensus and information ordering, we can define admissible sets, grounded, complete and preferred models similarly as for AFs

## Definition

Let  $D = (S, C)$  be an ADF and  $v$  a three-valued interpretation over  $S$ , then

- $v$  is admissible in  $D$  if  $v \leq_i \Gamma_D(v)$
- $v$  is the grounded model of  $D$  if  $v$  is the lfp of  $\Gamma_D$  wrt  $<_i$
- $v$  is complete in  $D$  if  $v = \Gamma_D(v)$
- $v$  is preferred in  $D$  if  $v$  is  $<_i$ -maximal admissible

# Example

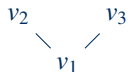
## Example

Let  $C_a = \top$ ,  $C_b = a$ ,  $C_c = c \wedge b$ ,  $C_d = \neg d$



Then the complete models are given by:

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	
$v_1$	<b>t</b>	<b>t</b>	<b>u</b>	<b>u</b>	<i>grd, com</i>
$v_2$	<b>t</b>	<b>t</b>	<b>t</b>	<b>u</b>	<i>com, prf</i>
$v_3$	<b>t</b>	<b>t</b>	<b>f</b>	<b>u</b>	<i>com, prf</i>



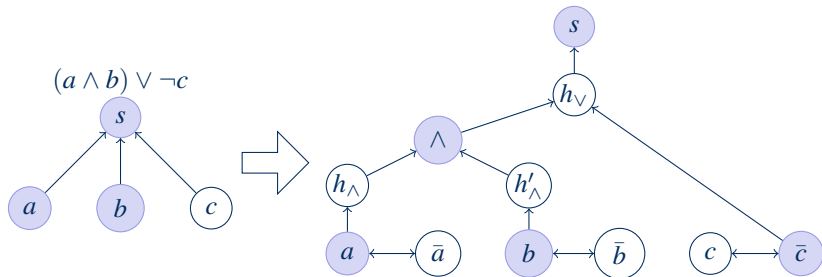


# Remarks about Expressibility

- Acceptance conditions of ADFs also allow definitions of preference relations
- Argument  $A$  has a higher priority than  $B$ :  $C_B = \varphi \wedge (B \rightarrow A)$
- In general: given preferences can be “compiled” to an ADF
- “Joint attacks” can be modeled: set of statements  $X$  attack  $a$  if  $C_a = \neg(\bigwedge_{x \in X} x)$

# ADF Simulation via AF

- Every ADF can be simulated by an AF such that the models of the ADF are in correspondence to the stable extensions of the AF [4].
- Idea from boolean circuits: for each statement  $s$  we construct its  $C_s$ :



- The size of the resulting AF is polynomially bounded wrt to size of ADF.



[1] Leila Amgoud and Claudette Cayrol and Marie-Christine Lagasquie and Pierre Livet,  
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International Journal of Intelligent Systems 23(10): 1062–1093 (2008)



[2] P. Baroni, F. Cerutti, M. Giacomin and G. Guida.  
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Int. J. Approx. Reasoning 52(1): 19–37 (2011).



[3] G. Brewka and S. Woltran.  
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Proceedings of the 12th International Conference on the Principles of Knowledge Representation and Reasoning (KR'10), pp. 102–111, AAAI Press, 2010.



[4] G. Brewka, P. Dunne and S. Woltran.  
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Proceedings of the 22nd International Joint Conference on Artificial Intelligence (IJCAI'11), 2011.



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[6] S. Modgil.  
**Reasoning about Preferences in Argumentation Frameworks.**  
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[7] H. Strass.  
**Approximating Operators and Semantics for Abstract Dialectical Frameworks.**  
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