Class 3
Noise, point operations

What is noise?

- Noise is a disturbance of the image data (projection of the scene) by image acquisition or the transmission of images
- In statistical methods, the term noise is also used generally for data that does not fit the underlying model
- Main goal of image enhancement: removal of noise
- Several algorithms for noise removal in this course
- Computer vision: rather than removing noise, choose a model that can deal with noise

Additive noise

- Gray values and noise are independent: \( I_{ij} = I_{ij}^s + n_{ij} \)
- Noise distribution depends on the sensor
  - Gaussian noise (very common)
  - Rayleigh distribution (radar)
  - Gamma distribution (laser imaging)
  - Exponential distribution (laser imaging)

Gaussian noise

- Density function
  \[
  g(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left( \frac{(x - \mu)^2}{2\sigma^2} \right)
  \]
- Usually zero-mean Gaussian noise
- Good approximation in many practical situations
- In particular: thermic sensor noise in CCD cameras

- \( 1\sigma \) interval: 68% of values
- \( 2\sigma \) interval: 95.5% of values
- \( 3\sigma \) interval: 99.7% of values

From Gonzales-Woods 2002
Simulation of Gaussian noise

- Box-Muller method creates a Gaussian distributed random variable with $\mu = 0, \sigma = 1$

- Create independent random variables $U$ and $V$ with uniform distribution in $[0,1]$

- Compute $N = \sqrt{-2 \ln U \cos(2\pi V)}$ and $M = \sqrt{-2 \ln U \sin(2\pi V)}$

- $N$ and $M$ are independent Gaussian distributed random variables with $\mu = 0, \sigma = 1$

- Degradation of grayscale image: $\tilde{I} = I + \sigma N$

- When saving the degraded image to disk (e.g. PGM format)
  - Ensure not to leave the interval $[0,255]$
  - Remember there is quantization

Lena test image without noise
Gaussian noise with $\sigma = 20$ added

Multiplicative noise

- Signal dependent: $I_{ij} = I_{ij}^* (1 + n_{ij})$

- More difficult to handle in image enhancement algorithms

- Useful trick:
  - Transform image by applying the logarithm $\log I_{ij} = \log I_{ij}^* + \log(1 + n_{ij})$
  - Noise becomes additive noise (can be removed by standard algorithms)
  - Apply backtransform to denoised image $I_{ij}^* = \exp I_{ij}^*$

Lena Söderberg

- Original image from the November 1972 issue of Playboy
- Only the top part has been used for testing image processing algorithms
- Became probably the most popular test image in image processing
- Lena Söderberg was invited to the 50th annual Conference of the Society for Imaging Science and Technology in 1997

Lena in 1997
Lena at the IS&T 1997
Full version
Impulse noise

• A certain percentage of pixels is replaced by one (unipolar impulse noise) or two (bipolar impulse noise) fixed values

• Caused, for instance, by pixel defects of CCD chips

• Special case salt-and-pepper noise: some pixels replaced by white or black values

Original 5% salt-and-pepper noise 20% salt-and-pepper noise

Uniform noise

• A certain percentage of pixels is replaced by uniformly distributed random variables

• Very unpleasant noise, no a-priori knowledge in the noise model

Original 5% uniform noise 20% uniform noise

Signal-to-noise ratio (SNR)

• Quantitative measure for the degradation of an image $I$ versus a noise-free version $I^*$ (ground truth)

• Based on the variance of the image versus the variance of the noise

• Variance of the image: $\sigma_I = \frac{1}{N} \sum_i (I_i^* - \mu)^2$

• Additive, zero-mean noise model: $I_i = I_i^* + n_i$

• Variance of the noise: $\sigma_n = \frac{1}{N} \sum_i (I_i^* - I_i)^2$

• Signal-to-noise ratio: $\text{SNR} = 10 \log_{10} \left( \frac{\sigma_I}{\sigma_n} \right) = 10 \log_{10} \left( \frac{\sum_i (I_i^* - \mu)^2}{\sum_i (I_i^* - I_i)^2} \right)$

• Peak signal-to-noise ratio: $\text{PSNR} = 10 \log_{10} \left( \frac{\max_i I_i^* - \min_i I_i^*}{\sum_i (I_i^* - I_i)^2} \right)$

• Their unit is decibel (dB), the higher the better

Original 5% uniform noise 20% uniform noise
Point operations

- The most simple way to enhance an image is to treat each pixel independently:
  \[ u_{ij} = f(I_{ij}) \]

- This kind of operation mainly transforms intensities in a way that relevant structures are in a range that can be well observed.

- Often \( f(s) \) is a monotonically increasing function:
  \[ s_1 \leq s_2 \Rightarrow f(s_1) \leq f(s_2) \]

- If \( s_1 < s_2 \Rightarrow f(s_1) < f(s_2) \), then \( f(s) \) is strictly monotonic increasing.

- Strictly monotonic increasing transformations are invertible, i.e. the original image can be restored from the transformed image (if there is no quantization!)

Brightness and contrast enhancement

- Brightening of an image
  \[ u(x, y) = I(x, y) + b \]
  - Darkening for \( b < 0 \)
  - Clipping of values that exceed the allowed range

- Contrast enhancement
  \[ u(x, y) = aI(x, y) \]
  - \( a > 0 \)
  - Contrast attenuation for \( a < 1 \)
  - Clipping of values that exceed the allowed range

Gamma correction

- Many cameras transform light intensities \( I \) into values that are proportional to \( I^\gamma \)

- Reason: Camera chips have different response curves than the human eye

- Similar transformations appear with computer monitors

- Compensation of these effects by a gamma correction
  \[ f(I(x, y)) = I_{max} \left( \frac{I(x, y)}{I_{max}} \right)^{\frac{1}{\gamma}} \quad \gamma > 0 \]

- The range \([0, I_{max}]\) is not affected

Especially dark areas become brighter and, hence, better visible
Thresholding

- Converts a grayscale image into a binary image
  \[ u(x, y) = \begin{cases} 
  255 & I(x, y) > \theta \\
  0 & I(x, y) \leq \theta 
  \end{cases} \]

- Yields two regions: the simplest form of image segmentation

- Most difficult: choosing an appropriate threshold \( \theta \)

- There are many adaptive methods that set the threshold automatically
  - Otsu 1979: Minimize the variance of intensities in the two regions
  - Minimum of the smoothed grayscale histogram
  - Iteratively (two-means clustering):
    - Compute means \( \mu_1, \mu_2 \) of the two regions
    - Threshold the image with \( \theta = \frac{\mu_1 + \mu_2}{2} \)

**Advantages:**
- Very fast
- Very simple

**Disadvantages:**
- Often there is no threshold that separates the objects
  - double thresholding, local thresholds
- The spatial context is ignored (one could assume that two neighboring pixels are more likely to belong to the same region)
- The adaptive choice of the threshold misses mathematical rigor and statistical foundation (clustering methods)

**Outlook:** variational image segmentation, graph cut methods (in course Computer Vision)

- The image histogram contains the number of pixels that have a certain gray value
**Histogram equalization**

- Transform the image grayvalues in a way such that all gray values are equally frequent

- Given:
  - \( p_i \) : Number of pixels of input image having gray value \( v_i \), \( i = 1, \ldots, m \)
  - \( q_j \) : Desired number of pixels having gray value \( w_j \), \( j = 1, \ldots, n \)
  (for \( N \) pixels and 256 gray scales \( q_j = N/256 \))

- Algorithm:
  - \( k_0 = 0 \)
  - For \( r = 1, \ldots, n \):
    - Search the largest index \( k_r \leq m \) with
    \[
    \sum_{i=1}^{k_r} p_i \leq \sum_{j=1}^{r} q_j
    \]
  - and map the grey values \( v_{k_r+1}, \ldots, v_{k_r} \) to \( w_r \)

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**Difference image**

- Subtracting the grayvalues of two images from each other yields a difference image:
  \[
  I_\Delta = |I_1 - I_2|
  \]

- Can be used for detecting parts of moving objects in static scenes

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**Background subtraction**

- Special difference image: **background subtraction**
  - Take one image of the static background without the object
  - Difference image to this background image indicates the object
  - Can be used for object tracking and segmentation of a person in front of a static background

- Difference image of color images:
  \[
  I_\Delta = \frac{1}{3} \sum_{k=1}^{3} |I_{k,1} - I_{k,2}|
  \]
Image averaging

- Two successive images from a static scene can be added to remove noise
  \[ \bar{I} = \frac{1}{N} \sum_{i=1}^{N} I_i \]

- Can be regarded as a temporal smoothing filter

- In case of additive, zero mean noise, the expectation of \( \bar{I} \) is
  \[ E(\bar{I}) = E(I + n) = E(I) + E(n) = E(I) \]

- In case of Gaussian noise, the noise variance is reduced by
  \[ \tilde{\sigma}^2 = \frac{\sigma^2}{N} \]

- 16 images are necessary to reduce the standard deviation to 25% of the original standard deviation

Summary

- There are different noise models depending on the source of noise

- Zero-mean Gaussian noise is the most frequent model

- The quality of an image enhancement method can be measured by the signal-to-noise ratio

- Point operations neglect the spatial relationship between pixels

- Popular point operations are gamma correction, thresholding, histogram equalization

- Difference images can detect moving objects

- Summation of static images removes noise

Literature


Programming assignment - downsampling

- Add Gaussian noise with standard deviation 10 and 20 to the Lena image in `ImageProcessingEx02.zip`. Use the Box-Muller method to simulate the noise.

- Measure the PSNR of the noisy images.

- Create a sequence of 20 noisy images. Average these images. Measure the PSNR of the averaged image.

- Compute the difference image of `Sidenbladh.ppm` and `SidenbladhBG.ppm`. Threshold the difference image. Play with the threshold.

- Hint: Color images can be handled with the class `CTensor`.

- Extra assignment: Implement automatic thresholding by the two-means method.