

Determinization of Weighted Tree Automata using Factorizations

Matthias Büchse Jonathan May Heiko Vogler

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Machine Translation (MT)

le cas a suscité ...

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- 0.25 the case has aroused ...
- 0.22 the case has caused ...
- 0.21 the case has arouse ...
- 0.16 the case has cause ...
- 0.15 the case has attracted ...

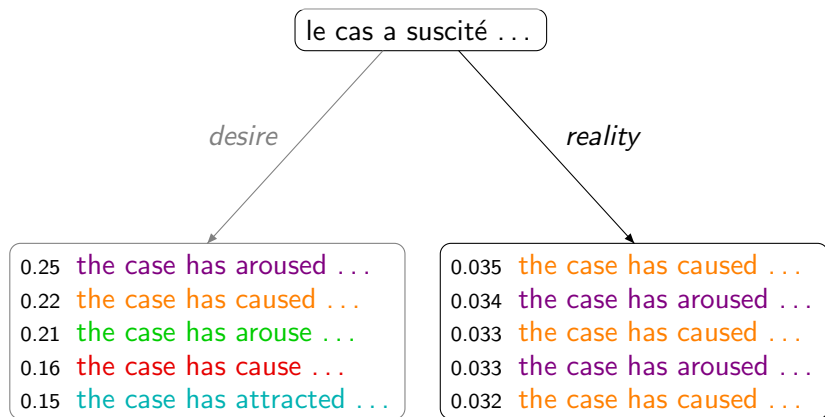
Machine Translation (MT)

le cas a suscité ...

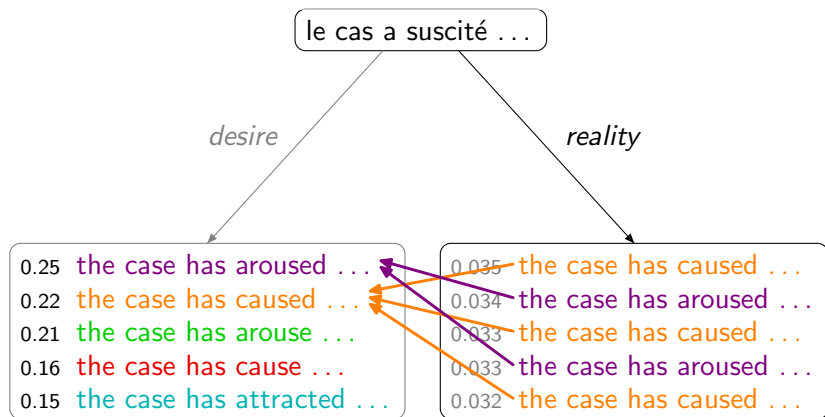
desire

- 0.25 the case has aroused ...
- 0.22 the case has caused ...
- 0.21 the case has arouse ...
- 0.16 the case has cause ...
- 0.15 the case has attracted ...

Machine Translation (MT)



Machine Translation (MT)



Bridging the gap

Actual output: list of best derivations

0.035: the case has caused ...

0.034: the case has aroused ...

0.033: the case has caused ...

0.033: the case has aroused ...

0.032: the case has caused ...

- ▶ merge derivations in the output list

Bridging the gap

Modified actual output

0.1: the case has caused ...

0.034: the case has aroused ...

0.033: the case has aroused ...

Bridging the gap

Modified actual output

0.1: the case has caused . . .

0.067: the case has aroused . . .

Bridging the gap

Modified actual output

0.1: the case has caused ...

0.067: the case has aroused ...

0.032: the case has caused ...

0.032: the case has aroused ...

0.032: the case has aroused ...

0.031: the case has arouse ...

0.03: the case has aroused ...

Bridging the gap

Modified actual output

0.132: the case has caused ...

0.067: the case has aroused ...

0.032: the case has aroused ...

0.032: the case has aroused ...

0.031: the case has arouse ...

0.03: the case has aroused ...

Bridging the gap

Modified actual output

0.161: the case has aroused ...

0.132: the case has caused ...

0.031: the case has arouse ...

Bridging the gap

Modified actual output

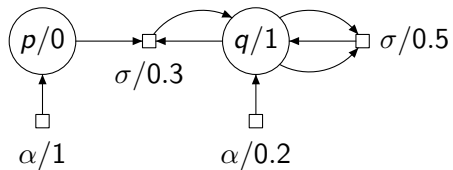
0.161: the case has aroused ...

0.132: the case has caused ...

0.031: the case has arouse ...

- ▶ still more derivations needed!
- ▶ actual weights unknown
- ▶ reengineer the system

MT system \approx weighted tree automaton (wta)

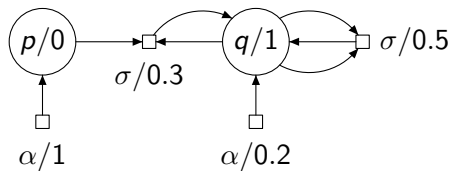


Definition of wta

$M = (Q, \mu, \nu)$ wta (over Σ and A)

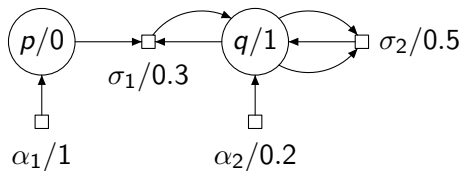
- ▶ Q is a nonempty finite set (of **states**),
- ▶ μ is a family $(\mu_k \mid k \in \mathbb{N})$ (of **transition mappings**) with $\mu_k : \Sigma^{(k)} \rightarrow A^{Q^k \times Q}$ for every $k \in \mathbb{N}$, and
- ▶ $\nu \in A^Q$ (**final weights**).

Unfoldings in wta I



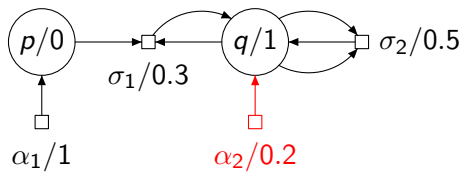
10 best unfoldings:

Unfoldings in wta I



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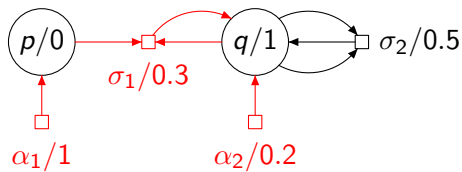


10 best unfoldings:

α_2

0.2

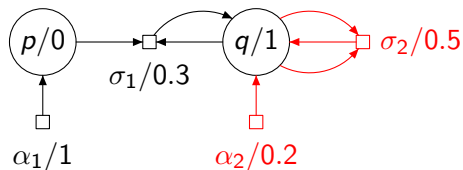
Unfoldings in wta I



10 best unfoldings:

$$\begin{array}{ll} \alpha_2 & 0.2 \\ \sigma_1(\alpha_1, \alpha_2) & 0.06 \end{array}$$

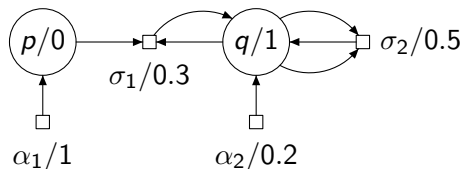
Unfoldings in wta I



10 best unfoldings:

α_2	0.2
$\sigma_1(\alpha_1, \alpha_2)$	0.06
$\sigma_2(\alpha_2, \alpha_2)$	0.02

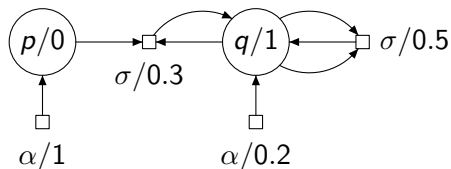
Unfoldings in wta I



10 best unfoldings:

α_2	0.2
$\sigma_1(\alpha_1, \alpha_2)$	0.06
$\sigma_2(\alpha_2, \alpha_2)$	0.02
$\sigma_1(\alpha_1, \sigma_1(\alpha_1, \alpha_2))$	0.018
$\sigma_1(\alpha_1, \sigma_2(\alpha_2, \alpha_2))$	0.006
$\sigma_2(\alpha_2, \sigma_1(\alpha_1, \alpha_2))$	0.006
$\sigma_2(\sigma_1(\alpha_1, \alpha_2), \alpha_2)$	0.006
$\sigma_1(\alpha_1, \sigma_1(\alpha_1, \sigma_1(\alpha_1, \alpha_2)))$	0.0054
$\sigma_2(\alpha_2, \sigma_2(\alpha_2, \alpha_2))$	0.002
$\sigma_2(\sigma_2(\alpha_2, \alpha_2), \alpha_2)$	0.002

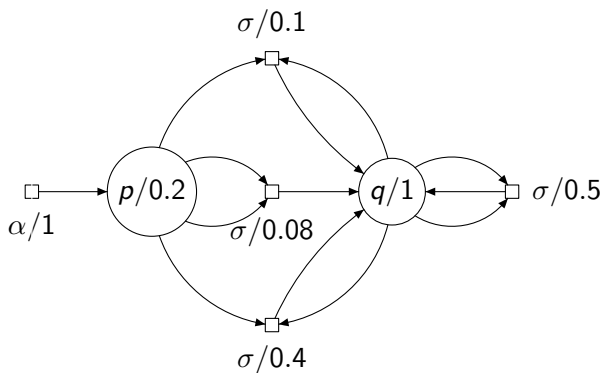
Unfoldings in wta I



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Unfoldings in wta ii

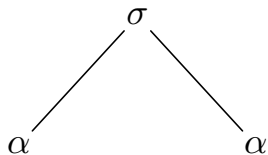
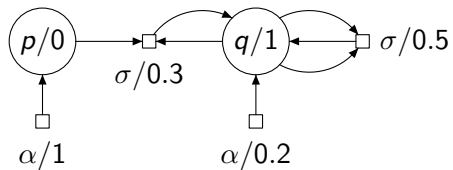


- ▶ list of best unfoldings matches list of best trees

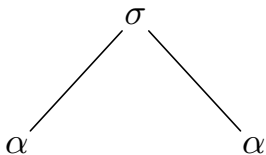
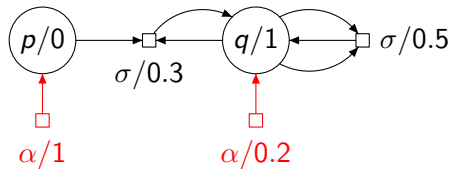
Deterministic wta

- ▶ A wta is called **deterministic** if every tree can be unfolded from at most one state.
- ▶ If a wta is deterministic, it allows at most one unfolding for every tree.
- ▶ Then: list of best trees and list of best derivations coincide.
- ▶ **Determinization**: Try to construct equivalent deterministic wta.

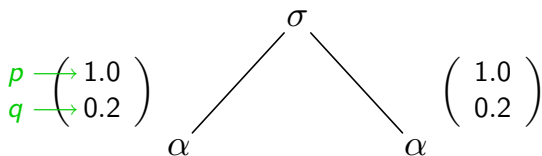
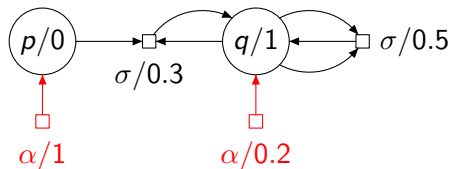
Semantics: example



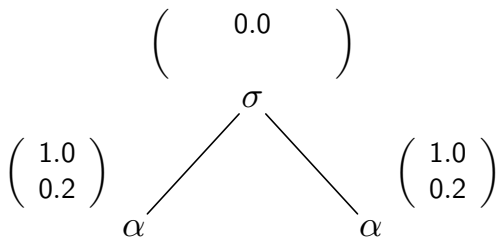
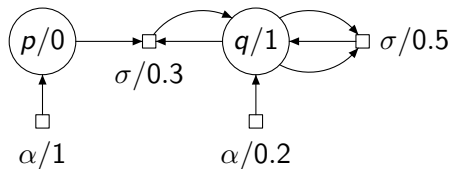
Semantics: example



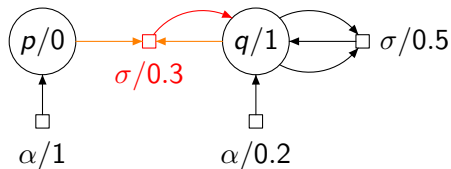
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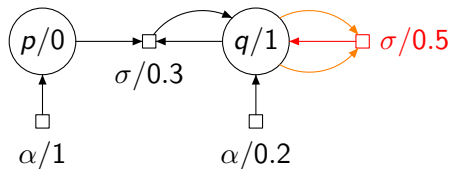


Semantics: example



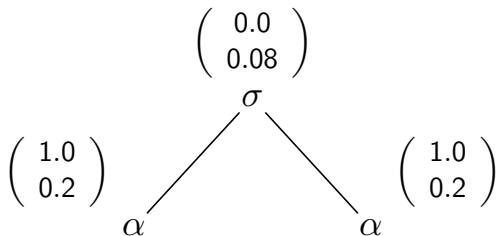
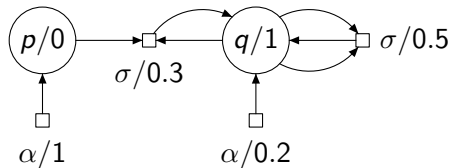
$$1.0 \cdot 0.2 \cdot 0.3 \rightarrow \begin{pmatrix} 0.0 \\ 0.06 \end{pmatrix}$$
$$\begin{matrix} & \sigma & \\ \begin{pmatrix} 1.0 \\ 0.2 \end{pmatrix} \alpha & & \alpha \begin{pmatrix} 1.0 \\ 0.2 \end{pmatrix} \end{matrix}$$

Semantics: example

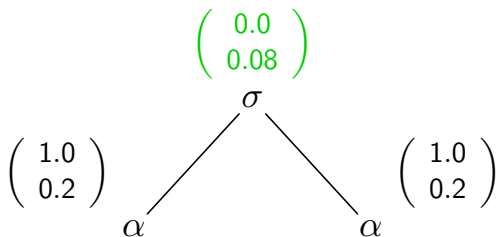
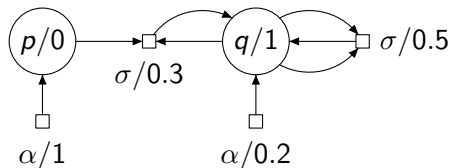


$$\begin{pmatrix} 0.0 \\ 0.06 + 0.02 \end{pmatrix} \leftarrow 0.2 \cdot 0.2 \cdot 0.5$$
$$\begin{matrix} & \sigma & \\ \alpha & / & \backslash & \alpha \\ \begin{pmatrix} 1.0 \\ 0.2 \end{pmatrix} & & & \begin{pmatrix} 1.0 \\ 0.2 \end{pmatrix} \end{matrix}$$

Semantics: example



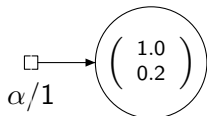
Semantics: example



every tree has a uniquely determined **weight vector**!

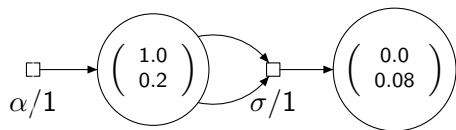
Determinization

according to Borchardt (2004)



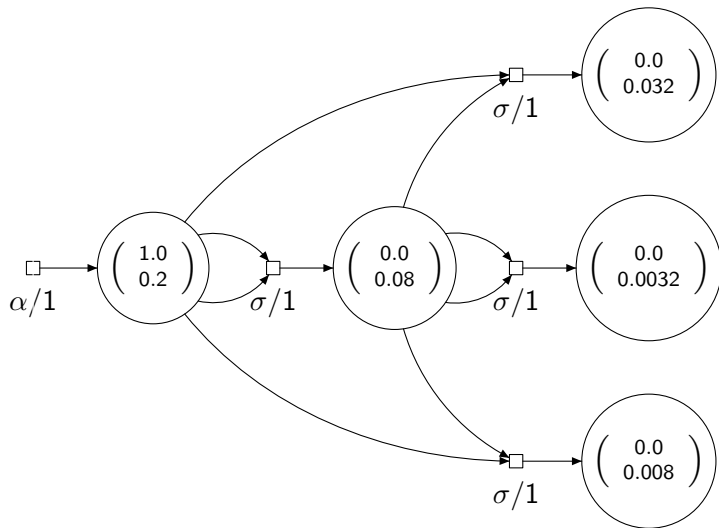
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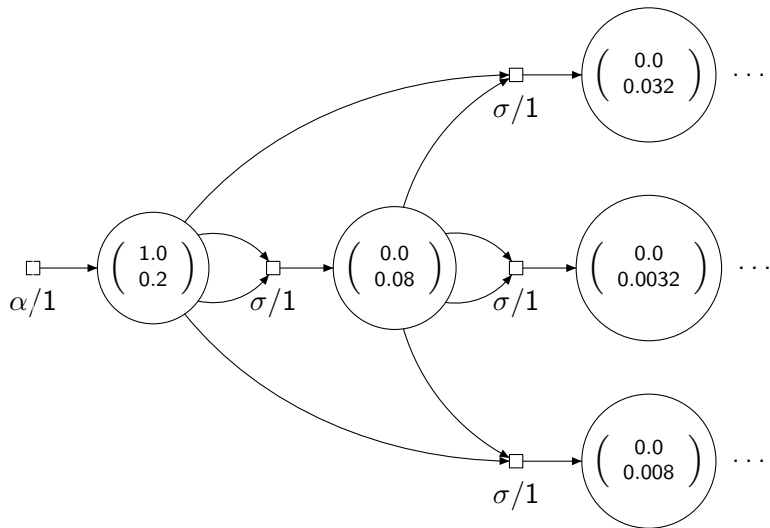
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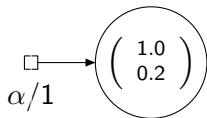


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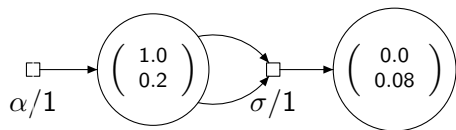
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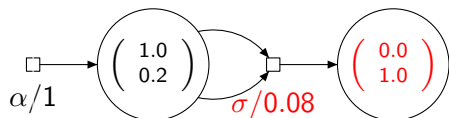
Introducing factorizations



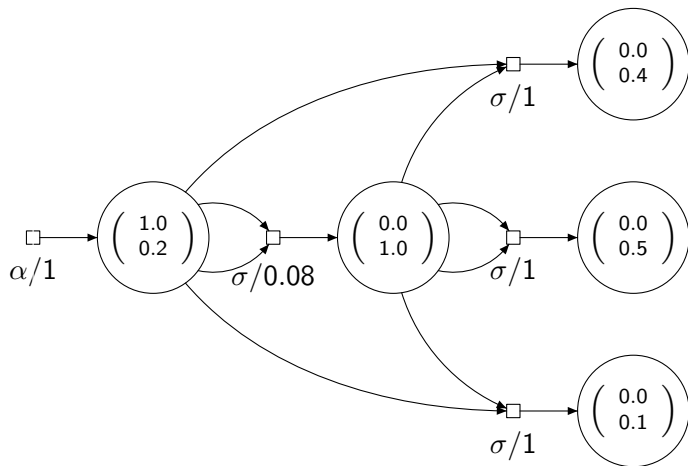
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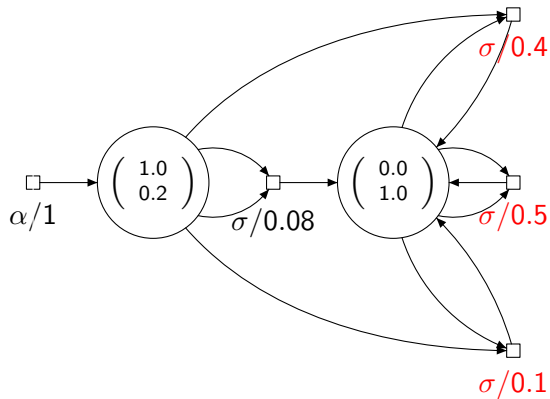
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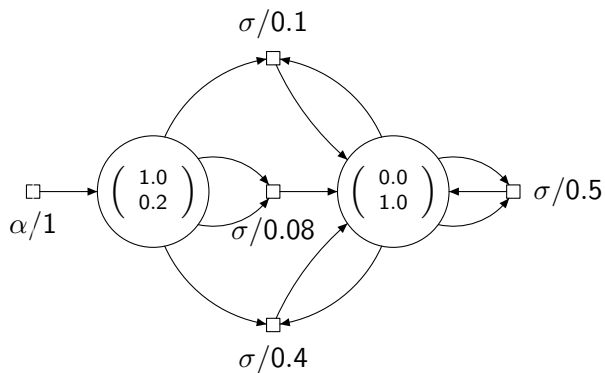
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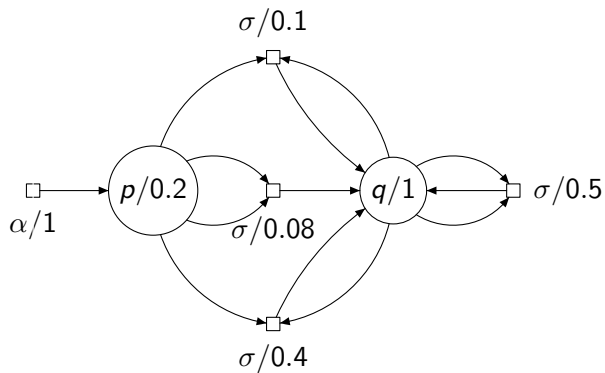
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History of factorizations

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- ▶ Mohri 1997

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Weight domains: commutative semirings

Definition

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- ▶ natural numbers: $(\mathbb{N}, +, \cdot, 0, 1)$,
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- ▶ Viterbi semiring: $([0, 1], \max, \cdot, 0, 1)$,
- ▶ non-negative reals: $(\mathbb{R}^{\geq 0}, +, \cdot, 0, 1)$.

Factorizations

(Kirsten and Mäurer 2005)

Definition

A pair (f, g) is a **factorization** if

1. $f : A^Q \setminus \{0^Q\} \rightarrow A^Q$,
2. $g : A^Q \setminus \{0^Q\} \rightarrow A$, and
3. $u = g(u) \cdot f(u)$ for every $u \in A^Q \setminus \{0^Q\}$.

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- ▶ $g(u) = \min\{u_q \mid q \in Q\}$,
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$$g\left(\begin{pmatrix} 2 \\ 3 \end{pmatrix}\right) = 2, \quad f\left(\begin{pmatrix} 2 \\ 3 \end{pmatrix}\right) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

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Example

non-negative reals: $A = (\mathbb{R}^{\geq 0}, +, \cdot, 0, 1)$.

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$$g\left(\begin{pmatrix} 2 \\ 3 \end{pmatrix}\right) = 5, \quad f\left(\begin{pmatrix} 2 \\ 3 \end{pmatrix}\right) = \begin{pmatrix} 0.4 \\ 0.6 \end{pmatrix}$$

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Definition

A factorization (f, g) is **maximal** if

$$f(u) = f(a \cdot u)$$

for every $a \in A$ and $u \in A^Q$ with $a \cdot u \neq 0^Q$.

Construction of deterministic wta

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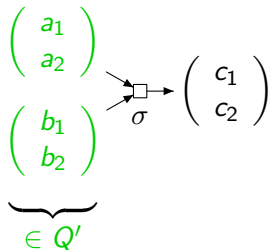
1. start with empty automaton, $Q' = \emptyset$

Construction of deterministic wta

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2. close off under:

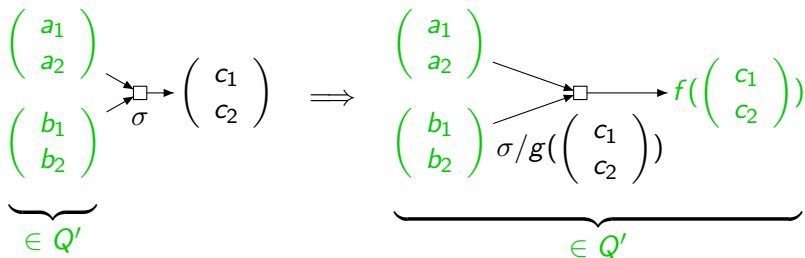
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Formal construction

$\det_{(f,g)}(M) = (Q', \mu', \nu')$ where

- ▶ Q' is the smallest set $P \subseteq A^Q$ such that for every $k \in \mathbb{N}$, $\sigma \in \Sigma^{(k)}$, $u_1, \dots, u_k \in P$: if $\mu_M(\sigma)(u_1, \dots, u_k) \neq \tilde{0}$, then $f(\mu_M(\sigma)(u_1, \dots, u_k)) \in P$.

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▶ $\mu'_k(\sigma)_{u_1 \dots u_k, u} =$
$$\begin{cases} g(\mu_M(\sigma)(u_1, \dots, u_k)) & \text{if } \mu_M(\sigma)(u_1, \dots, u_k) \neq \tilde{0} \text{ and} \\ & u = f(\mu_M(\sigma)(u_1, \dots, u_k)), \\ 0 & \text{otherwise,} \end{cases}$$

for every $k \in \mathbb{N}$, $\sigma \in \Sigma^{(k)}$, $u_1, \dots, u_k \in Q'$, and $u \in Q'$,

Formal construction

$\det_{(f,g)}(M) = (Q', \mu', \nu')$ where

- ▶ Q' is the smallest set $P \subseteq A^Q$ such that for every $k \in \mathbb{N}$, $\sigma \in \Sigma^{(k)}$, $u_1, \dots, u_k \in P$: if $\mu_M(\sigma)(u_1, \dots, u_k) \neq \tilde{0}$, then $f(\mu_M(\sigma)(u_1, \dots, u_k)) \in P$.
- ▶ $\mu'_k(\sigma)_{u_1 \dots u_k, u} = \begin{cases} g(\mu_M(\sigma)(u_1, \dots, u_k)) & \text{if } \mu_M(\sigma)(u_1, \dots, u_k) \neq \tilde{0} \text{ and} \\ & u = f(\mu_M(\sigma)(u_1, \dots, u_k)), \\ 0 & \text{otherwise,} \end{cases}$
for every $k \in \mathbb{N}$, $\sigma \in \Sigma^{(k)}$, $u_1, \dots, u_k \in Q'$, and $u \in Q'$,
- ▶ $\nu'_u = \sum_{q \in Q} u_q \cdot \nu_q$ for every $u \in Q'$.

Results and problems

Results

- ▶ determinization construction for wta over commutative semirings
- ▶ resulting automaton not necessarily finite
- ▶ correctness
- ▶ sufficient conditions for finiteness:
maximal factorization and
 - ▶ nonrecursive wta (acyclic hypergraph) *or*
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only the best derivation counts

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Twins property

undecidable in general

Twins property

A wta $M = (Q, \mu, \nu)$ is said to have the **twins property** if for every $p, q \in Q$, $\xi \in T_\Sigma$, and $\zeta \in C_\Sigma$, we have that if $\text{wt}_M(R_M^p(\zeta \cdot \xi)) \neq 0$ and $\text{wt}_M(R_M^q(\zeta \cdot \xi)) \neq 0$, then $\text{wt}_M(R_M^{p;p}(\zeta)) = \text{wt}_M(R_M^{q;q}(\zeta))$.

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