Determinization of Weighted Tree Automata using Factorizations

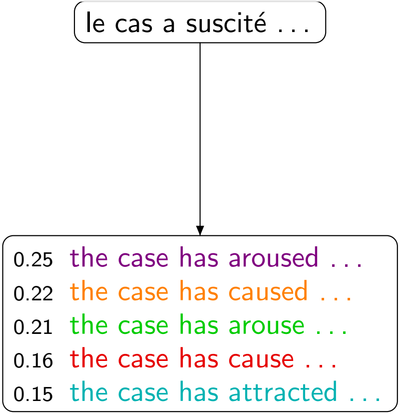
Matthias Büchse   Jonathan May   Heiko Vogler

2009-09-01

Machine Translation (MT)

le cas a suscité . . .

Machine Translation (MT)



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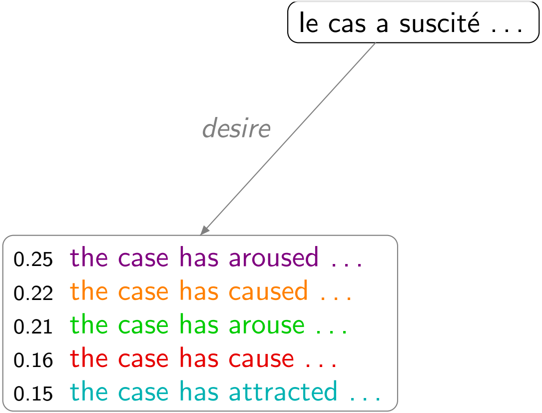

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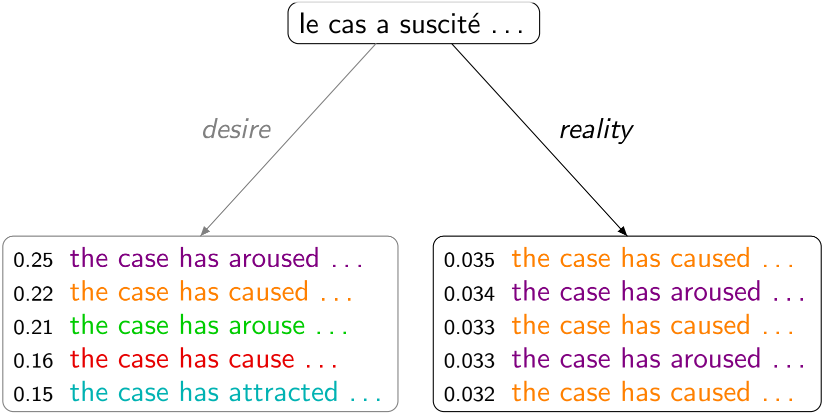

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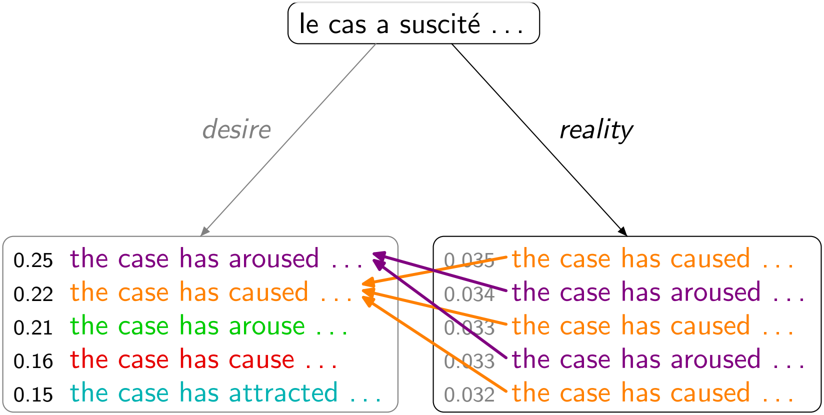


Bridging the gap

Actual output: list of best derivations

0.035: the case has caused . . .
0.034: the case has aroused . . .
0.033: the case has caused . . .
0.033: the case has aroused . . .
0.032: the case has caused . . .

- merge derivations in the output list

Bridging the gap

Modified actual output

0.1: the case has caused . . .
0.034: the case has aroused . . .
0.033: the case has aroused . . .

Bridging the gap

Modified actual output

0.1: the case has caused . . .
0.067: the case has aroused . . .

Bridging the gap

Modified actual output

0.1: the case has caused ...
0.067: the case has aroused ...
0.032: the case has caused ...
0.032: the case has aroused ...
0.032: the case has aroused ...
0.031: the case has arouse ...
0.03: the case has aroused ...

Bridging the gap

Modified actual output

0.132: the case has caused . . .
0.067: the case has aroused . . .
0.032: the case has aroused . . .
0.032: the case has aroused . . .
0.031: the case has arouse . . .
0.03: the case has aroused . . .

Bridging the gap

Modified actual output

0.161: the case has aroused ...

0.132: the case has caused ...

0.031: the case has arouse ...

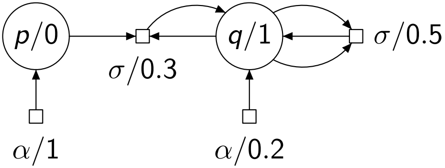
Bridging the gap

Modified actual output

0.161: the case has aroused . . .
0.132: the case has caused . . .
0.031: the case has arouse . . .

- still more derivations needed!
- actual weights unknown
- reengineer the system

MT system ≈ weighted tree automaton (wta)

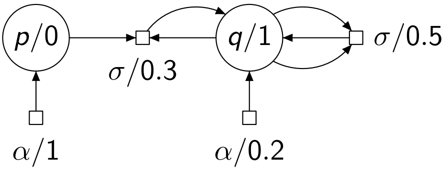


Definition of wta

$M = (Q, \mu, \nu)$ wta (over Σ and A)

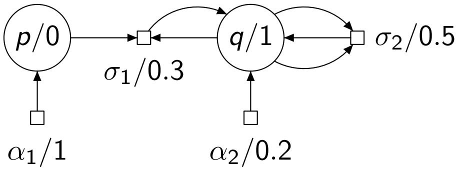
- Q is a nonempty finite set (of states),
- μ is a family $(\mu_k \mid k \in \mathbb{N})$ (of transition mappings) with $\mu_k : \Sigma^{(k)} \to A^{Q^k \times Q}$ for every $k \in \mathbb{N}$, and
- $\nu \in A^Q$ (final weights).

Unfoldings in wta I



10 best unfoldings:

Unfoldings in wta I



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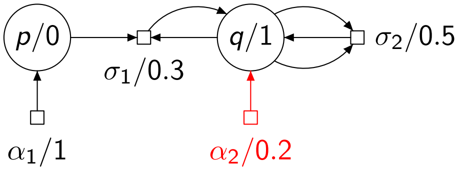
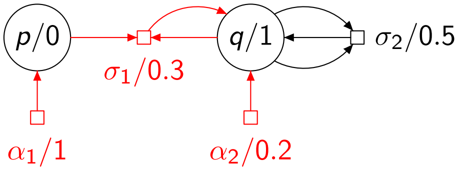


10 best unfoldings:

α_2 0.2

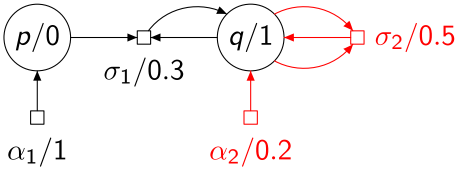
Unfoldings in wta I



10 best unfoldings:

α_2	0.2
$\sigma_1(\alpha_1, \alpha_2)$	0.06

Unfoldings in wta I



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α_2	0.2
$\sigma_1(\alpha_1, \alpha_2)$	0.06
$\sigma_2(\alpha_2, \alpha_2)$	0.02

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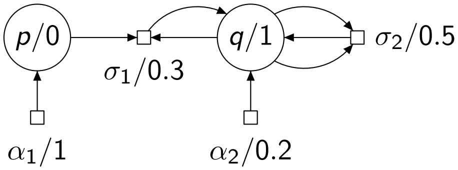
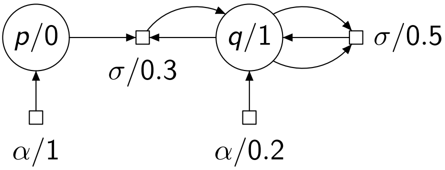


10 best unfoldings:

α_2	0.2
$\sigma_1(\alpha_1, \alpha_2)$	0.06
$\sigma_2(\alpha_2, \alpha_2)$	0.02
$\sigma_1(\alpha_1, \sigma_1(\alpha_1, \alpha_2))$	0.018
$\sigma_1(\alpha_1, \sigma_2(\alpha_2, \alpha_2))$	0.006
$\sigma_2(\alpha_2, \sigma_1(\alpha_1, \alpha_2))$	0.006
$\sigma_2(\sigma_1(\alpha_1, \alpha_2), \alpha_2)$	0.006
$\sigma_1(\alpha_1, \sigma_1(\alpha_1, \sigma_1(\alpha_1, \alpha_2)))$	0.0054
$\sigma_2(\alpha_2, \sigma_2(\alpha_2, \alpha_2))$	0.002
$\sigma_2(\sigma_2(\alpha_2, \alpha_2), \alpha_2)$	0.002

Unfoldings in wta I



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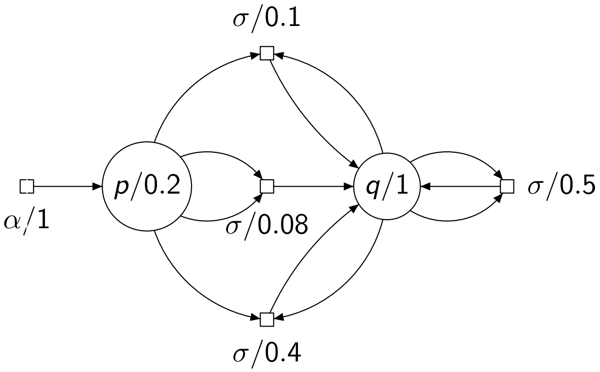
Unfoldings in wta ii

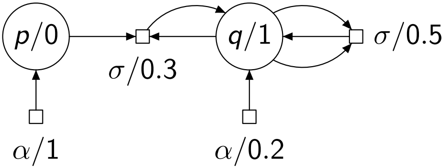


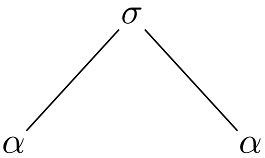
- list of best unfoldings matches list of best trees

Deterministic wta

- A wta is called deterministic if every tree can be unfolded from at most one state.
- If a wta is deterministic, it allows at most one unfolding for every tree.
- Then: list of best trees and list of best derivations coincide.
- Determinization: Try to construct equivalent deterministic wta.

Semantics: example





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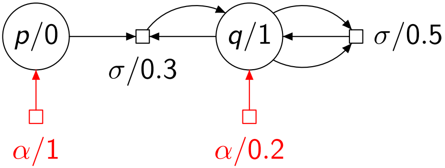

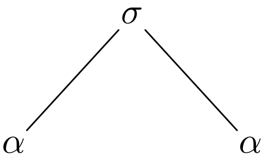

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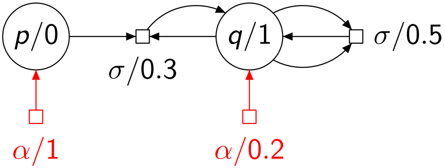

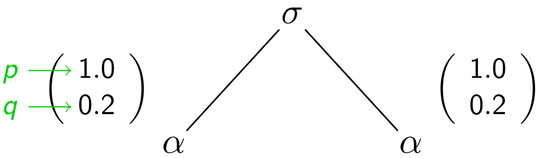

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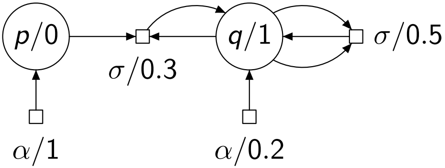

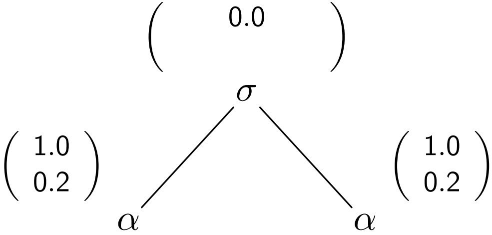

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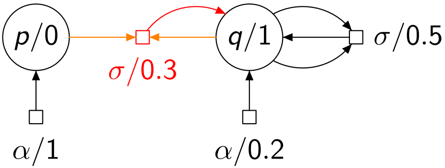

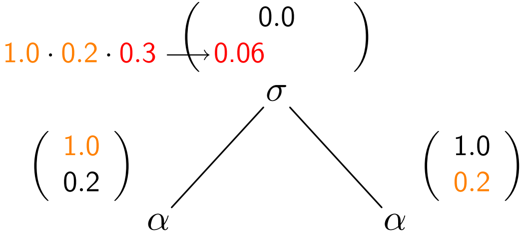

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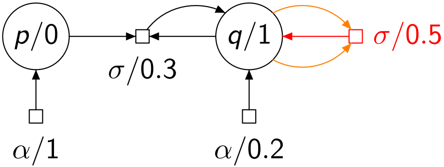

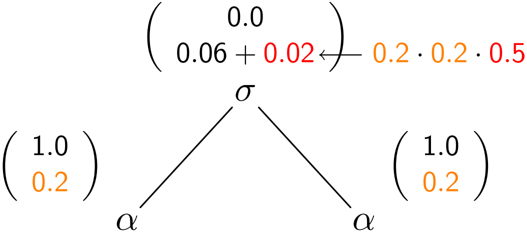

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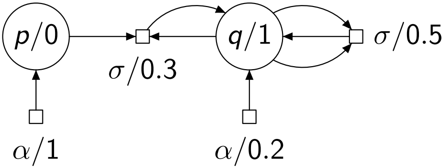

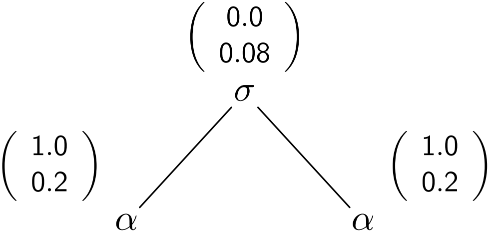

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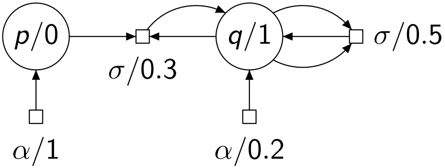
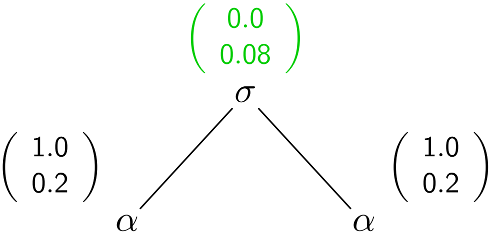
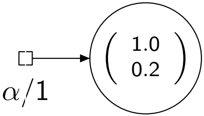




every tree has a uniquely determined weight vector!

Determinization

according to Borchardt (2004)



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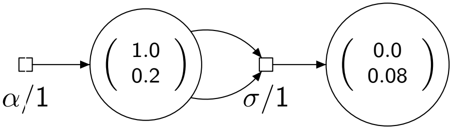

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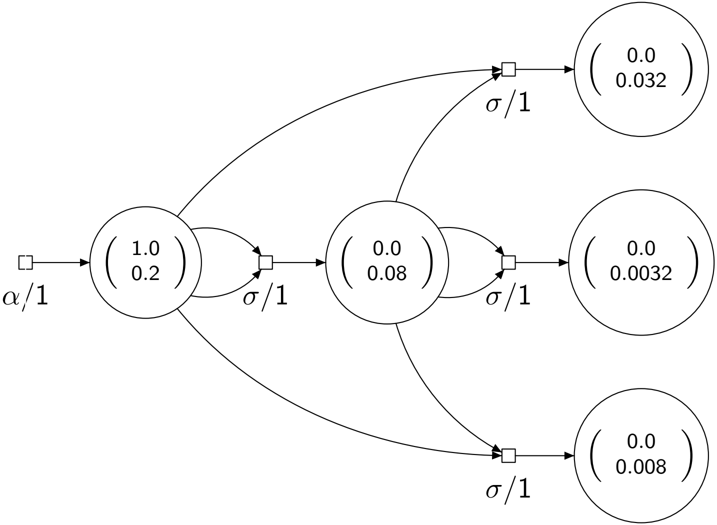

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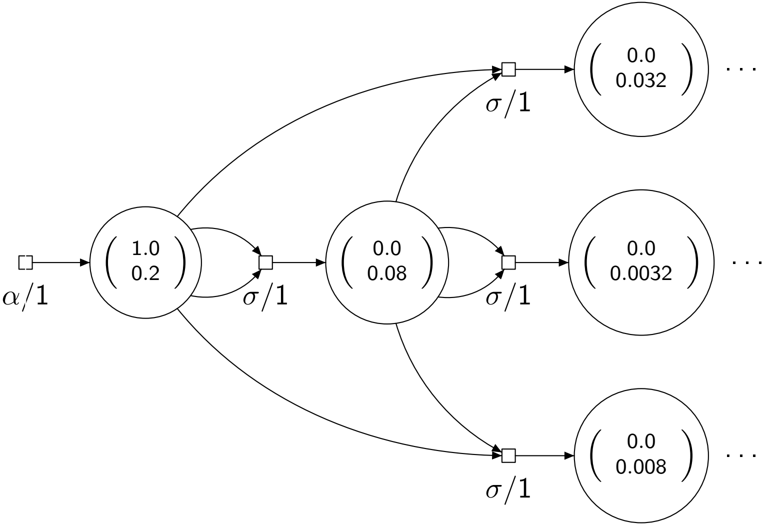
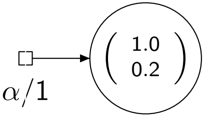


Introducing factorizations



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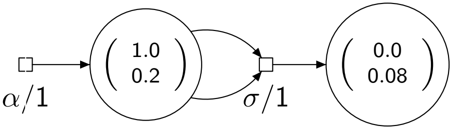

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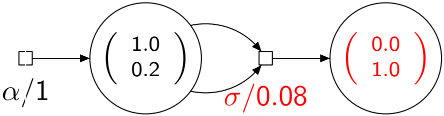

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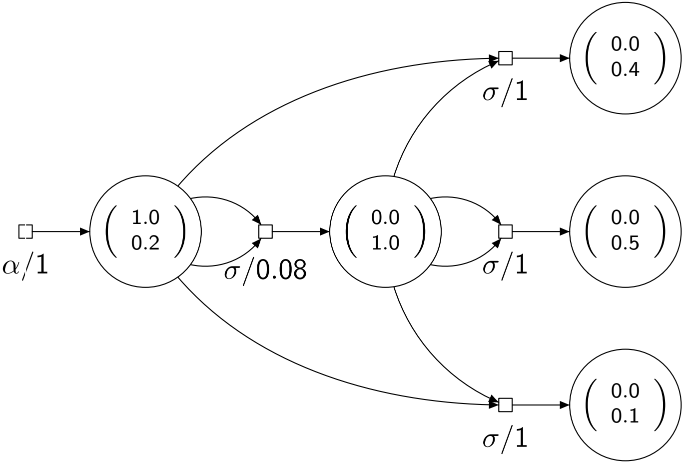

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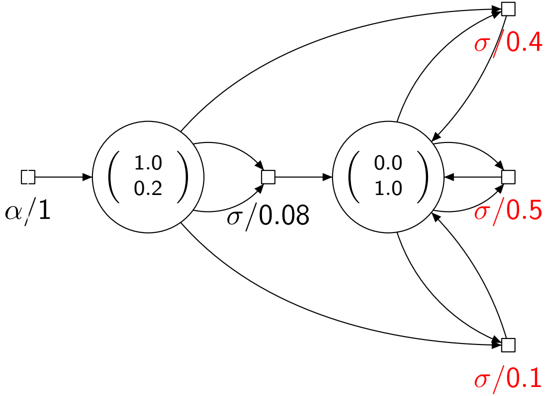

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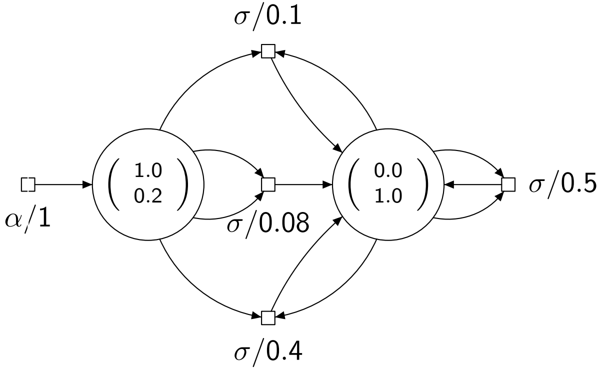

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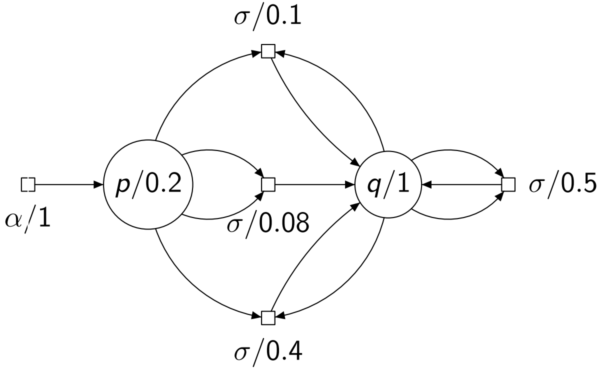

History of factorizations

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- Mohri 1997

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Weight domains: commutative semirings

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- Tropical semiring: $(\mathbb{R}^{\geq 0}_{\infty}, \min, +, \infty, 0)$,

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- Viterbi semiring: $([0, 1], \max, \cdot, 0, 1)$,
- non-negative reals: $(\mathbb{R}^{\geq 0}, +, \cdot, 0, 1)$.

Factorizations

(Kirsten and Mäurer 2005)

Definition

A pair (f, g) is a factorization if

1. $f : A^Q \setminus \{0^Q\} \to A^Q$,
2. $g : A^Q \setminus \{0^Q\} \to A$, and
3. $u = g(u) \cdot f(u)$ for every $u \in A^Q \setminus \{0^Q\}$.

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Example

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$$g(\begin{pmatrix} 2 \\ 3 \end{pmatrix}) = 2, \quad f(\begin{pmatrix} 2 \\ 3 \end{pmatrix}) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

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Example

non-negative reals: $A = (\mathbb{R}^{\geq 0}, +, \cdot, 0, 1)$.

- $g(u) = \sum_{q \in Q} u_q$,
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$$g(\begin{pmatrix} 2 \\ 3 \end{pmatrix}) = 5, \quad f(\begin{pmatrix} 2 \\ 3 \end{pmatrix}) = \begin{pmatrix} 0.4 \\ 0.6 \end{pmatrix}$$

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Definition

A factorization (f, g) is maximal if

$$f(u) = f(a \cdot u)$$

for every $a \in A$ and $u \in A^Q$ with $a \cdot u \neq 0^Q$.

Construction of deterministic wta

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1. start with empty automaton, $Q' = \emptyset$

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2. close off under:

Construction of deterministic wta

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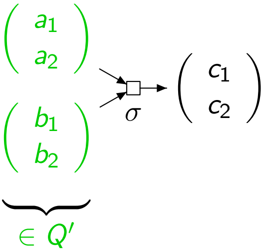
2. close off under:



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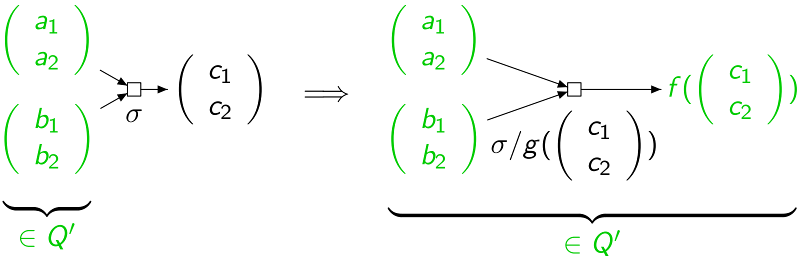

Formal construction

$\det_{(f,g)}(M) = (Q', \mu', \nu')$ where

- Q' is the smallest set $P \subseteq A^Q$ such that for every $k \in \mathbb{N}$, $\sigma \in \Sigma^{(k)}$, $u_1, \ldots, u_k \in P$: if $\mu_M(\sigma)(u_1, \ldots, u_k) \neq \tilde{0}$, then $f(\mu_M(\sigma)(u_1, \ldots, u_k)) \in P$.

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- $\mu'_k(\sigma)_{u_1 \ldots u_k, u} =$

$$\begin{cases} g(\mu_M(\sigma)(u_1, \ldots, u_k)) & \text{if } \mu_M(\sigma)(u_1, \ldots, u_k) \neq \tilde{0} \text{ and} \\ & \quad u = f(\mu_M(\sigma)(u_1, \ldots, u_k)) , \\ 0 & \text{otherwise ,} \end{cases}$$

for every $k \in \mathbb{N}$, $\sigma \in \Sigma^{(k)}$, $u_1, \ldots, u_k \in Q'$, and $u \in Q'$,

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- $\nu'_u = \sum_{q \in Q} u_q \cdot \nu_q$ for every $u \in Q'$.

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- determinization construction for wta over commutative semirings
- resulting automaton not necessarily finite
- correctness
- sufficient conditions for finiteness:
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Twins property

undecidable in general

Twins property

A wta $M = (Q, \mu, \nu)$ is said to have the twins property if for every $p, q \in Q$, $\xi \in T_\Sigma$, and $\zeta \in C_\Sigma$, we have that if $\mathrm{wt}_M(R_M^p(\zeta \cdot \xi)) \neq 0$ and $\mathrm{wt}_M(R_M^q(\zeta \cdot \xi)) \neq 0$, then $\mathrm{wt}_M(R_M^{p,p}(\zeta)) = \mathrm{wt}_M(R_M^{q,q}(\zeta))$.

References

B. Borchardt.
A pumping lemma and decidability problems for recognizable tree series.
Acta Cybernet., 16(4):509–544, 2004.

D. Kirsten and I. Mäurer.
On the determinization of weighted automata.
J. Autom. Lang. Comb., 10:287–312, 2005.

J. May and K. Knight.
A better N-best list: practical determinization of weighted finite tree automata.
In *Proc. NAACL-HLT*, 2006.

M. Mohri.
Finite-state transducers in language and speech processing.
Computational Linguistics, 23(2):269–311, 1997.