Statistical Machine Translation with Weighted Grammars

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September 12, 2011

Outline

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Ambiguity in Natural Language

Example (Chiang 2007)

- e = "Australia is one of the few countries that have diplomatic relations with North Korea"
- f = "Aozhou shi yu Beihan you bangjiao de shaoshu guojia zhiyi"

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Is *e* an adequate translation of *f*?

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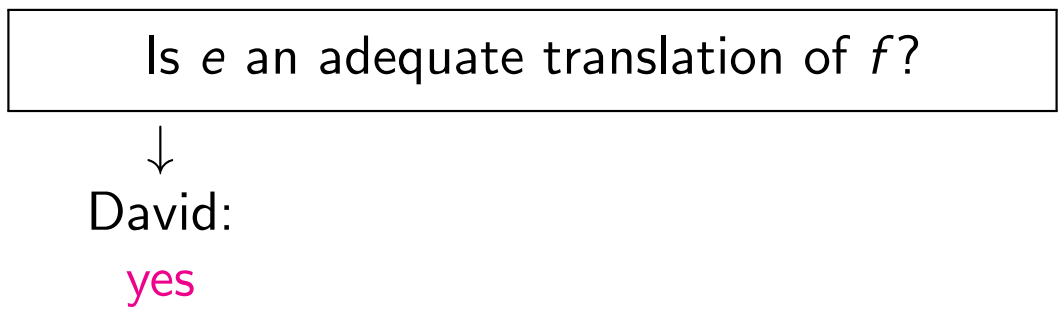

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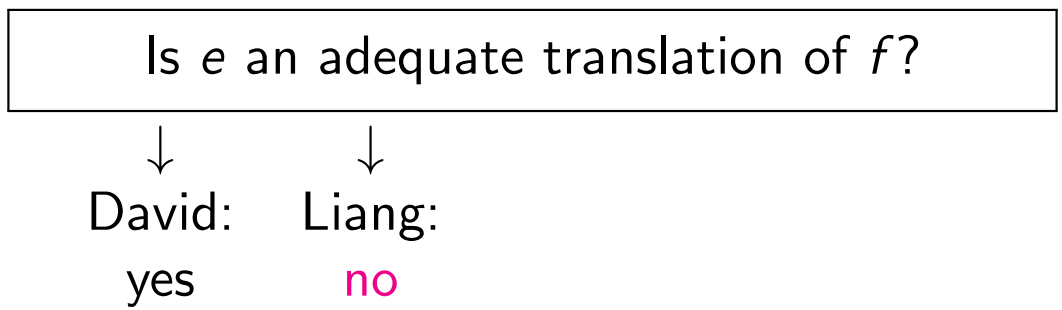

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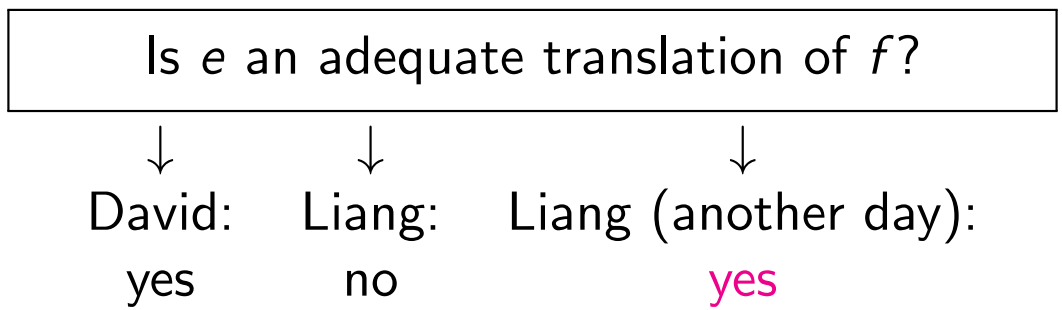

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$\downarrow$

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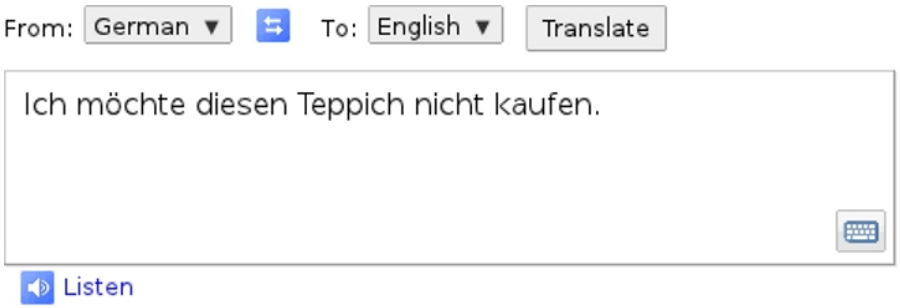
Search (aka Decoding) in SMT



given $p \in \mathcal{M}(E \mid F)$

Search (aka Decoding) in SMT

given $p \in \mathcal{M}(E \mid F)$ $(\sum_e p(e \mid f) = 1$ for every $f)$

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for short

$$\hat{e} = \operatorname{argmax}_{e \in E} p(e \mid f) \,.$$

Outline

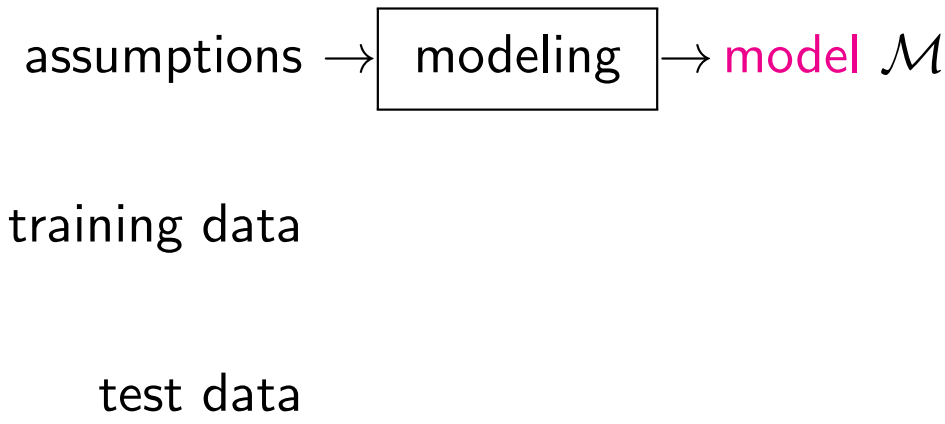
Building an SMT System

assumptions

training data

test data

Building an SMT System



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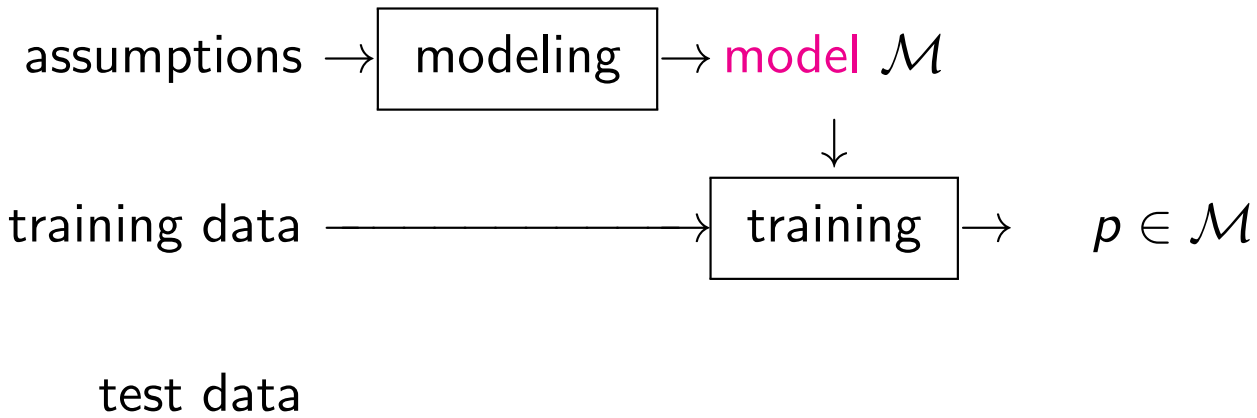

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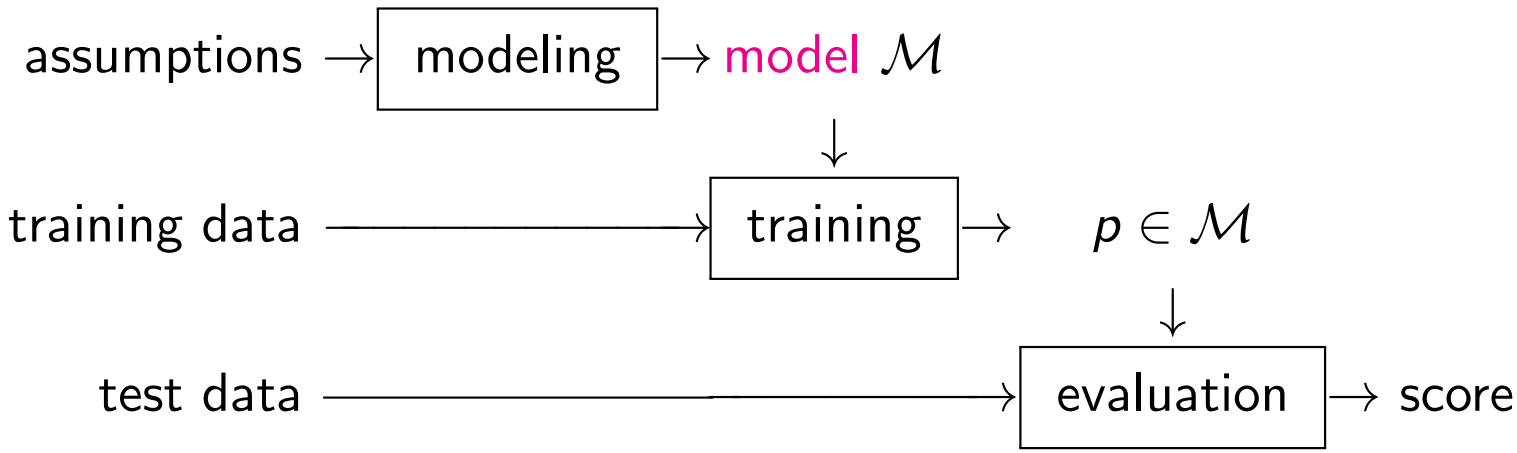

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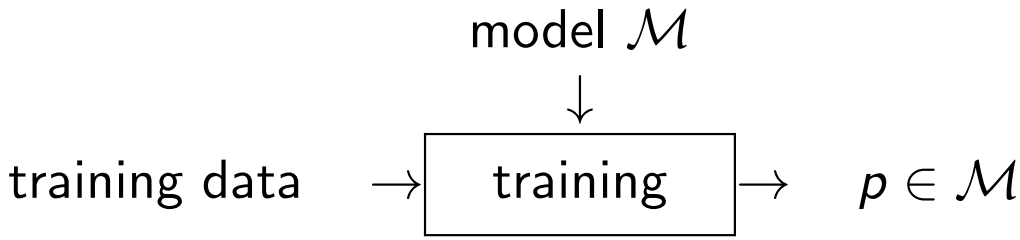
Training = Point Estimation



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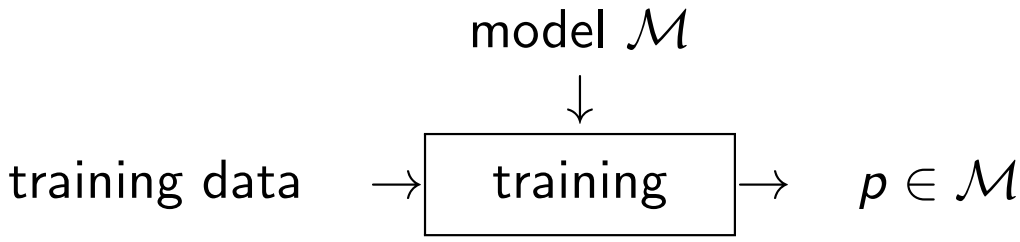

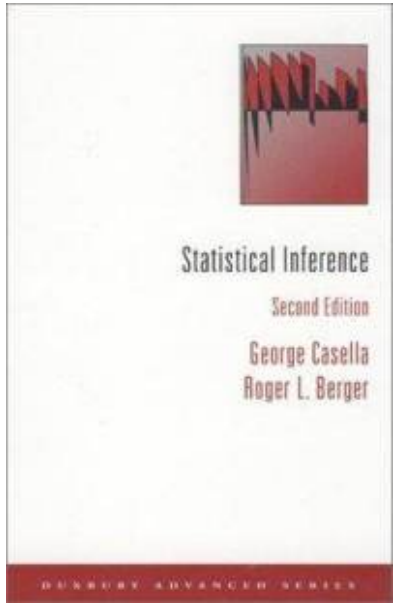

Statistical Inference, Ch. 7: Point Estimation (Casella and Berger 2002)

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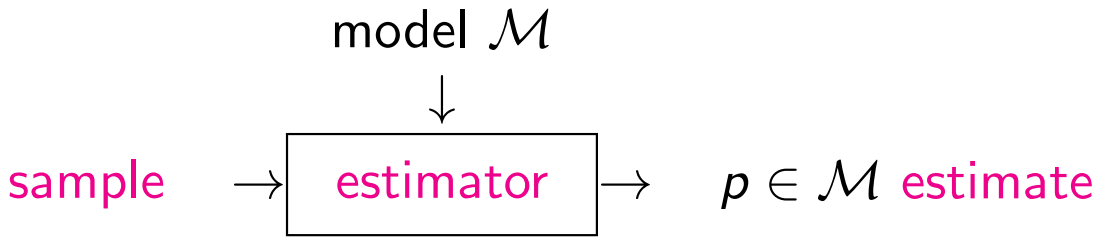

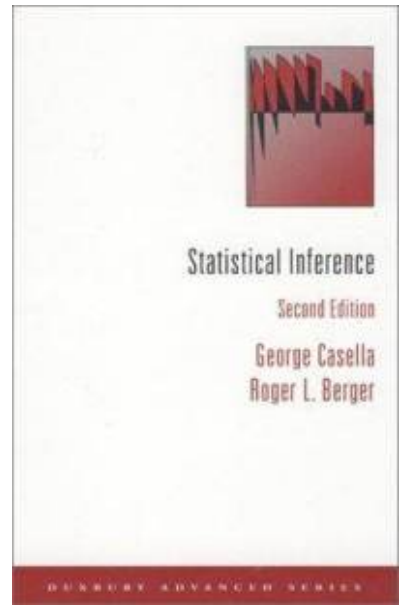

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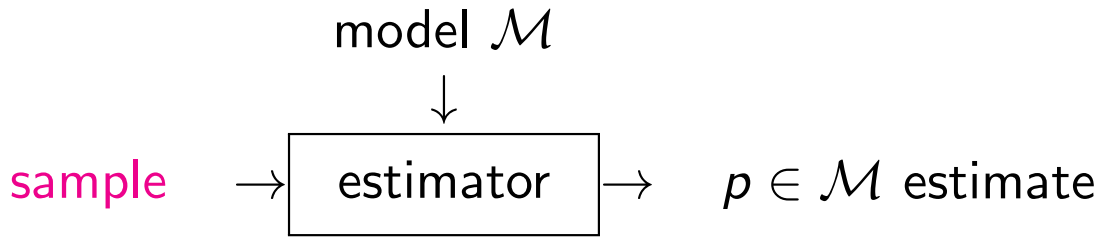
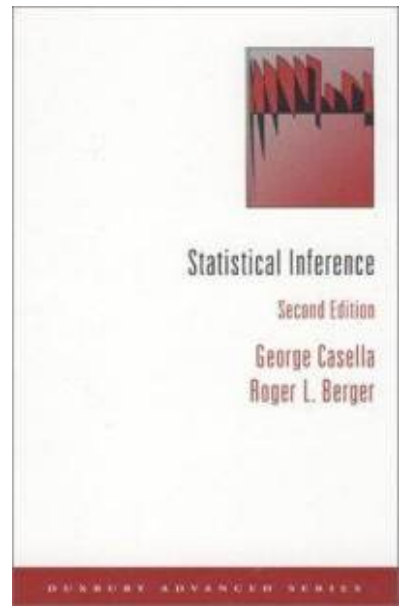


sample

- representative of all translation situations
- e. g., Hong Kong Hansards (parliament proceedings)



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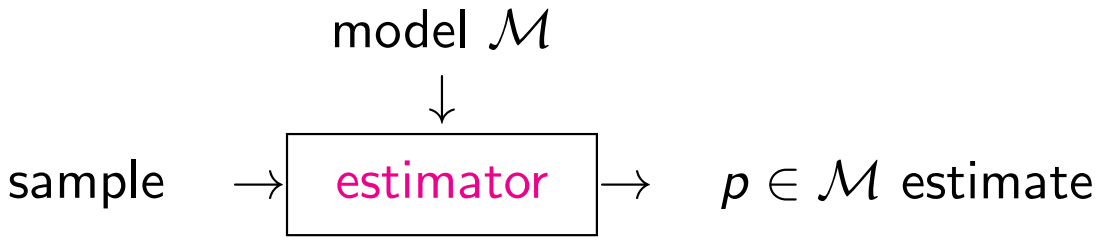

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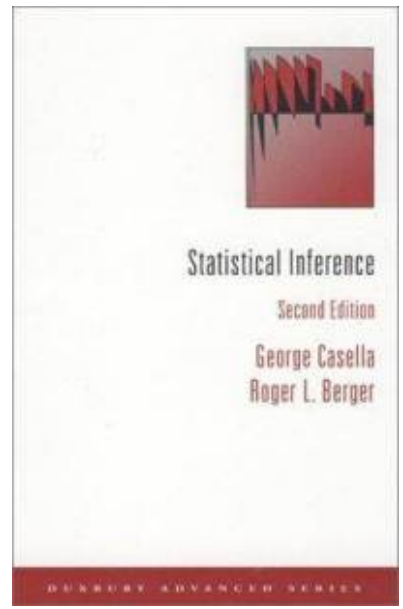
- maximum entropy



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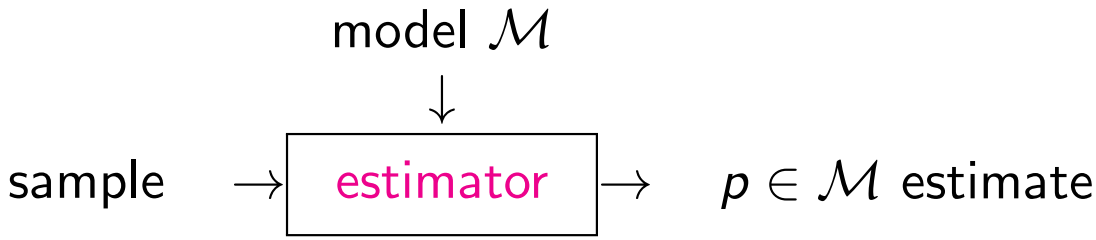

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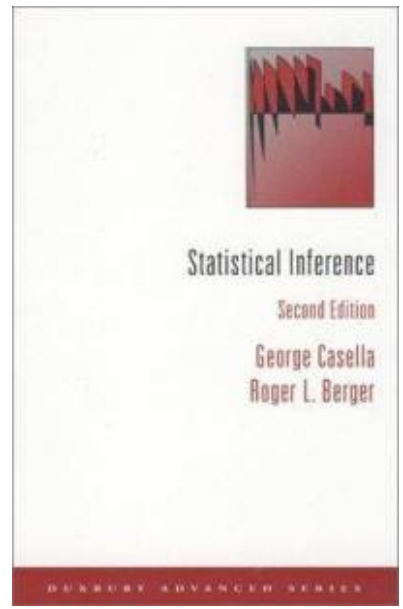

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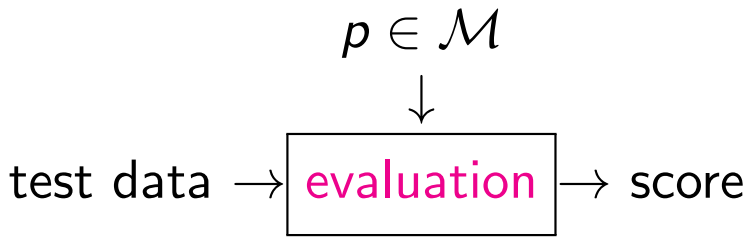
Evaluation



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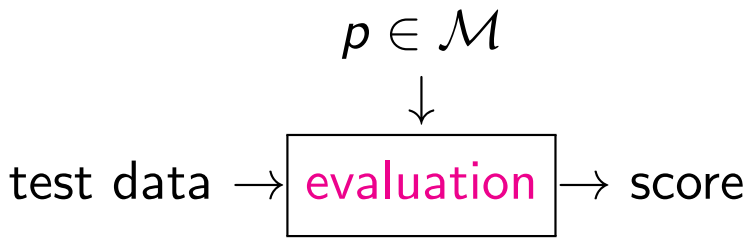

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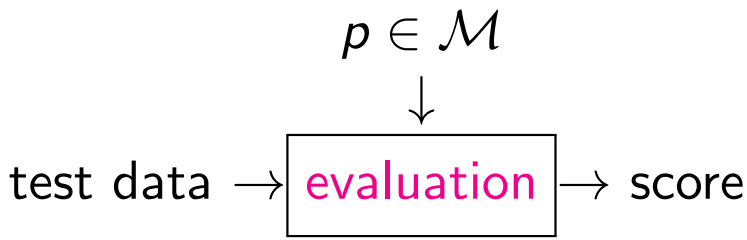

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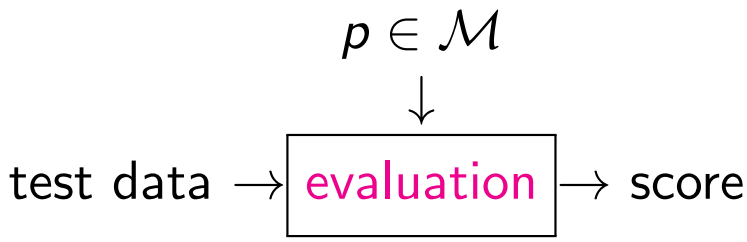

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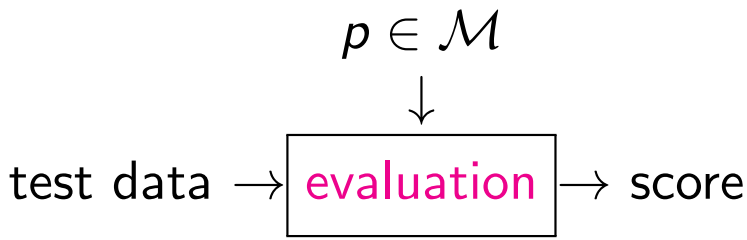

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Meteor (Banerjee and Lavie 2005)	alignments, unigram precision, unigram recall, penalty	higher values

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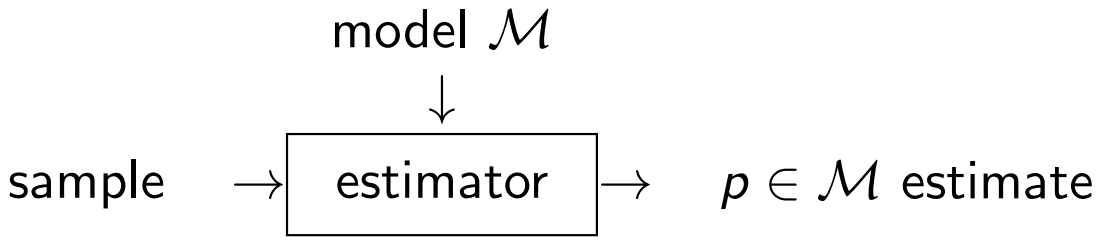

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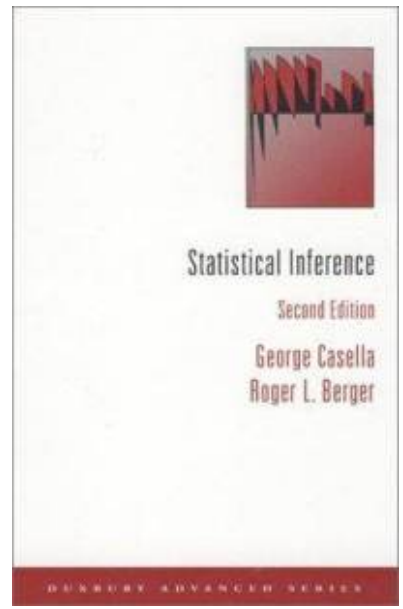

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- model appropriately

Log-linear Model

(Berger, Della Pietra, and Della Pietra 1996; Och and Ney 2002)

- $h_1, \ldots, h_n \colon E \times F \to \mathbb{R}$ features

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- $h_1, \ldots, h_n \colon E \times F \to \mathbb{R}$ features
- maximum-entropy formulation:

$$\mathcal{M} = \{p \in \mathcal{M}(E \mid F) \;\; \Big| \;\; \forall i \colon \mathbb{E}_p[h_i] = \mathbb{E}_{\tilde{p}}[h_i]\}$$

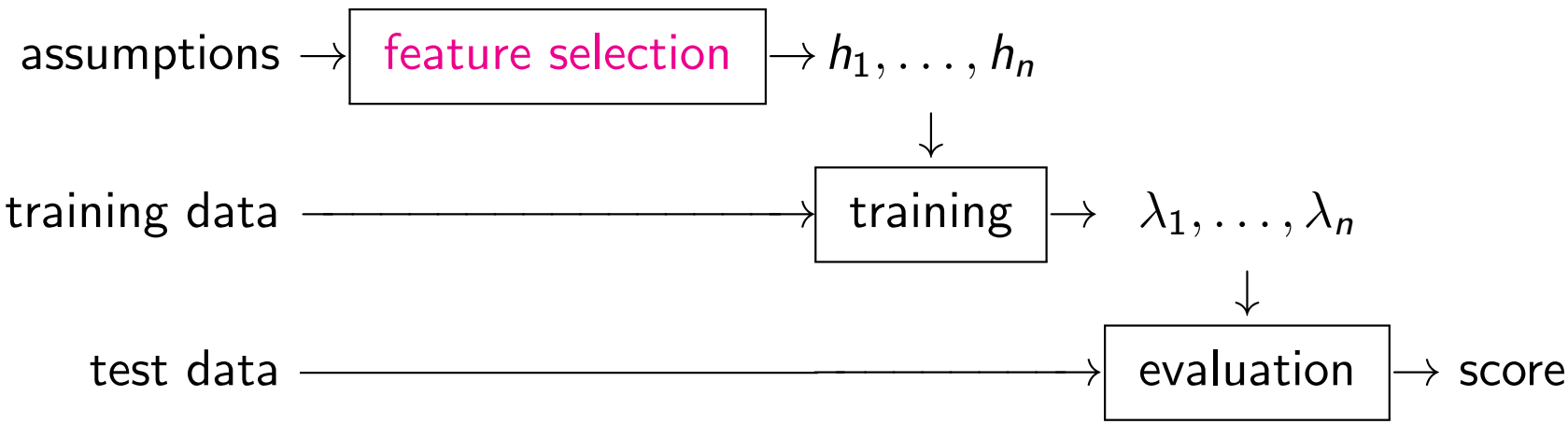
Log-linear Model

(Berger, Della Pietra, and Della Pietra 1996; Och and Ney 2002)

- $h_1, \ldots, h_n \colon E \times F \to \mathbb{R}$ features
- maximum-likelihood formulation:

$$\mathcal{M} = \{p \in \mathcal{M}(E \mid F) \;\Big|\; \exists \lambda_1, \ldots, \lambda_n \in \mathbb{R} \colon p(e \mid f) = \frac{\exp \sum_i \lambda_i \cdot h_i(e, f)}{\sum_{e' \in E} \exp \sum_i \lambda_i \cdot h_i(e', f)}\}$$

Building an SMT System with Log-linear Models



Feature Selection (Examples)

- translation model feature

$$h_1(e, f) = \log p_{\mathrm{TM}}(e \mid f)$$

estimated from a model $\mathcal{M}_{\mathrm{TM}} \subseteq \mathcal{M}(E \mid F)$ via a sample of translations

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- length feature

$$h_3(e, f) = |e|$$

More Recent Estimators

- minimum-error-rate training (Och 2003)
- minimum-risk annealing (Smith and Eisner 2006)
- large-margin methods (Chiang)

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Example (Chiang 2007)

Recall

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Synchronous Context-Free Grammar G:

π_1: $\mathrm{S} \rightarrow \langle \mathrm{S}\ \mathrm{X},\ \mathrm{S}\ \mathrm{X} \rangle$

π_2: $\mathrm{S} \rightarrow \langle \mathrm{X},\ \mathrm{X} \rangle$

π_3: $\mathrm{X} \rightarrow \langle \text{yu}\ \mathrm{X}_{\boxed{1}}\ \text{you}\ \mathrm{X}_{\boxed{2}},\ \text{have}\ \mathrm{X}_{\boxed{2}}\ \text{with}\ \mathrm{X}_{\boxed{1}} \rangle$

π_4: $\mathrm{X} \rightarrow \langle \mathrm{X}_{\boxed{1}}\ \text{de}\ \mathrm{X}_{\boxed{2}},\ \text{the}\ \mathrm{X}_{\boxed{2}}\ \text{that}\ \mathrm{X}_{\boxed{1}} \rangle$

π_5: $\mathrm{X} \rightarrow \langle \mathrm{X}\ \text{zhiyi},\ \text{one of}\ \mathrm{X} \rangle$

π_6: $\mathrm{X} \rightarrow \langle \text{Aozhou},\ \text{Australia} \rangle$

π_7: $\mathrm{X} \rightarrow \langle \text{Beihan},\ \text{North Korea} \rangle$

π_8: $\mathrm{X} \rightarrow \langle \text{shi},\ \text{is} \rangle$

π_9: $\mathrm{X} \rightarrow \langle \text{bangjiao},\ \text{diplomatic relations} \rangle$

π_{10}: $\mathrm{X} \rightarrow \langle \text{shaoshu guojia},\ \text{few countries} \rangle$

Derivation

$\langle S, S \rangle$

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\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{S}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_2}{\Rightarrow}\ & \langle \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_6}{\Rightarrow}\ & \langle \mathrm{Aozhou}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{Australia}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle
\end{aligned}
$$

Derivation

$$
\begin{aligned}
& \langle \mathrm{S}, \mathrm{S} \rangle \\
\overset{\pi_1}{\Rightarrow} \; & \langle \mathrm{S}\ \mathrm{X}, \mathrm{S}\ \mathrm{X} \rangle \\
\overset{\pi_1}{\Rightarrow} \; & \langle \mathrm{S}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}}, \mathrm{S}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_2}{\Rightarrow} \; & \langle \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}}, \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_6}{\Rightarrow} \; & \langle \text{Aozhou}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}}, \text{Australia}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_8}{\Rightarrow} \; & \langle \text{Aozhou shi X}, \text{Australia is X} \rangle
\end{aligned}
$$

Derivation

$$
\begin{aligned}
& \langle \mathrm{S},\ \mathrm{S} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S}\ \mathrm{X},\ \mathrm{S}\ \mathrm{X} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{S}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_2}{\Rightarrow}\ & \langle \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_6}{\Rightarrow}\ & \langle \text{Aozhou}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \text{Australia}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_8}{\Rightarrow}\ & \langle \text{Aozhou shi X},\ \text{Australia is X} \rangle
\end{aligned}
$$

Derivation

$$
\begin{aligned}
& \langle \mathrm{S},\ \mathrm{S} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X},\ \mathrm{S\ X} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{S\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_2}{\Rightarrow}\ & \langle \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_6}{\Rightarrow}\ & \langle \mathrm{Aozhou\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{Australia\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_8}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X},\ \mathrm{Australia\ is\ X} \rangle \\
\overset{\pi_5}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X\ zhiyi},\ \mathrm{Australia\ is\ one\ of\ X} \rangle
\end{aligned}
$$

Derivation

$$
\begin{aligned}
& \langle \mathrm{S},\ \mathrm{S} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S}\ \mathrm{X},\ \mathrm{S}\ \mathrm{X} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{S}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_2}{\Rightarrow}\ & \langle \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_6}{\Rightarrow}\ & \langle \text{Aozhou}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \text{Australia}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_8}{\Rightarrow}\ & \langle \text{Aozhou shi X},\ \text{Australia is X} \rangle \\
\overset{\pi_5}{\Rightarrow}\ & \langle \text{Aozhou shi X zhiyi},\ \text{Australia is one of X} \rangle
\end{aligned}
$$

Derivation

$$
\begin{aligned}
& \langle \mathrm{S},\ \mathrm{S} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X},\ \mathrm{S\ X} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{S\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_2}{\Rightarrow}\ & \langle \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_6}{\Rightarrow}\ & \langle \mathrm{Aozhou\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{Australia\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_8}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X},\ \mathrm{Australia\ is\ X} \rangle \\
\overset{\pi_5}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X\ zhiyi},\ \mathrm{Australia\ is\ one\ of\ X} \rangle \\
\overset{\pi_4}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X}_{\boxed{1}}\ \mathrm{de\ X}_{\boxed{2}}\ \mathrm{zhiyi}, \\
& \ \mathrm{Australia\ is\ one\ of\ the\ X}_{\boxed{2}}\ \mathrm{that\ X}_{\boxed{1}} \rangle
\end{aligned}
$$

Derivation

$$
\begin{aligned}
& \langle \mathrm{S},\ \mathrm{S} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X},\ \mathrm{S\ X} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{S\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_2}{\Rightarrow}\ & \langle \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_6}{\Rightarrow}\ & \langle \mathrm{Aozhou\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{Australia\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_8}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X},\ \mathrm{Australia\ is\ X} \rangle \\
\overset{\pi_5}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X\ zhiyi},\ \mathrm{Australia\ is\ one\ of\ X} \rangle \\
\overset{\pi_4}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X}_{\boxed{1}}\ \mathrm{de\ X}_{\boxed{2}}\ \mathrm{zhiyi}, \\
& \quad \mathrm{Australia\ is\ one\ of\ the\ X}_{\boxed{2}}\ \mathrm{that\ X}_{\boxed{1}} \rangle
\end{aligned}
$$

Derivation

$$
\begin{aligned}
& \langle \mathrm{S},\ \mathrm{S} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X},\ \mathrm{S\ X} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{S\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_2}{\Rightarrow}\ & \langle \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_6}{\Rightarrow}\ & \langle \mathrm{Aozhou\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{Australia\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_8}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X},\ \mathrm{Australia\ is\ X} \rangle \\
\overset{\pi_5}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X\ zhiyi},\ \mathrm{Australia\ is\ one\ of\ X} \rangle \\
\overset{\pi_4}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X}_{\boxed{1}}\ \mathrm{de\ X}_{\boxed{2}}\ \mathrm{zhiyi}, \\
& \quad \mathrm{Australia\ is\ one\ of\ the\ X}_{\boxed{2}}\ \mathrm{that\ X}_{\boxed{1}} \rangle \\
\overset{\pi_3}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ yu\ X}_{\boxed{1}}\ \mathrm{you\ X}_{\boxed{0}}\ \mathrm{de\ X}_{\boxed{2}}\ \mathrm{zhiyi}, \\
& \quad \mathrm{Australia\ is\ one\ of\ the\ X}_{\boxed{2}}\ \mathrm{that\ have\ X}_{\boxed{0}}\ \mathrm{with\ X}_{\boxed{1}} \rangle
\end{aligned}
$$

Derivation

$$
\begin{aligned}
& \langle \mathrm{S},\ \mathrm{S} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X},\ \mathrm{S\ X} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{S\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_2}{\Rightarrow}\ & \langle \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_6}{\Rightarrow}\ & \langle \mathrm{Aozhou\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{Australia\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_8}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X},\ \mathrm{Australia\ is\ X} \rangle \\
\overset{\pi_5}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X\ zhiyi},\ \mathrm{Australia\ is\ one\ of\ X} \rangle \\
\overset{\pi_4}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X}_{\boxed{1}}\ \mathrm{de\ X}_{\boxed{2}}\ \mathrm{zhiyi}, \\
& \quad \mathrm{Australia\ is\ one\ of\ the\ X}_{\boxed{2}}\ \mathrm{that\ X}_{\boxed{1}} \rangle \\
\overset{\pi_3}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ yu\ X}_{\boxed{1}}\ \mathrm{you\ X}_{\boxed{0}}\ \mathrm{de\ X}_{\boxed{2}}\ \mathrm{zhiyi}, \\
& \quad \mathrm{Australia\ is\ one\ of\ the\ X}_{\boxed{2}}\ \mathrm{that\ have\ X}_{\boxed{0}}\ \mathrm{with\ X}_{\boxed{1}} \rangle
\end{aligned}
$$

Derivation

$$
\begin{aligned}
& \langle \mathrm{S},\ \mathrm{S} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X},\ \mathrm{S\ X} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X_{\boxed{1}}\ X_{\boxed{2}}},\ \mathrm{S\ X_{\boxed{1}}\ X_{\boxed{2}}} \rangle \\
\overset{\pi_2}{\Rightarrow}\ & \langle \mathrm{X_{\boxed{0}}\ X_{\boxed{1}}\ X_{\boxed{2}}},\ \mathrm{X_{\boxed{0}}\ X_{\boxed{1}}\ X_{\boxed{2}}} \rangle \\
\overset{\pi_6}{\Rightarrow}\ & \langle \mathrm{Aozhou\ X_{\boxed{1}}\ X_{\boxed{2}}},\ \mathrm{Australia\ X_{\boxed{1}}\ X_{\boxed{2}}} \rangle \\
\overset{\pi_8}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X},\ \mathrm{Australia\ is\ X} \rangle \\
\overset{\pi_5}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X\ zhiyi},\ \mathrm{Australia\ is\ one\ of\ X} \rangle \\
\overset{\pi_4}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X_{\boxed{1}}\ de\ X_{\boxed{2}}\ zhiyi}, \\
& \quad \mathrm{Australia\ is\ one\ of\ the\ X_{\boxed{2}}\ that\ X_{\boxed{1}}} \rangle \\
\overset{\pi_3}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ yu\ X_{\boxed{1}}\ you\ X_{\boxed{0}}\ de\ X_{\boxed{2}}\ zhiyi}, \\
& \quad \mathrm{Australia\ is\ one\ of\ the\ X_{\boxed{2}}\ that\ have\ X_{\boxed{0}}\ with\ X_{\boxed{1}}} \rangle \\
\overset{\pi_7}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ yu\ Beihan\ you\ X_{\boxed{0}}\ de\ X_{\boxed{2}}\ zhiyi}, \\
& \quad \mathrm{Australia\ is\ one\ of\ the\ X_{\boxed{2}}\ that\ have\ X_{\boxed{0}}\ with\ North\ Korea} \rangle
\end{aligned}
$$

Derivation

$$
\begin{aligned}
& \langle \mathrm{S},\ \mathrm{S} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X},\ \mathrm{S\ X} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X_{\boxed{1}}\ X_{\boxed{2}}},\ \mathrm{S\ X_{\boxed{1}}\ X_{\boxed{2}}} \rangle \\
\overset{\pi_2}{\Rightarrow}\ & \langle \mathrm{X_{\boxed{0}}\ X_{\boxed{1}}\ X_{\boxed{2}}},\ \mathrm{X_{\boxed{0}}\ X_{\boxed{1}}\ X_{\boxed{2}}} \rangle \\
\overset{\pi_6}{\Rightarrow}\ & \langle \mathrm{Aozhou\ X_{\boxed{1}}\ X_{\boxed{2}}},\ \mathrm{Australia\ X_{\boxed{1}}\ X_{\boxed{2}}} \rangle \\
\overset{\pi_8}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X},\ \mathrm{Australia\ is\ X} \rangle \\
\overset{\pi_5}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X\ zhiyi},\ \mathrm{Australia\ is\ one\ of\ X} \rangle \\
\overset{\pi_4}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X_{\boxed{1}}\ de\ X_{\boxed{2}}\ zhiyi}, \\
& \quad \mathrm{Australia\ is\ one\ of\ the\ X_{\boxed{2}}\ that\ X_{\boxed{1}}} \rangle \\
\overset{\pi_3}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ yu\ X_{\boxed{1}}\ you\ X_{\boxed{0}}\ de\ X_{\boxed{2}}\ zhiyi}, \\
& \quad \mathrm{Australia\ is\ one\ of\ the\ X_{\boxed{2}}\ that\ have\ X_{\boxed{0}}\ with\ X_{\boxed{1}}} \rangle \\
\overset{\pi_7}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ yu\ Beihan\ you\ X_{\boxed{0}}\ de\ X_{\boxed{2}}\ zhiyi}, \\
& \quad \mathrm{Australia\ is\ one\ of\ the\ X_{\boxed{2}}\ that\ have\ X_{\boxed{0}}\ with\ North\ Korea} \rangle
\end{aligned}
$$

Derivation

$$
\begin{aligned}
& \langle \mathrm{S},\ \mathrm{S} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X},\ \mathrm{S\ X} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{S\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_2}{\Rightarrow}\ & \langle \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_6}{\Rightarrow}\ & \langle \mathrm{Aozhou\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{Australia\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_8}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X},\ \mathrm{Australia\ is\ X} \rangle \\
\overset{\pi_5}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X\ zhiyi},\ \mathrm{Australia\ is\ one\ of\ X} \rangle \\
\overset{\pi_4}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X}_{\boxed{1}}\ \mathrm{de\ X}_{\boxed{2}}\ \mathrm{zhiyi}, \\
& \quad \mathrm{Australia\ is\ one\ of\ the\ X}_{\boxed{2}}\ \mathrm{that\ X}_{\boxed{1}} \rangle \\
\overset{\pi_3}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ yu\ X}_{\boxed{1}}\ \mathrm{you\ X}_{\boxed{0}}\ \mathrm{de\ X}_{\boxed{2}}\ \mathrm{zhiyi}, \\
& \quad \mathrm{Australia\ is\ one\ of\ the\ X}_{\boxed{2}}\ \mathrm{that\ have\ X}_{\boxed{0}}\ \mathrm{with\ X}_{\boxed{1}} \rangle \\
\overset{\pi_7}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ yu\ Beihan\ you\ X}_{\boxed{0}}\ \mathrm{de\ X}_{\boxed{2}}\ \mathrm{zhiyi}, \\
& \quad \mathrm{Australia\ is\ one\ of\ the\ X}_{\boxed{2}}\ \mathrm{that\ have\ X}_{\boxed{0}}\ \mathrm{with\ North\ Korea} \rangle \\
\overset{\pi_9}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ yu\ Beihan\ you\ bangjiao\ de\ X}_{\boxed{2}}\ \mathrm{zhiyi}, \\
& \quad \mathrm{Australia\ is\ one\ of\ the\ X}_{\boxed{2}}\ \mathrm{that\ have\ diplomatic\ relations\ with\ North\ Korea} \rangle
\end{aligned}
$$

Derivation

$\langle S, S \rangle$

$\overset{\pi_1}{\Rightarrow} \langle S\ X, S\ X \rangle$

$\overset{\pi_1}{\Rightarrow} \langle S\ X_{\boxed{1}}\ X_{\boxed{2}}, S\ X_{\boxed{1}}\ X_{\boxed{2}} \rangle$

$\overset{\pi_2}{\Rightarrow} \langle X_{\boxed{0}}\ X_{\boxed{1}}\ X_{\boxed{2}}, X_{\boxed{0}}\ X_{\boxed{1}}\ X_{\boxed{2}} \rangle$

$\overset{\pi_6}{\Rightarrow} \langle \text{Aozhou } X_{\boxed{1}}\ X_{\boxed{2}}, \text{Australia } X_{\boxed{1}}\ X_{\boxed{2}} \rangle$

$\overset{\pi_8}{\Rightarrow} \langle \text{Aozhou shi X, Australia is X} \rangle$

$\overset{\pi_5}{\Rightarrow} \langle \text{Aozhou shi X zhiyi, Australia is one of X} \rangle$

$\overset{\pi_4}{\Rightarrow} \langle \text{Aozhou shi } X_{\boxed{1}} \text{ de } X_{\boxed{2}} \text{ zhiyi,}$
$\text{Australia is one of the } X_{\boxed{2}} \text{ that } X_{\boxed{1}} \rangle$

$\overset{\pi_3}{\Rightarrow} \langle \text{Aozhou shi yu } X_{\boxed{1}} \text{ you } X_{\boxed{0}} \text{ de } X_{\boxed{2}} \text{ zhiyi,}$
$\text{Australia is one of the } X_{\boxed{2}} \text{ that have } X_{\boxed{0}} \text{ with } X_{\boxed{1}} \rangle$

$\overset{\pi_7}{\Rightarrow} \langle \text{Aozhou shi yu Beihan you } X_{\boxed{0}} \text{ de } X_{\boxed{2}} \text{ zhiyi,}$
$\text{Australia is one of the } X_{\boxed{2}} \text{ that have } X_{\boxed{0}} \text{ with North Korea} \rangle$

$\overset{\pi_9}{\Rightarrow} \langle \text{Aozhou shi yu Beihan you bangjiao de } X_{\boxed{2}} \text{ zhiyi,}$
$\text{Australia is one of the } X_{\boxed{2}} \text{ that have diplomatic relations with North Korea} \rangle$

Derivation

$\langle$S, S$\rangle$

$\overset{\pi_1}{\Rightarrow}$ $\langle$S X, S X$\rangle$

$\overset{\pi_1}{\Rightarrow}$ $\langle$S $X_{\boxed{1}}$ $X_{\boxed{2}}$, S $X_{\boxed{1}}$ $X_{\boxed{2}}$$\rangle$

$\overset{\pi_2}{\Rightarrow}$ $\langle$$X_{\boxed{0}}$ $X_{\boxed{1}}$ $X_{\boxed{2}}$, $X_{\boxed{0}}$ $X_{\boxed{1}}$ $X_{\boxed{2}}$$\rangle$

$\overset{\pi_6}{\Rightarrow}$ $\langle$Aozhou $X_{\boxed{1}}$ $X_{\boxed{2}}$, Australia $X_{\boxed{1}}$ $X_{\boxed{2}}$$\rangle$

$\overset{\pi_8}{\Rightarrow}$ $\langle$Aozhou shi X, Australia is X$\rangle$

$\overset{\pi_5}{\Rightarrow}$ $\langle$Aozhou shi X zhiyi, Australia is one of X$\rangle$

$\overset{\pi_4}{\Rightarrow}$ $\langle$Aozhou shi $X_{\boxed{1}}$ de $X_{\boxed{2}}$ zhiyi,
Australia is one of the $X_{\boxed{2}}$ that $X_{\boxed{1}}$$\rangle$

$\overset{\pi_3}{\Rightarrow}$ $\langle$Aozhou shi yu $X_{\boxed{1}}$ you $X_{\boxed{0}}$ de $X_{\boxed{2}}$ zhiyi,
Australia is one of the $X_{\boxed{2}}$ that have $X_{\boxed{0}}$ with $X_{\boxed{1}}$$\rangle$

$\overset{\pi_7}{\Rightarrow}$ $\langle$Aozhou shi yu Beihan you $X_{\boxed{0}}$ de $X_{\boxed{2}}$ zhiyi,
Australia is one of the $X_{\boxed{2}}$ that have $X_{\boxed{0}}$ with North Korea$\rangle$

$\overset{\pi_9}{\Rightarrow}$ $\langle$Aozhou shi yu Beihan you bangjiao de $X_{\boxed{2}}$ zhiyi,
Australia is one of the $X_{\boxed{2}}$ that have diplomatic relations with North Korea$\rangle$

$\overset{\pi_{10}}{\Rightarrow}$ $\langle$Aozhou shi yu Beihan you bangjiao de shaoshu guojia zhiyi,
Australia is one of the few countries that have diplomatic relations with N. K.$\rangle$

Derivation

$\langle$S, S$\rangle$

$\stackrel{\pi_1}{\Rightarrow}$ $\langle$S X, S X$\rangle$

$\stackrel{\pi_1}{\Rightarrow}$ $\langle$S $X_{\boxed{1}}$ $X_{\boxed{2}}$, S $X_{\boxed{1}}$ $X_{\boxed{2}}$$\rangle$

$\stackrel{\pi_2}{\Rightarrow}$ $\langle$$X_{\boxed{0}}$ $X_{\boxed{1}}$ $X_{\boxed{2}}$, $X_{\boxed{0}}$ $X_{\boxed{1}}$ $X_{\boxed{2}}$$\rangle$

$\stackrel{\pi_6}{\Rightarrow}$ $\langle$Aozhou $X_{\boxed{1}}$ $X_{\boxed{2}}$, Australia $X_{\boxed{1}}$ $X_{\boxed{2}}$$\rangle$

$\stackrel{\pi_8}{\Rightarrow}$ $\langle$Aozhou shi X, Australia is X$\rangle$

$\stackrel{\pi_5}{\Rightarrow}$ $\langle$Aozhou shi X zhiyi, Australia is one of X$\rangle$

$\stackrel{\pi_4}{\Rightarrow}$ $\langle$Aozhou shi $X_{\boxed{1}}$ de $X_{\boxed{2}}$ zhiyi,
Australia is one of the $X_{\boxed{2}}$ that $X_{\boxed{1}}$$\rangle$

$\stackrel{\pi_3}{\Rightarrow}$ $\langle$Aozhou shi yu $X_{\boxed{1}}$ you $X_{\boxed{0}}$ de $X_{\boxed{2}}$ zhiyi,
Australia is one of the $X_{\boxed{2}}$ that have $X_{\boxed{0}}$ with $X_{\boxed{1}}$$\rangle$

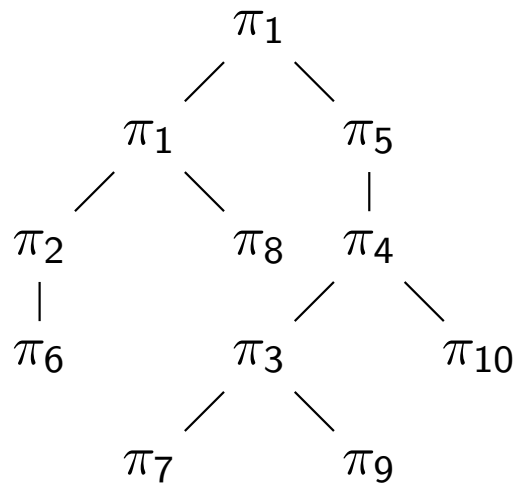
$\stackrel{\pi_7}{\Rightarrow}$ $\langle$Aozhou shi yu Beihan you $X_{\boxed{0}}$ de $X_{\boxed{2}}$ zhiyi,
Australia is one of the $X_{\boxed{2}}$ that have $X_{\boxed{0}}$ with North Korea$\rangle$

$\stackrel{\pi_9}{\Rightarrow}$ $\langle$Aozhou shi yu Beihan you bangjiao de $X_{\boxed{2}}$ zhiyi,
Australia is one of the $X_{\boxed{2}}$ that have diplomatic relations with North Korea$\rangle$

$\stackrel{\pi_{10}}{\Rightarrow}$ $\langle$Aozhou shi yu Beihan you bangjiao de shaoshu guojia zhiyi,
Australia is one of the few countries that have diplomatic relations with N. K.$\rangle$

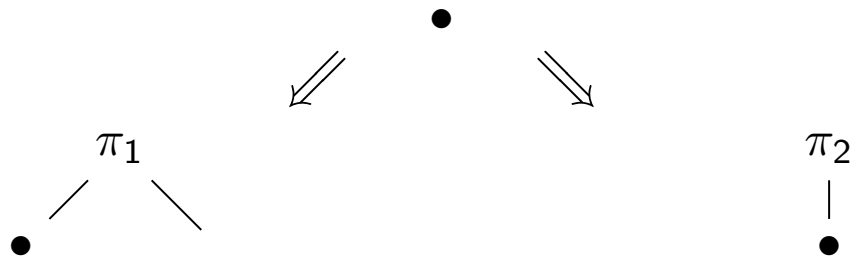
Inducing a Model

Derivation Tree



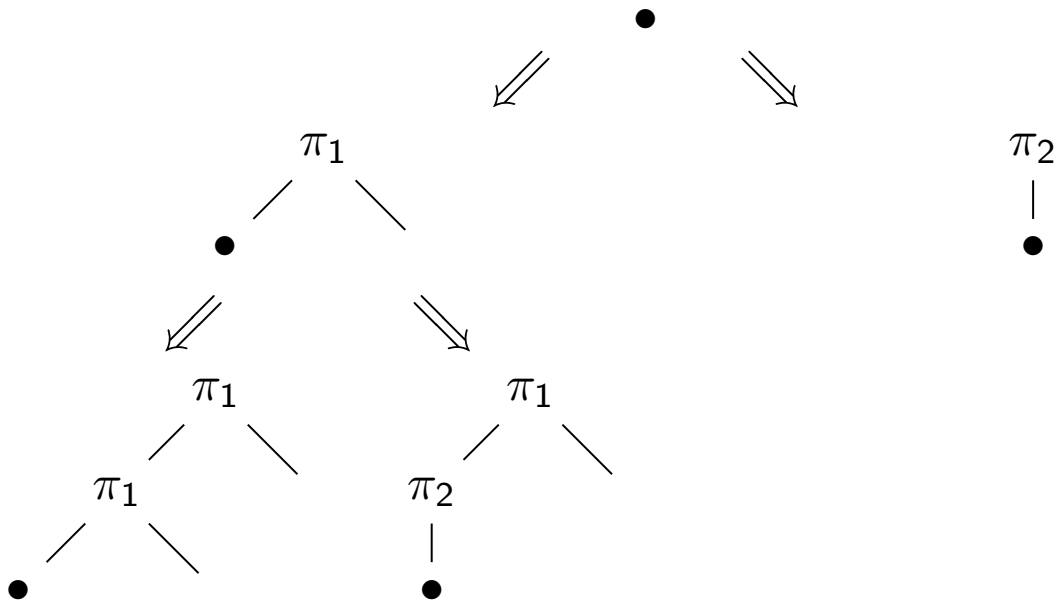
Inducing a Model

Decision Tree



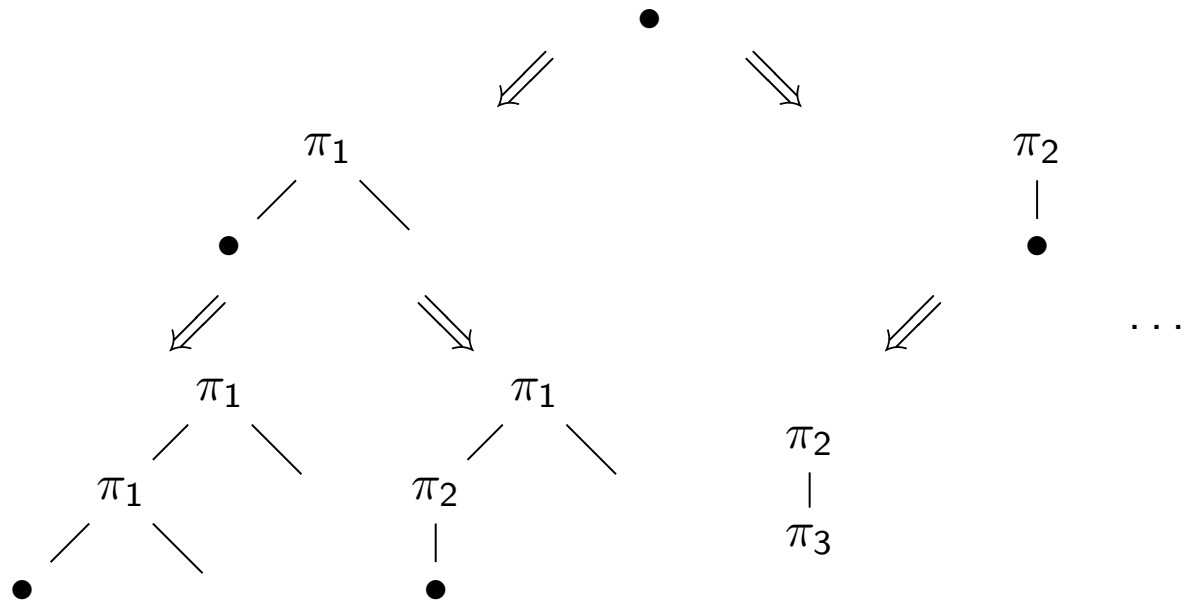
Inducing a Model

Decision Tree



Inducing a Model

Decision Tree



Inducing a Model

Assignments

$$\left.\begin{array}{r}\text{grammar } G \\ \text{assignment } p'\end{array}\right\} \text{“distribution”} \;\; p'(e, d, f)$$

$$\text{model } \mathcal{M}_G = \{p \in \mathcal{M}(E, F) \mid \exists p' \colon p(e, f) = \textstyle\sum_d p'(e, d, f)\}$$

Training: Rule Extraction

Word-aligned Training Pair

⟨30 duonianlai de youhao hezuo, friendly cooperation over the last 30 years⟩

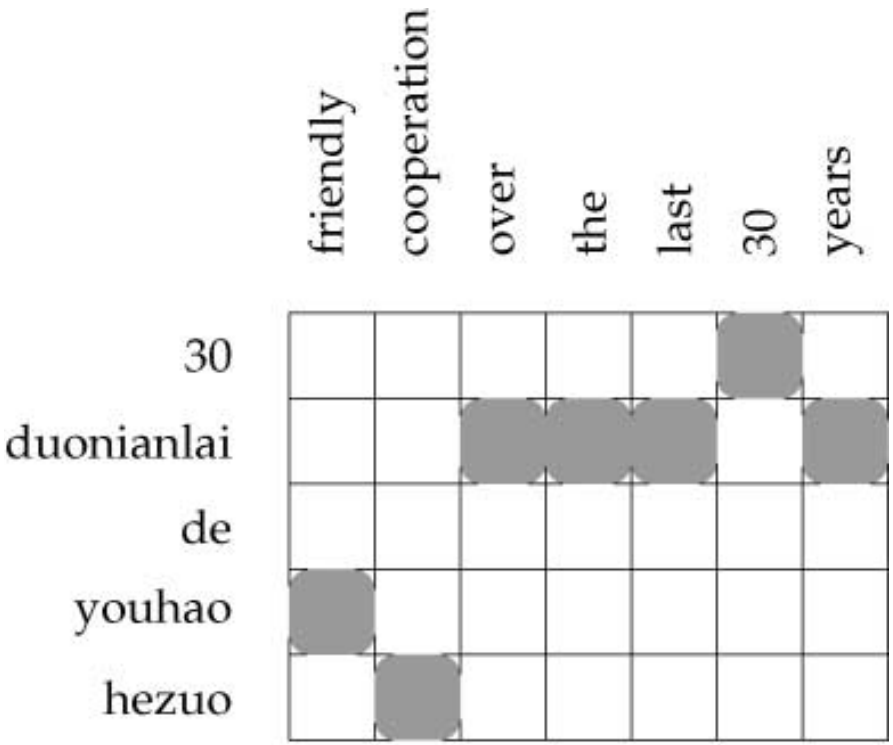


Figure 2 of (Chiang 2007)

Training: Rule Extraction

Initial Rules

$X \rightarrow \langle$30 duonianlai de youhao hezuo, friendly cooperation over the last 30 years$\rangle$

$X \rightarrow \langle$duonianlai, over the last 30 years$\rangle$

$X \rightarrow \langle$youhao hezuo, friendly cooperation$\rangle$

$X \rightarrow \langle 30, 30 \rangle$

$X \rightarrow \langle$youhao, friendly$\rangle$ $\quad$ $X \rightarrow \langle$hezuo, cooperation$\rangle$

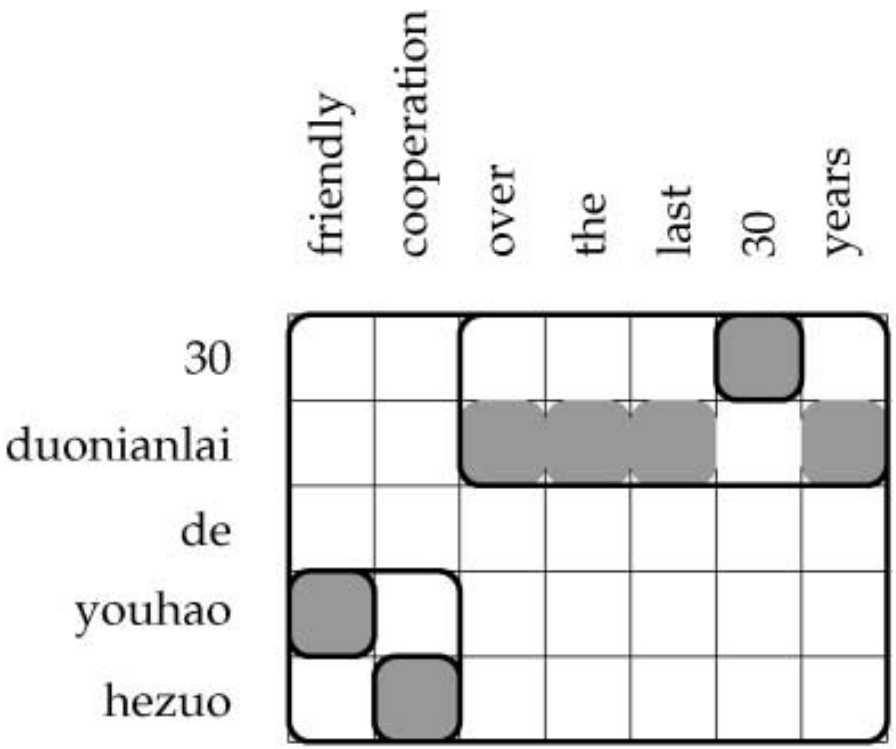


Figure 2 of (Chiang 2007)

Training: Rule Extraction

Example Rule

$X \rightarrow \langle X_{[1]}$ duonianlai de $X_{[2]}$, $X_{[2]}$ over the last $X_{[1]}$ years$\rangle$

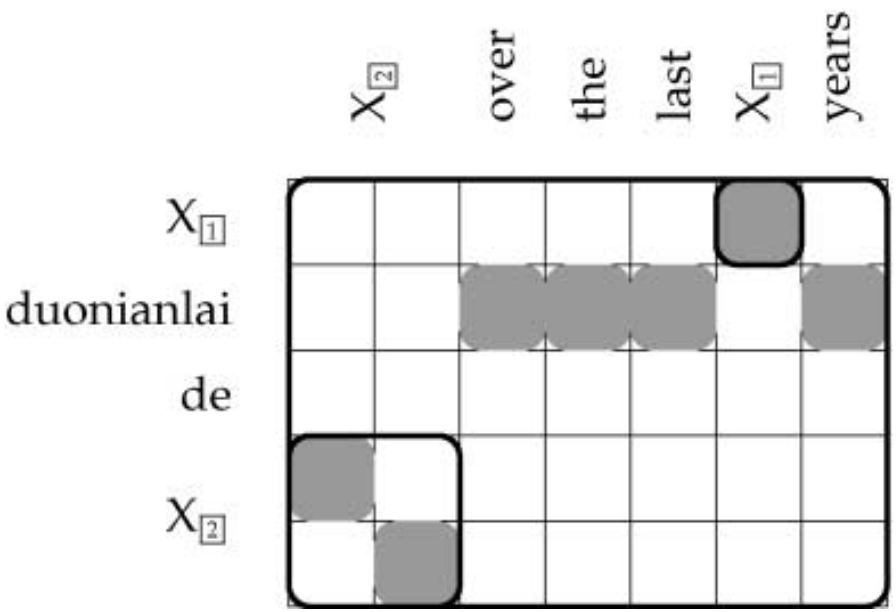


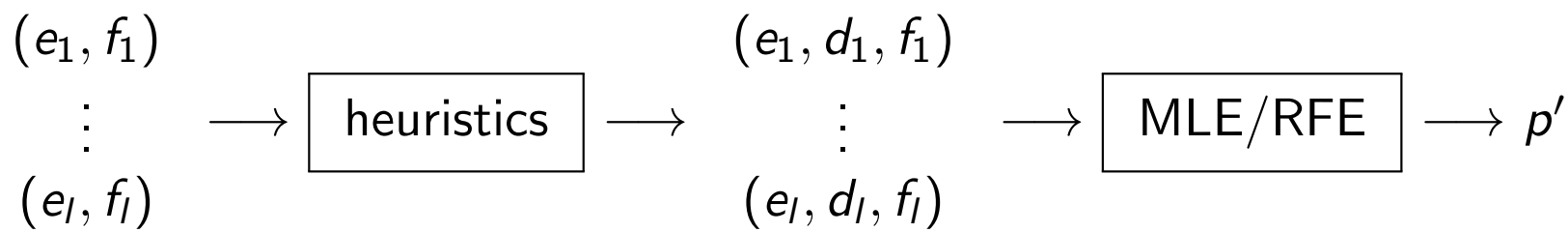
Figure 2 of (Chiang 2007)

Training: Maximum-Likelihood Estimation

- training data: sequence $\mathcal{D} = (e_1, f_1), \ldots, (e_l, f_l)$
- model: $\mathcal{M}_G = \{p \in \mathcal{M}(E, F) \mid \exists p' : p(e, f) = \sum_d p'(e, d, f)\}$
- likelihood: $P(\mathcal{D} \mid p') = \prod_j \sum_d p'(e_j, d, f_j)$
- use EM algorithm

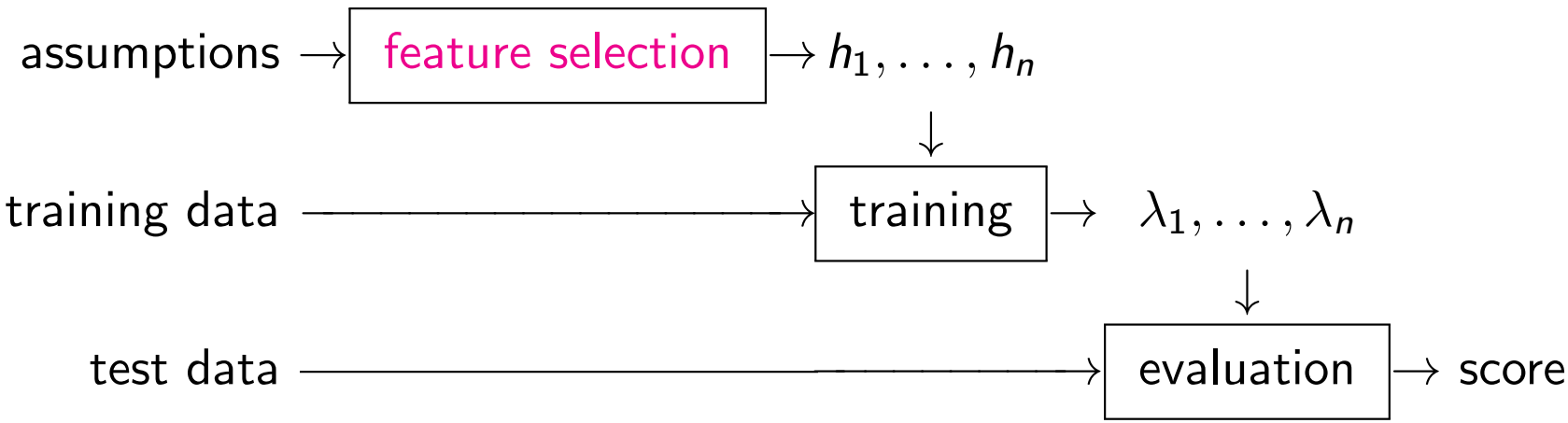
Training: Reality

Chiang (2007), DeNeefe and Knight (2009):



Outline

Building an SMT System with Log-linear Models



Feature Selection

Hidden variable: derivation of (e, f) in G

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- $\mathcal{M} \subseteq \mathcal{M}(E \mid F)$
- $h_1(e, f) = \log \sum_d p_G(e, d, f)$
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- $h_6(e, d, f) = \#_{\text{ex}} d$
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Outline

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$$\hat{e} = \text{argmax}_e \; p(e \mid f)$$

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NP-hard! (Casacuberta and Higuera 2000; Li, Eisner, and Khudanpur 2009)

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where $D_{\text{fin}} = \text{argmax}_d^{(n)} \exp \sum_i \lambda_i \cdot h_i(\text{e}(d), d, f)$

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$$\max\{\ 0.9,\ 0.99,\ 0.999,\ \ldots\ \}\ ?$$

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$$G' \rightarrow \boxed{\text{n-best}} \rightarrow \text{argmax}_d^{(n)}\, p_{G'}(d)$$

n-best $\approx O(|R'| + |N'| \cdot n \cdot \log n)$ (Huang and Chiang 2005)

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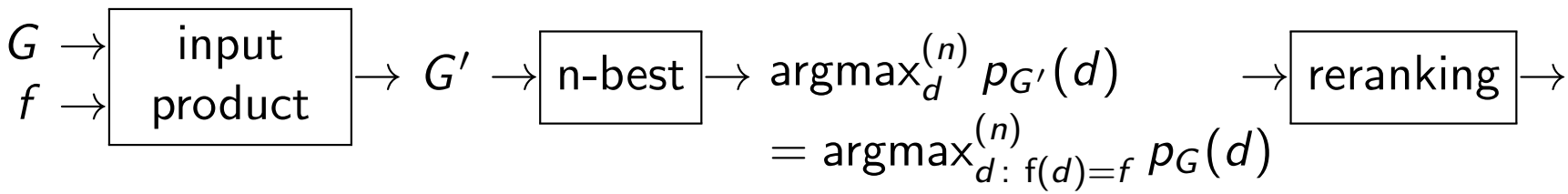

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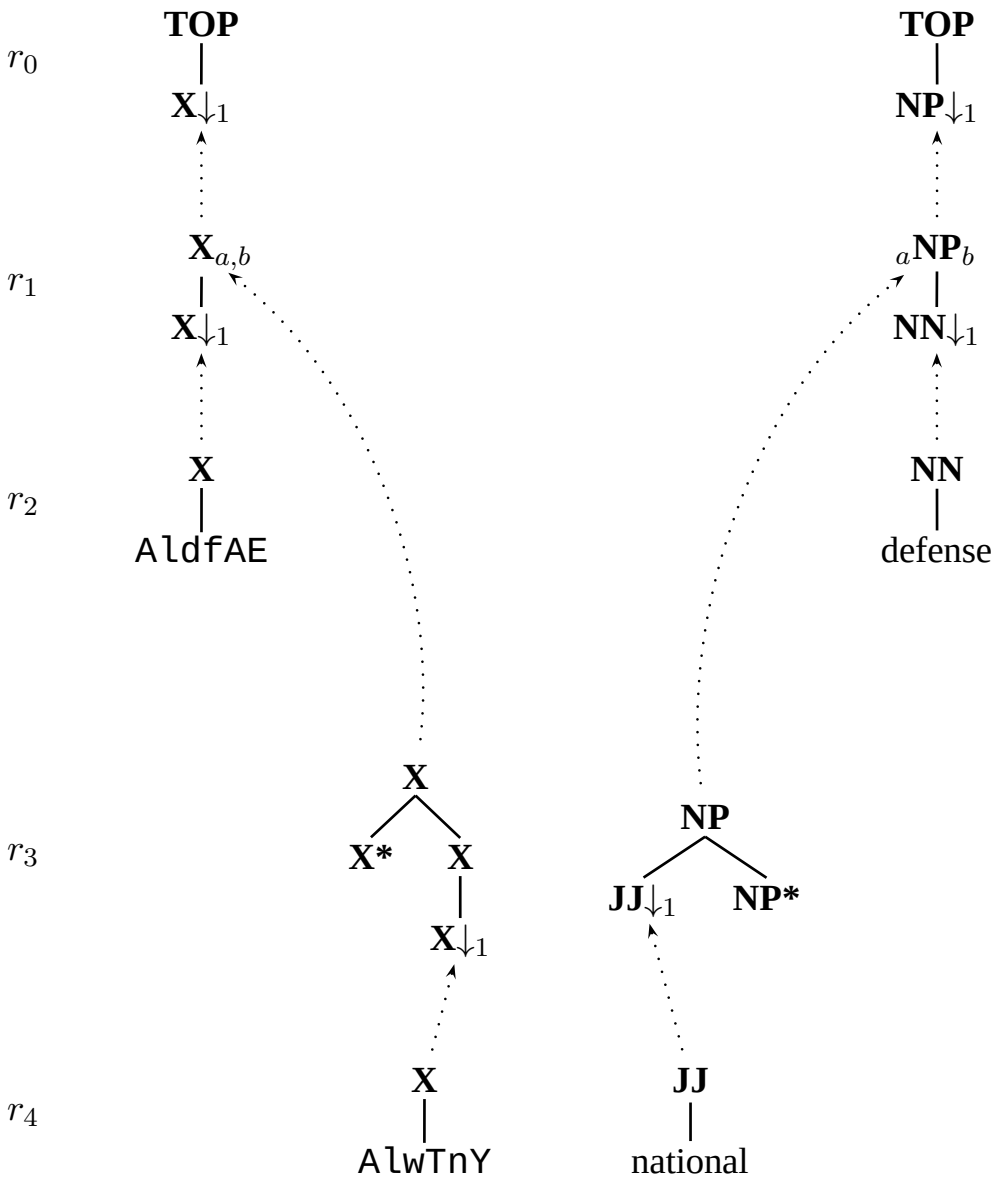
Outline

Example: Synchronous Tree-Insertion Grammar



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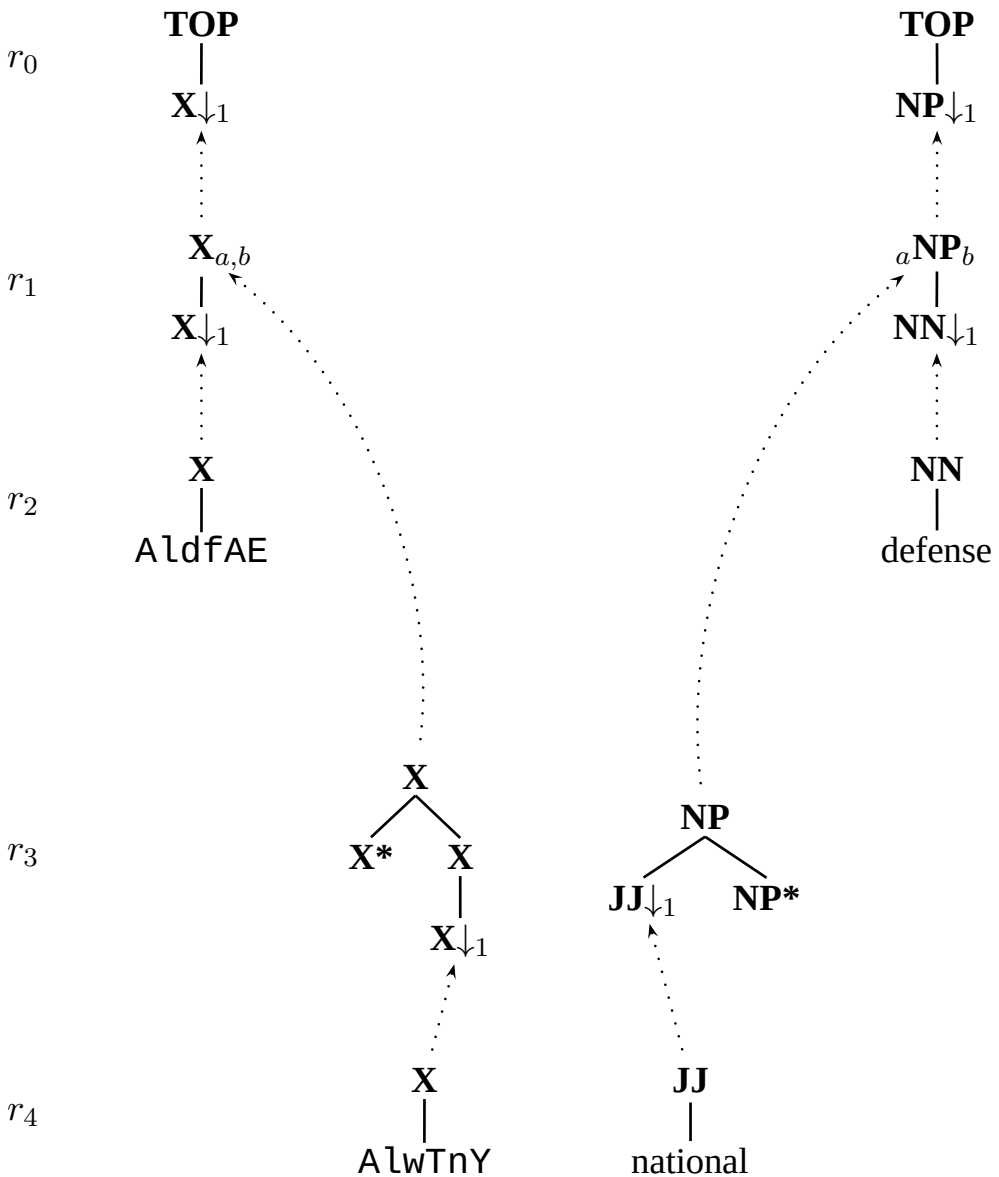

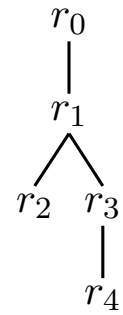

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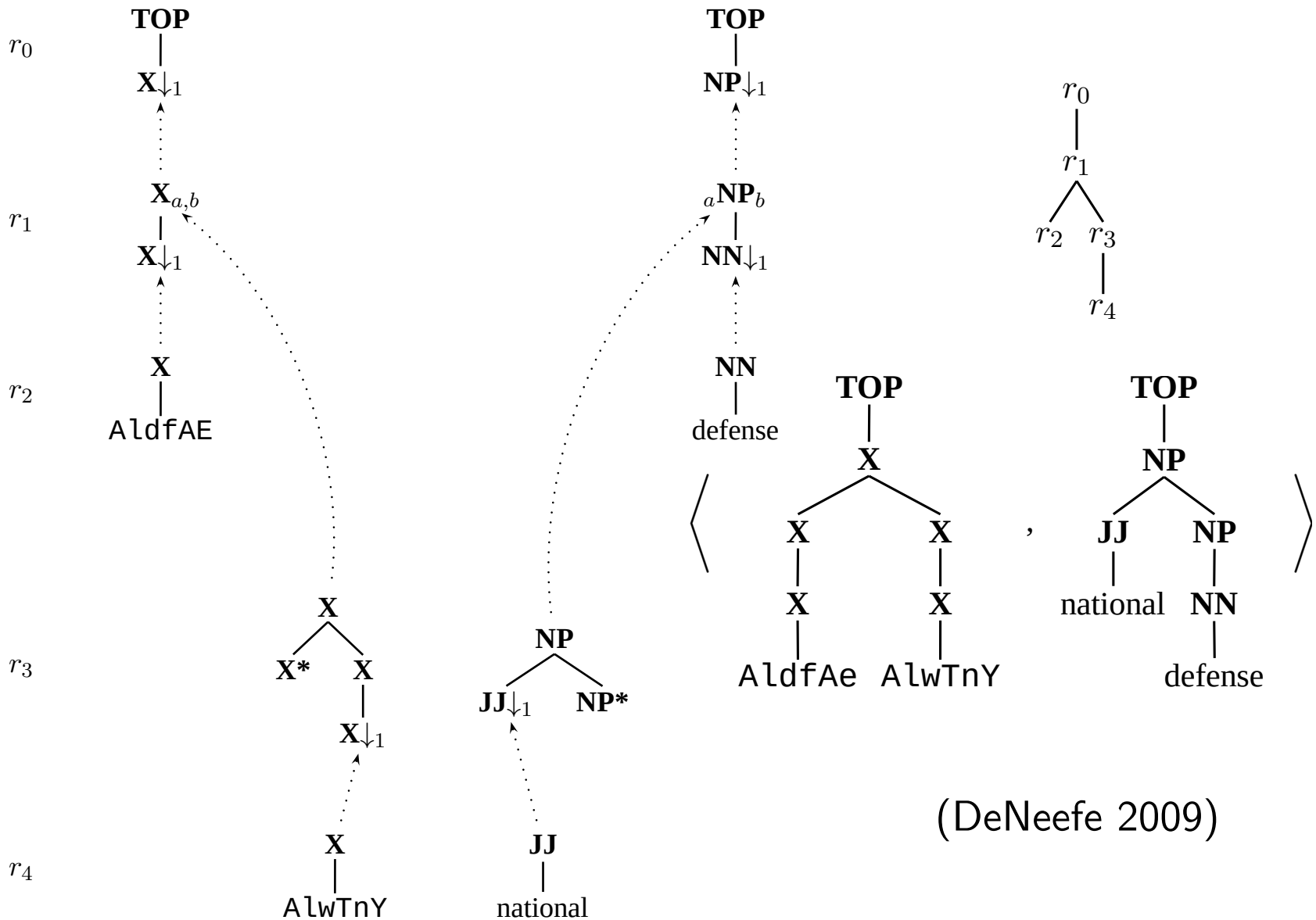

(DeNeefe 2009)

Overview

- Extended Top-down Tree-to-String Transducers (Galley et al. 2004; Huang, Knight, and Joshi 2006; Graehl, Knight, and May 2008)
- Extended Multi-Bottom-up Tree Transducers (Engelfriet, Lilin, and Maletti 2008)
- Synchronous Context-Free Grammars (Chiang 2007)
- Synchronous Tree-Substitution Grammars (Eisner 2003)
- Synchronous Tree-Sequence-Substitution Grammars (Zhang et al. 2008)
- Synchronous Tree-Insertion Grammars (Nesson, Shieber, and Rush 2006; DeNeefe and Knight 2009)

Outline

Key Aspects

- Translation Model: Synchronous Tree-Adjoining Grammars
  - formulation with states and explicit sites
  - formal properties: closure under input/output product
  - algorithms for training, decoding

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- Auxiliary Algorithms for Decoding
  - n-best algorithm
  - determinization of weighted tree automata

Outline

Publications so far

- MB, Jonathan May, and Heiko Vogler (2010).
  "Determinization of Weighted Tree Automata using Factorizations".
  In: Journal of Automata, Languages and Combinatorics. (accepted for publication)
- MB, Daniel Geisler, Torsten Stüber, and Heiko Vogler (2010).
  "n-Best Parsing Revisited".
  In: Proceedings of the 2010 Workshop on Applications of Tree Automata in Natural Language Processing, ACL 2010. Uppsala, Sweden, 16 July 2010, pp. 46–54.
- MB, Mark-Jan Nederhof, and Heiko Vogler (2011).
  "Tree Parsing with Synchronous Tree-Adjoining Grammars".
  In: Proceedings of IWPT. (accepted for publication)