Statistical Machine Translation with Weighted Grammars

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Outline

Ambiguity in Natural Language

Example (Chiang 2007)

- e = "Australia is one of the few countries that have diplomatic relations with North Korea"
- f = "Aozhou shi yu Beihan you bangjiao de shaoshu guojia zhiyi"

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Is e a correct translation of f?

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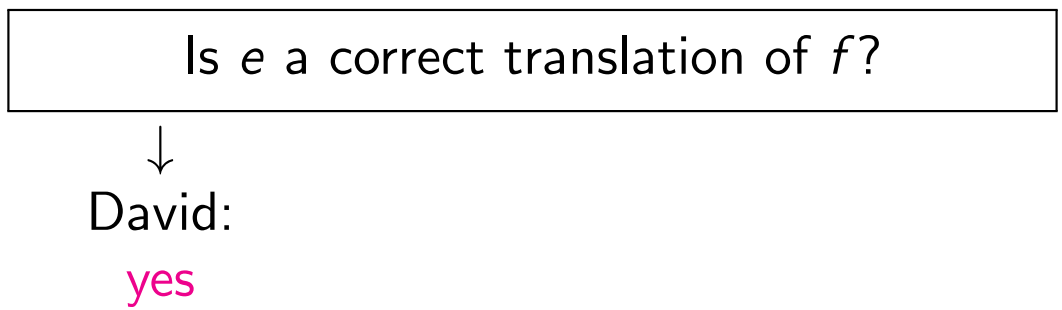

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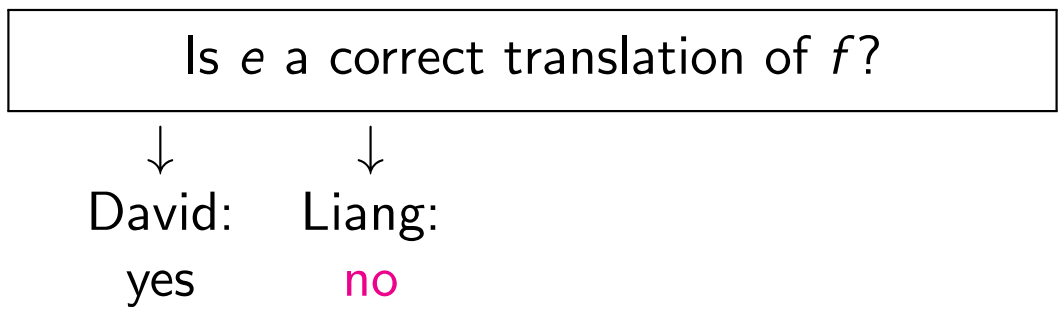

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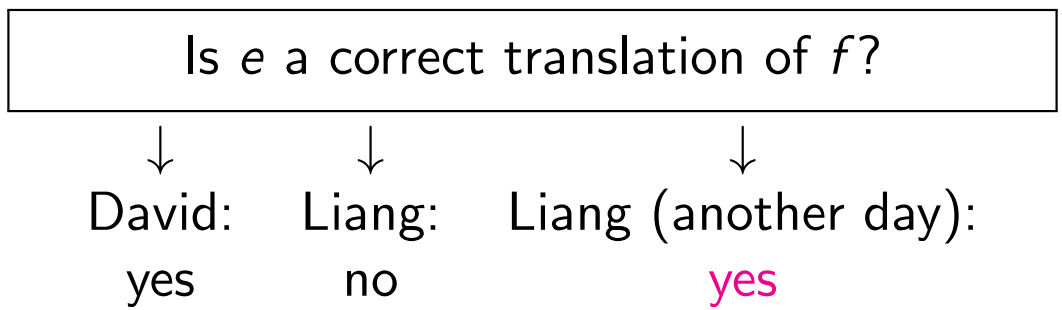

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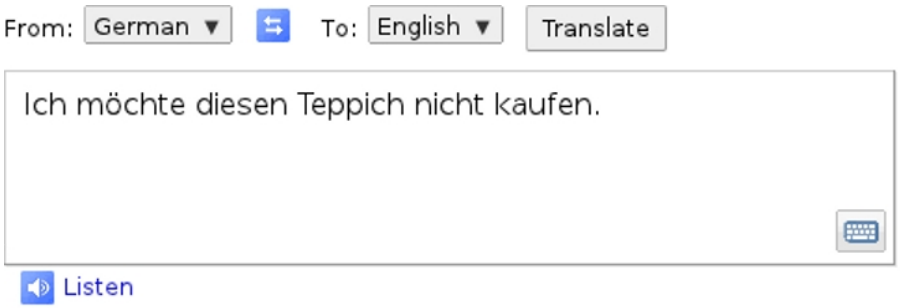
frequentist limit of relative frequencies

Bayesian degree of belief (subjective)

Search (aka Decoding) in SMT



given $p \in \mathcal{M}(E \mid F)$

Search (aka Decoding) in SMT

given $p \in \mathcal{M}(E \mid F)$ $(\sum_e p(e \mid f) = 1$ for every $f)$

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determine $\hat{e}$ such that for every $e \in E$:

$$p(\hat{e} \mid f) \geq p(e \mid f)\ . \qquad \text{(not unique)}$$

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for short

$$\hat{e} = \operatorname{argmax}_{e \in E} p(e \mid f) \; .$$

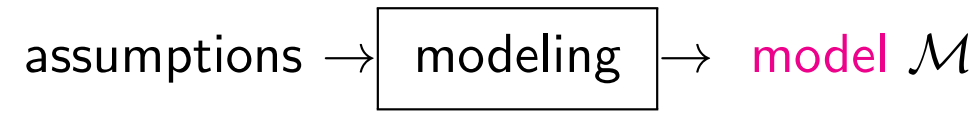
Building an SMT System

assumptions

training data

test data

Building an SMT System

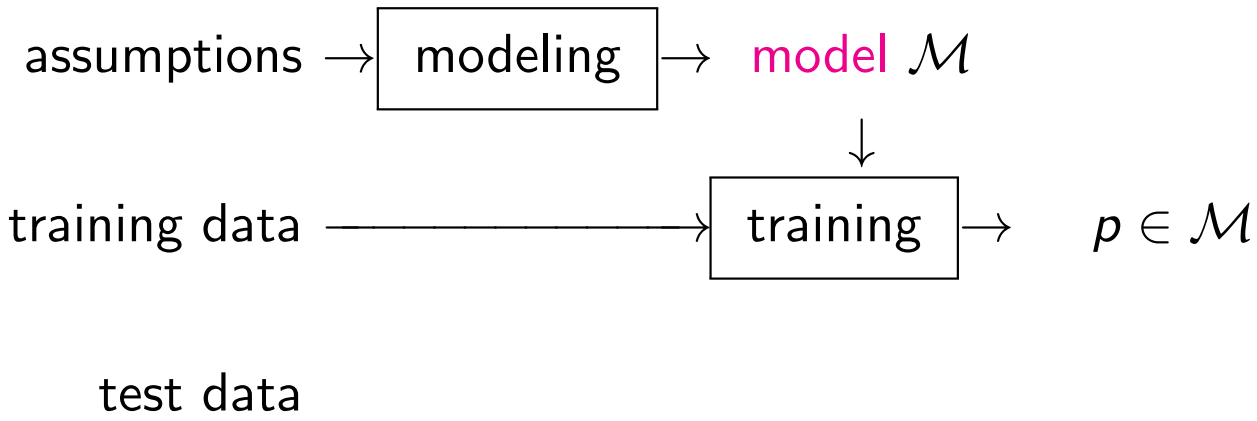


training data

test data

"Every time I fire a linguist, recognition rate goes up"
(attributed to Frederick Jelinek)

Building an SMT System



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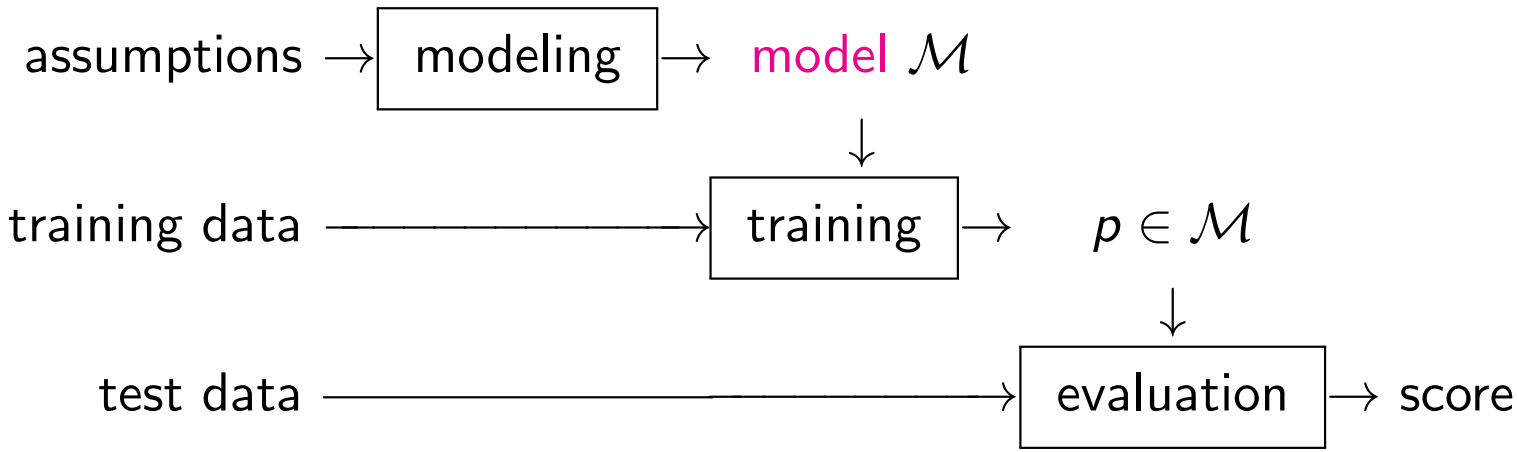
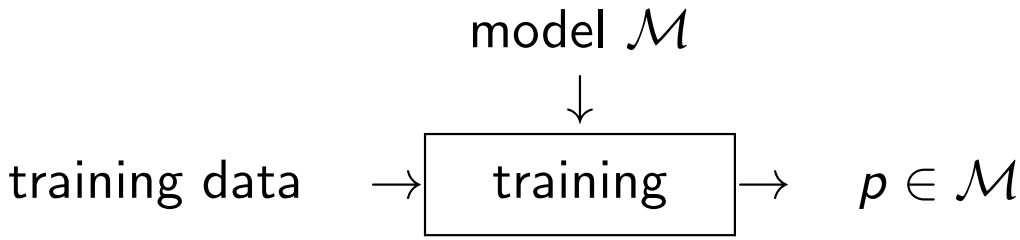


Training = Point Estimation



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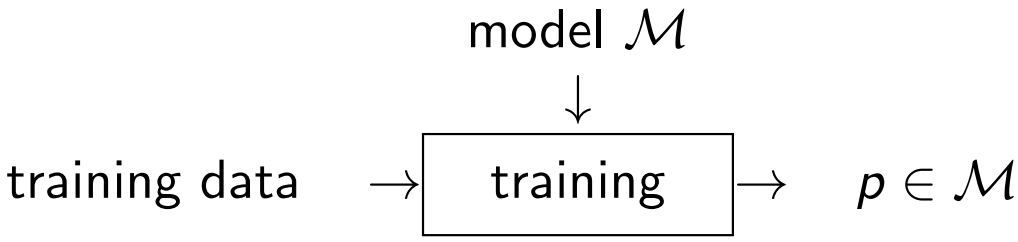

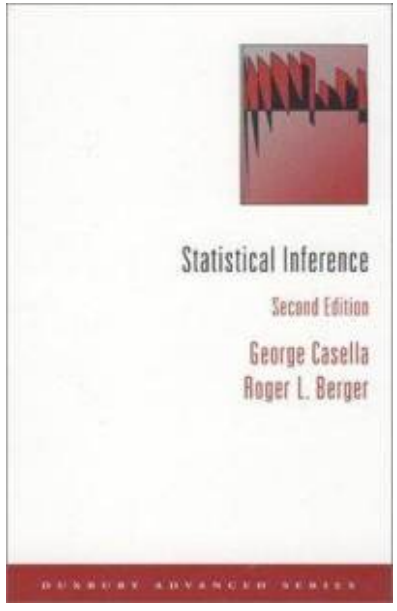

Statistical Inference, Ch. 7: Point Estimation (Casella and Berger 2002)

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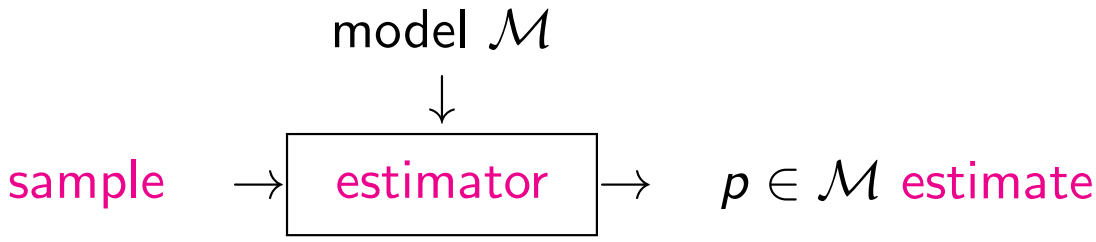

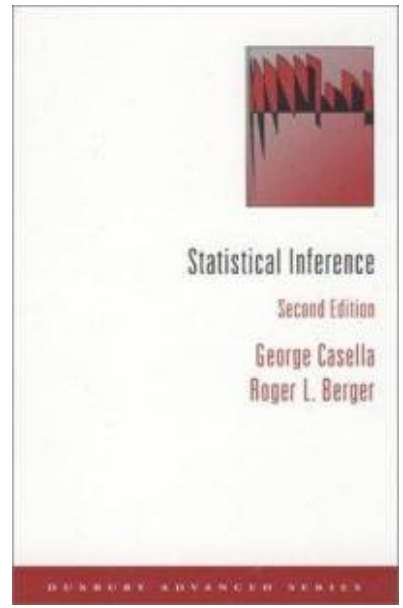

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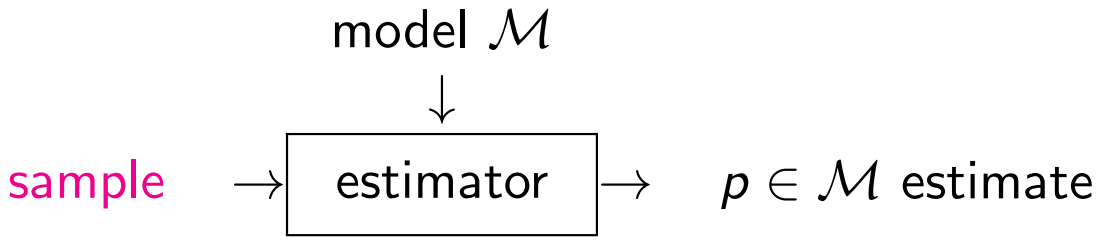
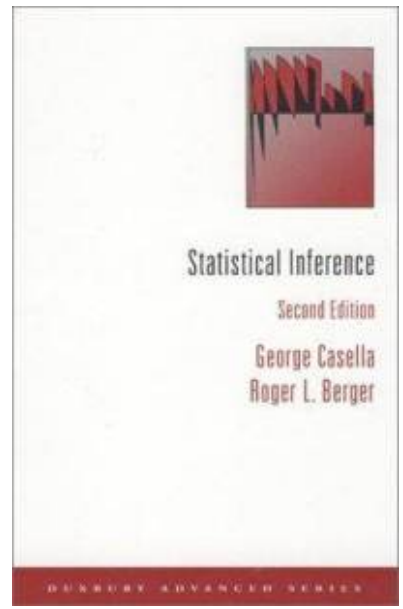


sample

- representative of all translation situations
- e. g., Hong Kong Hansards (parliament proceedings)



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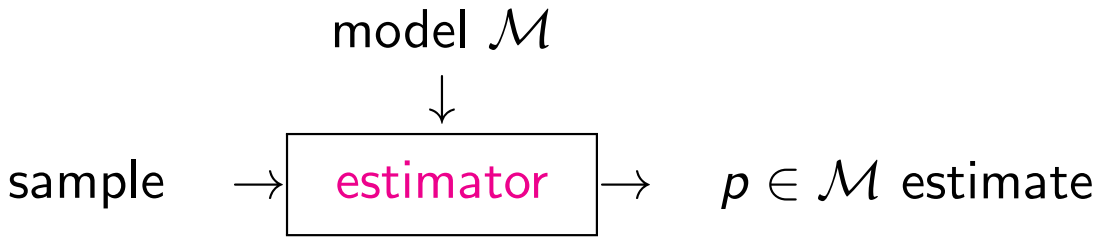

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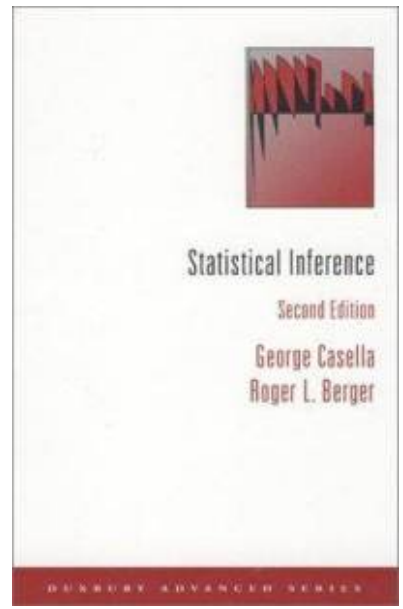
- maximum a-posteriori
- minimum risk
- maximum entropy



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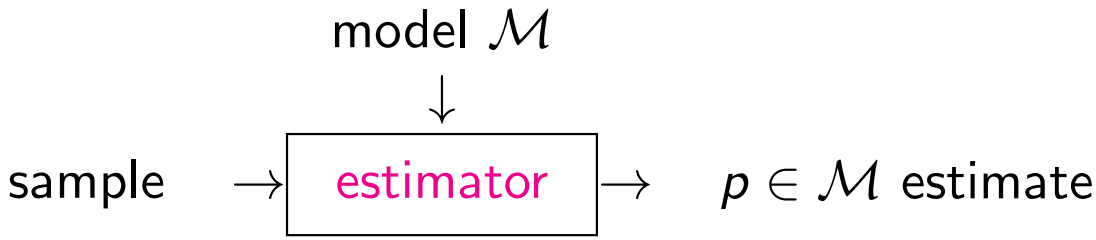

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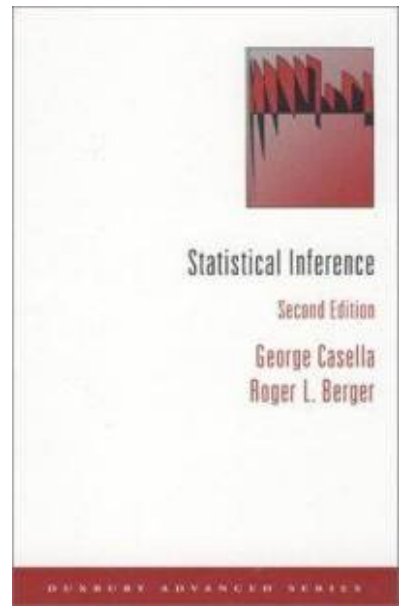

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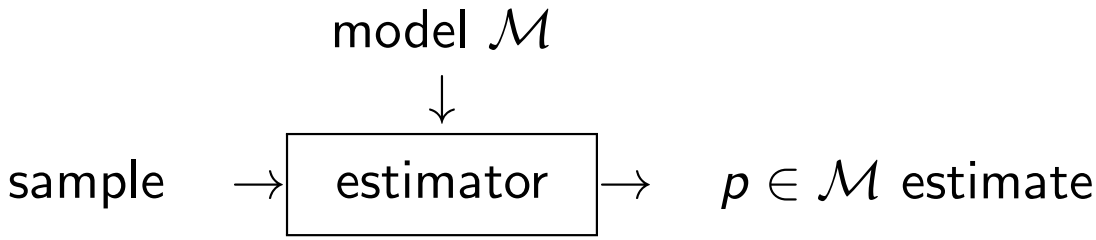

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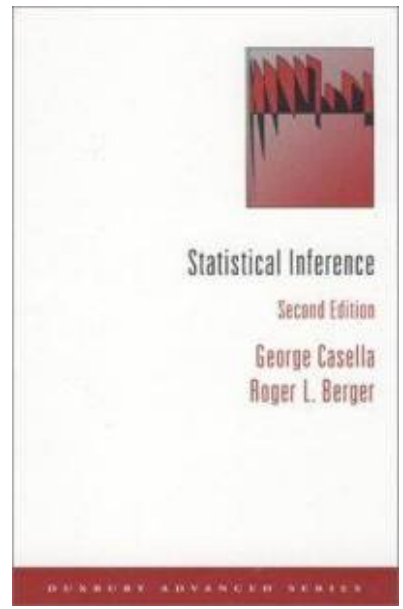

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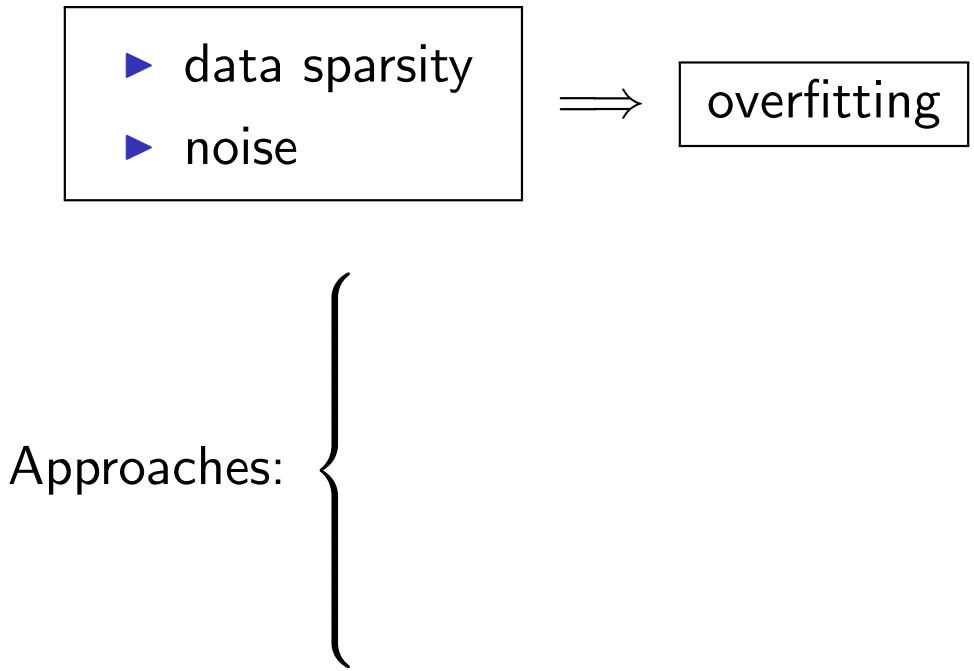

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Approaches:

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Approaches:

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- adjust the model

Adjusting the Model

Source-Channel Model (Brown et al. 1993)

- $\mathcal{M}_{\mathsf{LM}} \subseteq \mathcal{M}(E)$ language model
- $\mathcal{M}_{\mathsf{TM}} \subseteq \mathcal{M}(F \mid E)$ translation model

Adjusting the Model

Source-Channel Model (Brown et al. 1993)

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- then

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- estimate p_1 and p_2 independently on appropriate data

Adjusting the Model

Log-linear Model (Berger, Della Pietra, and Della Pietra 1996; Och and Ney 2002)

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- (and maximize entropy)

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- or consider

$$\mathcal{M} = \{p \in \mathcal{M}(E \mid F) \;\mid\; \exists \lambda_1, \ldots, \lambda_n \in \mathbb{R} \colon p(e \mid f) = \frac{\exp \sum_i \lambda_i \cdot h_i(e, f)}{\sum_{e' \in E} \exp \sum_i \lambda_i \cdot h_i(e', f)}\}$$

- (and maximize likelihood)

Adjusting the Model

Log-linear Model Example

- $h_1(e, f) = \log p_1(e)$,
- $h_2(e, f) = \log p_2(f \mid e)$,
- $\lambda_1 = 1$,
- $\lambda_2 = 1$,
- then

$$
\begin{aligned}
p(e \mid f) &= \frac{\exp\big(\lambda_1 \cdot h_1(e, f) + \lambda_2 \cdot h_2(e, f)\big)}{\sum_{e' \in E} \exp\big(\lambda_1 \cdot h_1(e', f) + \lambda_2 \cdot h_2(e, f)\big)} \\
&= \frac{\exp\big(\log p_1(e) + \log p_2(f \mid e)\big)}{\sum_{e' \in E} \exp\big(\log p_1(e') + \log p_2(f \mid e)\big)} \\
&= \frac{p_1(e) \cdot p_2(f \mid e)}{\sum_{e' \in E} p_1(e') \cdot p_2(f \mid e)}
\end{aligned}
$$

Estimating Conditional Models

- $\mathcal{M} \subseteq \mathcal{M}(E \mid F)$
- define

$$\mathcal{M}' = \{p' \in \mathcal{M}(E, F) \mid \exists p \in \mathcal{M}, p_F \in \mathcal{M}(F): \\ p'(e, f) = p(e \mid f) \cdot p_F(f)\}$$

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- bijection: $p' \leftrightarrow (p, p_F)$ iff $p'(e, f) = p(e \mid f) \cdot p_F(f)$
- if $P(e, f) = p'(e, f)$, then $P(e \mid f) = p(e \mid f)$,
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$\rightsquigarrow$ do estimation on $\mathcal{M}'$, throw p_F away

More Recent Estimators

- minimum-error-rate training (Och 2003)
- minimum-risk annealing (Smith and Eisner 2006)
- large-margin methods

Evaluation

(due to Saša Hasan)

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$$p \in \mathcal{M}$$

$$\text{test data} \rightarrow \boxed{\text{evaluation}} \rightarrow \text{score}$$

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method	how it works	goal
BLEU (Papineni et al. 2002)	n-gram precision, brevity penalty	higher values

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method	how it works	goal
BLEU (Papineni et al. 2002)	n-gram precision, brevity penalty	higher values
TER (Snover et al. 2006)	edit distance	lower values
Meteor (Banerjee and Lavie 2005)	alignments, unigram precision, unigram recall, penalty	higher values

Outline

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Synchronous Context-Free Grammar G:

π_1: $\mathrm{S} \rightarrow \langle \mathrm{S}\ \mathrm{X},\ \mathrm{S}\ \mathrm{X} \rangle$

π_2: $\mathrm{S} \rightarrow \langle \mathrm{X},\ \mathrm{X} \rangle$

π_3: $\mathrm{X} \rightarrow \langle \text{yu}\ \mathrm{X}_{\boxed{1}}\ \text{you}\ \mathrm{X}_{\boxed{2}},\ \text{have}\ \mathrm{X}_{\boxed{2}}\ \text{with}\ \mathrm{X}_{\boxed{1}} \rangle$

π_4: $\mathrm{X} \rightarrow \langle \mathrm{X}_{\boxed{1}}\ \text{de}\ \mathrm{X}_{\boxed{2}},\ \text{the}\ \mathrm{X}_{\boxed{2}}\ \text{that}\ \mathrm{X}_{\boxed{1}} \rangle$

π_5: $\mathrm{X} \rightarrow \langle \mathrm{X}\ \text{zhiyi},\ \text{one of}\ \mathrm{X} \rangle$

π_6: $\mathrm{X} \rightarrow \langle \text{Aozhou},\ \text{Australia} \rangle$

π_7: $\mathrm{X} \rightarrow \langle \text{Beihan},\ \text{North Korea} \rangle$

π_8: $\mathrm{X} \rightarrow \langle \text{shi},\ \text{is} \rangle$

π_9: $\mathrm{X} \rightarrow \langle \text{bangjiao},\ \text{diplomatic relations} \rangle$

π_{10}: $\mathrm{X} \rightarrow \langle \text{shaoshu guojia},\ \text{few countries} \rangle$

Derivation

$\langle S, S \rangle$

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$$\langle \mathsf{S},\ \mathsf{S} \rangle$$
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$$
\begin{aligned}
& \langle \mathsf{S},\ \mathsf{S} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathsf{S}\ \mathsf{X},\ \mathsf{S}\ \mathsf{X} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathsf{S}\ \mathsf{X}_{\boxed{1}}\ \mathsf{X}_{\boxed{2}},\ \mathsf{S}\ \mathsf{X}_{\boxed{1}}\ \mathsf{X}_{\boxed{2}} \rangle \\
\overset{\pi_2}{\Rightarrow}\ & \langle \mathsf{X}_{\boxed{0}}\ \mathsf{X}_{\boxed{1}}\ \mathsf{X}_{\boxed{2}},\ \mathsf{X}_{\boxed{0}}\ \mathsf{X}_{\boxed{1}}\ \mathsf{X}_{\boxed{2}} \rangle \\
\overset{\pi_6}{\Rightarrow}\ & \langle \text{Aozhou}\ \mathsf{X}_{\boxed{1}}\ \mathsf{X}_{\boxed{2}},\ \text{Australia}\ \mathsf{X}_{\boxed{1}}\ \mathsf{X}_{\boxed{2}} \rangle
\end{aligned}
$$

Derivation

$$
\begin{aligned}
& \langle \mathrm{S},\ \mathrm{S} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S}\ \mathrm{X},\ \mathrm{S}\ \mathrm{X} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{S}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_2}{\Rightarrow}\ & \langle \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_6}{\Rightarrow}\ & \langle \mathrm{Aozhou}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{Australia}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle
\end{aligned}
$$

Derivation

$$
\begin{aligned}
& \langle \mathrm{S},\ \mathrm{S} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S}\ \mathrm{X},\ \mathrm{S}\ \mathrm{X} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{S}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_2}{\Rightarrow}\ & \langle \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_6}{\Rightarrow}\ & \langle \text{Aozhou}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \text{Australia}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_8}{\Rightarrow}\ & \langle \text{Aozhou shi X},\ \text{Australia is X} \rangle
\end{aligned}
$$

Derivation

$$
\begin{aligned}
& \langle \mathrm{S}, \mathrm{S} \rangle \\
\overset{\pi_1}{\Rightarrow} \; & \langle \mathrm{S}\ \mathrm{X}, \mathrm{S}\ \mathrm{X} \rangle \\
\overset{\pi_1}{\Rightarrow} \; & \langle \mathrm{S}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}}, \mathrm{S}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_2}{\Rightarrow} \; & \langle \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}}, \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_6}{\Rightarrow} \; & \langle \text{Aozhou}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}}, \text{Australia}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_8}{\Rightarrow} \; & \langle \text{Aozhou shi X}, \text{Australia is X} \rangle
\end{aligned}
$$

Derivation

$$
\begin{aligned}
& \langle \mathrm{S},\ \mathrm{S} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X},\ \mathrm{S\ X} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{S\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_2}{\Rightarrow}\ & \langle \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_6}{\Rightarrow}\ & \langle \mathrm{Aozhou\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{Australia\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_8}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X},\ \mathrm{Australia\ is\ X} \rangle \\
\overset{\pi_5}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X\ zhiyi},\ \mathrm{Australia\ is\ one\ of\ X} \rangle
\end{aligned}
$$

Derivation

$$
\begin{aligned}
& \langle \mathrm{S}, \mathrm{S} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S}\ \mathrm{X}, \mathrm{S}\ \mathrm{X} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}}, \mathrm{S}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_2}{\Rightarrow}\ & \langle \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}}, \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_6}{\Rightarrow}\ & \langle \text{Aozhou}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}}, \text{Australia}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_8}{\Rightarrow}\ & \langle \text{Aozhou shi X}, \text{Australia is X} \rangle \\
\overset{\pi_5}{\Rightarrow}\ & \langle \text{Aozhou shi X zhiyi}, \text{Australia is one of X} \rangle
\end{aligned}
$$

Derivation

$$
\begin{aligned}
& \langle \mathrm{S},\ \mathrm{S} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X},\ \mathrm{S\ X} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{S\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_2}{\Rightarrow}\ & \langle \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_6}{\Rightarrow}\ & \langle \mathrm{Aozhou\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{Australia\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_8}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X},\ \mathrm{Australia\ is\ X} \rangle \\
\overset{\pi_5}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X\ zhiyi},\ \mathrm{Australia\ is\ one\ of\ X} \rangle \\
\overset{\pi_4}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X}_{\boxed{1}}\ \mathrm{de\ X}_{\boxed{2}}\ \mathrm{zhiyi}, \\
& \ \mathrm{Australia\ is\ one\ of\ the\ X}_{\boxed{2}}\ \mathrm{that\ X}_{\boxed{1}} \rangle
\end{aligned}
$$

Derivation

$$
\begin{aligned}
& \langle \mathrm{S},\ \mathrm{S} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X},\ \mathrm{S\ X} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{S\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_2}{\Rightarrow}\ & \langle \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_6}{\Rightarrow}\ & \langle \mathrm{Aozhou\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{Australia\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_8}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X},\ \mathrm{Australia\ is\ X} \rangle \\
\overset{\pi_5}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X\ zhiyi},\ \mathrm{Australia\ is\ one\ of\ X} \rangle \\
\overset{\pi_4}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X}_{\boxed{1}}\ \mathrm{de\ X}_{\boxed{2}}\ \mathrm{zhiyi}, \\
& \ \mathrm{Australia\ is\ one\ of\ the\ X}_{\boxed{2}}\ \mathrm{that\ X}_{\boxed{1}} \rangle
\end{aligned}
$$

Derivation

$$
\begin{aligned}
& \langle \mathrm{S},\ \mathrm{S} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X},\ \mathrm{S\ X} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{S\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_2}{\Rightarrow}\ & \langle \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_6}{\Rightarrow}\ & \langle \mathrm{Aozhou\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{Australia\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_8}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X},\ \mathrm{Australia\ is\ X} \rangle \\
\overset{\pi_5}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X\ zhiyi},\ \mathrm{Australia\ is\ one\ of\ X} \rangle \\
\overset{\pi_4}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X}_{\boxed{1}}\ \mathrm{de\ X}_{\boxed{2}}\ \mathrm{zhiyi}, \\
& \quad \mathrm{Australia\ is\ one\ of\ the\ X}_{\boxed{2}}\ \mathrm{that\ X}_{\boxed{1}} \rangle \\
\overset{\pi_3}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ yu\ X}_{\boxed{1}}\ \mathrm{you\ X}_{\boxed{0}}\ \mathrm{de\ X}_{\boxed{2}}\ \mathrm{zhiyi}, \\
& \quad \mathrm{Australia\ is\ one\ of\ the\ X}_{\boxed{2}}\ \mathrm{that\ have\ X}_{\boxed{0}}\ \mathrm{with\ X}_{\boxed{1}} \rangle
\end{aligned}
$$

Derivation

$$
\begin{aligned}
& \langle \mathrm{S},\ \mathrm{S} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X},\ \mathrm{S\ X} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{S\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_2}{\Rightarrow}\ & \langle \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_6}{\Rightarrow}\ & \langle \mathrm{Aozhou\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{Australia\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_8}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X},\ \mathrm{Australia\ is\ X} \rangle \\
\overset{\pi_5}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X\ zhiyi},\ \mathrm{Australia\ is\ one\ of\ X} \rangle \\
\overset{\pi_4}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X}_{\boxed{1}}\ \mathrm{de\ X}_{\boxed{2}}\ \mathrm{zhiyi}, \\
& \quad \mathrm{Australia\ is\ one\ of\ the\ X}_{\boxed{2}}\ \mathrm{that\ X}_{\boxed{1}} \rangle \\
\overset{\pi_3}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ yu\ X}_{\boxed{1}}\ \mathrm{you\ X}_{\boxed{0}}\ \mathrm{de\ X}_{\boxed{2}}\ \mathrm{zhiyi}, \\
& \quad \mathrm{Australia\ is\ one\ of\ the\ X}_{\boxed{2}}\ \mathrm{that\ have\ X}_{\boxed{0}}\ \mathrm{with\ X}_{\boxed{1}} \rangle
\end{aligned}
$$

Derivation

$$
\begin{aligned}
& \langle \mathrm{S},\ \mathrm{S} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X},\ \mathrm{S\ X} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{S\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_2}{\Rightarrow}\ & \langle \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_6}{\Rightarrow}\ & \langle \mathrm{Aozhou\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}},\ \mathrm{Australia\ X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle \\
\overset{\pi_8}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X},\ \mathrm{Australia\ is\ X} \rangle \\
\overset{\pi_5}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X\ zhiyi},\ \mathrm{Australia\ is\ one\ of\ X} \rangle \\
\overset{\pi_4}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X}_{\boxed{1}}\ \mathrm{de\ X}_{\boxed{2}}\ \mathrm{zhiyi}, \\
& \quad \mathrm{Australia\ is\ one\ of\ the\ X}_{\boxed{2}}\ \mathrm{that\ X}_{\boxed{1}} \rangle \\
\overset{\pi_3}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ yu\ X}_{\boxed{1}}\ \mathrm{you\ X}_{\boxed{0}}\ \mathrm{de\ X}_{\boxed{2}}\ \mathrm{zhiyi}, \\
& \quad \mathrm{Australia\ is\ one\ of\ the\ X}_{\boxed{2}}\ \mathrm{that\ have\ X}_{\boxed{0}}\ \mathrm{with\ X}_{\boxed{1}} \rangle \\
\overset{\pi_7}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ yu\ Beihan\ you\ X}_{\boxed{0}}\ \mathrm{de\ X}_{\boxed{2}}\ \mathrm{zhiyi}, \\
& \quad \mathrm{Australia\ is\ one\ of\ the\ X}_{\boxed{2}}\ \mathrm{that\ have\ X}_{\boxed{0}}\ \mathrm{with\ North\ Korea} \rangle
\end{aligned}
$$

Derivation

$$
\begin{aligned}
& \langle \mathrm{S},\ \mathrm{S} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X},\ \mathrm{S\ X} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X_{\boxed{1}}\ X_{\boxed{2}}},\ \mathrm{S\ X_{\boxed{1}}\ X_{\boxed{2}}} \rangle \\
\overset{\pi_2}{\Rightarrow}\ & \langle \mathrm{X_{\boxed{0}}\ X_{\boxed{1}}\ X_{\boxed{2}}},\ \mathrm{X_{\boxed{0}}\ X_{\boxed{1}}\ X_{\boxed{2}}} \rangle \\
\overset{\pi_6}{\Rightarrow}\ & \langle \mathrm{Aozhou\ X_{\boxed{1}}\ X_{\boxed{2}}},\ \mathrm{Australia\ X_{\boxed{1}}\ X_{\boxed{2}}} \rangle \\
\overset{\pi_8}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X},\ \mathrm{Australia\ is\ X} \rangle \\
\overset{\pi_5}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X\ zhiyi},\ \mathrm{Australia\ is\ one\ of\ X} \rangle \\
\overset{\pi_4}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X_{\boxed{1}}\ de\ X_{\boxed{2}}\ zhiyi}, \\
& \quad \mathrm{Australia\ is\ one\ of\ the\ X_{\boxed{2}}\ that\ X_{\boxed{1}}} \rangle \\
\overset{\pi_3}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ yu\ X_{\boxed{1}}\ you\ X_{\boxed{0}}\ de\ X_{\boxed{2}}\ zhiyi}, \\
& \quad \mathrm{Australia\ is\ one\ of\ the\ X_{\boxed{2}}\ that\ have\ X_{\boxed{0}}\ with\ X_{\boxed{1}}} \rangle \\
\overset{\pi_7}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ yu\ Beihan\ you\ X_{\boxed{0}}\ de\ X_{\boxed{2}}\ zhiyi}, \\
& \quad \mathrm{Australia\ is\ one\ of\ the\ X_{\boxed{2}}\ that\ have\ X_{\boxed{0}}\ with\ North\ Korea} \rangle
\end{aligned}
$$

Derivation

$$
\begin{aligned}
& \langle \mathrm{S},\ \mathrm{S} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X},\ \mathrm{S\ X} \rangle \\
\overset{\pi_1}{\Rightarrow}\ & \langle \mathrm{S\ X_{\boxed{1}}\ X_{\boxed{2}}},\ \mathrm{S\ X_{\boxed{1}}\ X_{\boxed{2}}} \rangle \\
\overset{\pi_2}{\Rightarrow}\ & \langle \mathrm{X_{\boxed{0}}\ X_{\boxed{1}}\ X_{\boxed{2}}},\ \mathrm{X_{\boxed{0}}\ X_{\boxed{1}}\ X_{\boxed{2}}} \rangle \\
\overset{\pi_6}{\Rightarrow}\ & \langle \mathrm{Aozhou\ X_{\boxed{1}}\ X_{\boxed{2}}},\ \mathrm{Australia\ X_{\boxed{1}}\ X_{\boxed{2}}} \rangle \\
\overset{\pi_8}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X},\ \mathrm{Australia\ is\ X} \rangle \\
\overset{\pi_5}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X\ zhiyi},\ \mathrm{Australia\ is\ one\ of\ X} \rangle \\
\overset{\pi_4}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ X_{\boxed{1}}\ de\ X_{\boxed{2}}\ zhiyi}, \\
& \quad \mathrm{Australia\ is\ one\ of\ the\ X_{\boxed{2}}\ that\ X_{\boxed{1}}} \rangle \\
\overset{\pi_3}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ yu\ X_{\boxed{1}}\ you\ X_{\boxed{0}}\ de\ X_{\boxed{2}}\ zhiyi}, \\
& \quad \mathrm{Australia\ is\ one\ of\ the\ X_{\boxed{2}}\ that\ have\ X_{\boxed{0}}\ with\ X_{\boxed{1}}} \rangle \\
\overset{\pi_7}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ yu\ Beihan\ you\ X_{\boxed{0}}\ de\ X_{\boxed{2}}\ zhiyi}, \\
& \quad \mathrm{Australia\ is\ one\ of\ the\ X_{\boxed{2}}\ that\ have\ X_{\boxed{0}}\ with\ North\ Korea} \rangle \\
\overset{\pi_9}{\Rightarrow}\ & \langle \mathrm{Aozhou\ shi\ yu\ Beihan\ you\ bangjiao\ de\ X_{\boxed{2}}\ zhiyi}, \\
& \quad \mathrm{Australia\ is\ one\ of\ the\ X_{\boxed{2}}\ that\ have\ diplomatic\ relations\ with\ North\ Korea} \rangle
\end{aligned}
$$

Derivation

$\langle \mathrm{S}, \mathrm{S} \rangle$

$\overset{\pi_1}{\Rightarrow} \langle \mathrm{S}\ \mathrm{X}, \mathrm{S}\ \mathrm{X} \rangle$

$\overset{\pi_1}{\Rightarrow} \langle \mathrm{S}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}}, \mathrm{S}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle$

$\overset{\pi_2}{\Rightarrow} \langle \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}}, \mathrm{X}_{\boxed{0}}\ \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle$

$\overset{\pi_6}{\Rightarrow} \langle \text{Aozhou } \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}}, \text{Australia } \mathrm{X}_{\boxed{1}}\ \mathrm{X}_{\boxed{2}} \rangle$

$\overset{\pi_8}{\Rightarrow} \langle \text{Aozhou shi X, Australia is X} \rangle$

$\overset{\pi_5}{\Rightarrow} \langle \text{Aozhou shi X zhiyi, Australia is one of X} \rangle$

$\overset{\pi_4}{\Rightarrow} \langle \text{Aozhou shi } \mathrm{X}_{\boxed{1}} \text{ de } \mathrm{X}_{\boxed{2}} \text{ zhiyi,}$
$\text{Australia is one of the } \mathrm{X}_{\boxed{2}} \text{ that } \mathrm{X}_{\boxed{1}} \rangle$

$\overset{\pi_3}{\Rightarrow} \langle \text{Aozhou shi yu } \mathrm{X}_{\boxed{1}} \text{ you } \mathrm{X}_{\boxed{0}} \text{ de } \mathrm{X}_{\boxed{2}} \text{ zhiyi,}$
$\text{Australia is one of the } \mathrm{X}_{\boxed{2}} \text{ that have } \mathrm{X}_{\boxed{0}} \text{ with } \mathrm{X}_{\boxed{1}} \rangle$

$\overset{\pi_7}{\Rightarrow} \langle \text{Aozhou shi yu Beihan you } \mathrm{X}_{\boxed{0}} \text{ de } \mathrm{X}_{\boxed{2}} \text{ zhiyi,}$
$\text{Australia is one of the } \mathrm{X}_{\boxed{2}} \text{ that have } \mathrm{X}_{\boxed{0}} \text{ with North Korea} \rangle$

$\overset{\pi_9}{\Rightarrow} \langle \text{Aozhou shi yu Beihan you bangjiao de } \mathrm{X}_{\boxed{2}} \text{ zhiyi,}$
$\text{Australia is one of the } \mathrm{X}_{\boxed{2}} \text{ that have diplomatic relations with North Korea} \rangle$

Derivation

$\langle S, S \rangle$

$\overset{\pi_1}{\Rightarrow} \langle S\ X, S\ X \rangle$

$\overset{\pi_1}{\Rightarrow} \langle S\ X_{\boxed{1}}\ X_{\boxed{2}}, S\ X_{\boxed{1}}\ X_{\boxed{2}} \rangle$

$\overset{\pi_2}{\Rightarrow} \langle X_{\boxed{0}}\ X_{\boxed{1}}\ X_{\boxed{2}}, X_{\boxed{0}}\ X_{\boxed{1}}\ X_{\boxed{2}} \rangle$

$\overset{\pi_6}{\Rightarrow} \langle \text{Aozhou } X_{\boxed{1}}\ X_{\boxed{2}}, \text{Australia } X_{\boxed{1}}\ X_{\boxed{2}} \rangle$

$\overset{\pi_8}{\Rightarrow} \langle \text{Aozhou shi X, Australia is X} \rangle$

$\overset{\pi_5}{\Rightarrow} \langle \text{Aozhou shi X zhiyi, Australia is one of X} \rangle$

$\overset{\pi_4}{\Rightarrow} \langle \text{Aozhou shi } X_{\boxed{1}} \text{ de } X_{\boxed{2}} \text{ zhiyi,}$
$\text{Australia is one of the } X_{\boxed{2}} \text{ that } X_{\boxed{1}} \rangle$

$\overset{\pi_3}{\Rightarrow} \langle \text{Aozhou shi yu } X_{\boxed{1}} \text{ you } X_{\boxed{0}} \text{ de } X_{\boxed{2}} \text{ zhiyi,}$
$\text{Australia is one of the } X_{\boxed{2}} \text{ that have } X_{\boxed{0}} \text{ with } X_{\boxed{1}} \rangle$

$\overset{\pi_7}{\Rightarrow} \langle \text{Aozhou shi yu Beihan you } X_{\boxed{0}} \text{ de } X_{\boxed{2}} \text{ zhiyi,}$
$\text{Australia is one of the } X_{\boxed{2}} \text{ that have } X_{\boxed{0}} \text{ with North Korea} \rangle$

$\overset{\pi_9}{\Rightarrow} \langle \text{Aozhou shi yu Beihan you bangjiao de } X_{\boxed{2}} \text{ zhiyi,}$
$\text{Australia is one of the } X_{\boxed{2}} \text{ that have diplomatic relations with North Korea} \rangle$

$\overset{\pi_{10}}{\Rightarrow} \langle \text{Aozhou shi yu Beihan you bangjiao de shaoshu guojia zhiyi,}$
$\text{Australia is one of the few countries that have diplomatic relations with N. K.} \rangle$

Derivation

$\langle S, S \rangle$

$\stackrel{\pi_1}{\Rightarrow} \langle S\ X, S\ X \rangle$

$\stackrel{\pi_1}{\Rightarrow} \langle S\ X_{\boxed{1}}\ X_{\boxed{2}}, S\ X_{\boxed{1}}\ X_{\boxed{2}} \rangle$

$\stackrel{\pi_2}{\Rightarrow} \langle X_{\boxed{0}}\ X_{\boxed{1}}\ X_{\boxed{2}}, X_{\boxed{0}}\ X_{\boxed{1}}\ X_{\boxed{2}} \rangle$

$\stackrel{\pi_6}{\Rightarrow} \langle \text{Aozhou } X_{\boxed{1}}\ X_{\boxed{2}}, \text{Australia } X_{\boxed{1}}\ X_{\boxed{2}} \rangle$

$\stackrel{\pi_8}{\Rightarrow} \langle \text{Aozhou shi X, Australia is X} \rangle$

$\stackrel{\pi_5}{\Rightarrow} \langle \text{Aozhou shi X zhiyi, Australia is one of X} \rangle$

$\stackrel{\pi_4}{\Rightarrow} \langle \text{Aozhou shi } X_{\boxed{1}} \text{ de } X_{\boxed{2}} \text{ zhiyi,}$
$\text{Australia is one of the } X_{\boxed{2}} \text{ that } X_{\boxed{1}} \rangle$

$\stackrel{\pi_3}{\Rightarrow} \langle \text{Aozhou shi yu } X_{\boxed{1}} \text{ you } X_{\boxed{0}} \text{ de } X_{\boxed{2}} \text{ zhiyi,}$
$\text{Australia is one of the } X_{\boxed{2}} \text{ that have } X_{\boxed{0}} \text{ with } X_{\boxed{1}} \rangle$

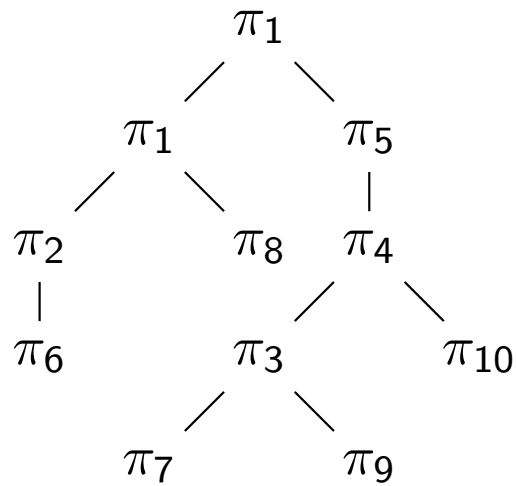
$\stackrel{\pi_7}{\Rightarrow} \langle \text{Aozhou shi yu Beihan you } X_{\boxed{0}} \text{ de } X_{\boxed{2}} \text{ zhiyi,}$
$\text{Australia is one of the } X_{\boxed{2}} \text{ that have } X_{\boxed{0}} \text{ with North Korea} \rangle$

$\stackrel{\pi_9}{\Rightarrow} \langle \text{Aozhou shi yu Beihan you bangjiao de } X_{\boxed{2}} \text{ zhiyi,}$
$\text{Australia is one of the } X_{\boxed{2}} \text{ that have diplomatic relations with North Korea} \rangle$

$\stackrel{\pi_{10}}{\Rightarrow} \langle \text{Aozhou shi yu Beihan you bangjiao de shaoshu guojia zhiyi,}$
$\text{Australia is one of the few countries that have diplomatic relations with N. K.} \rangle$

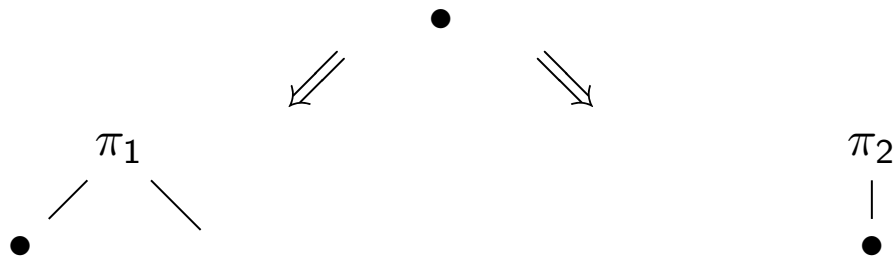
Inducing a Model

Derivation Tree



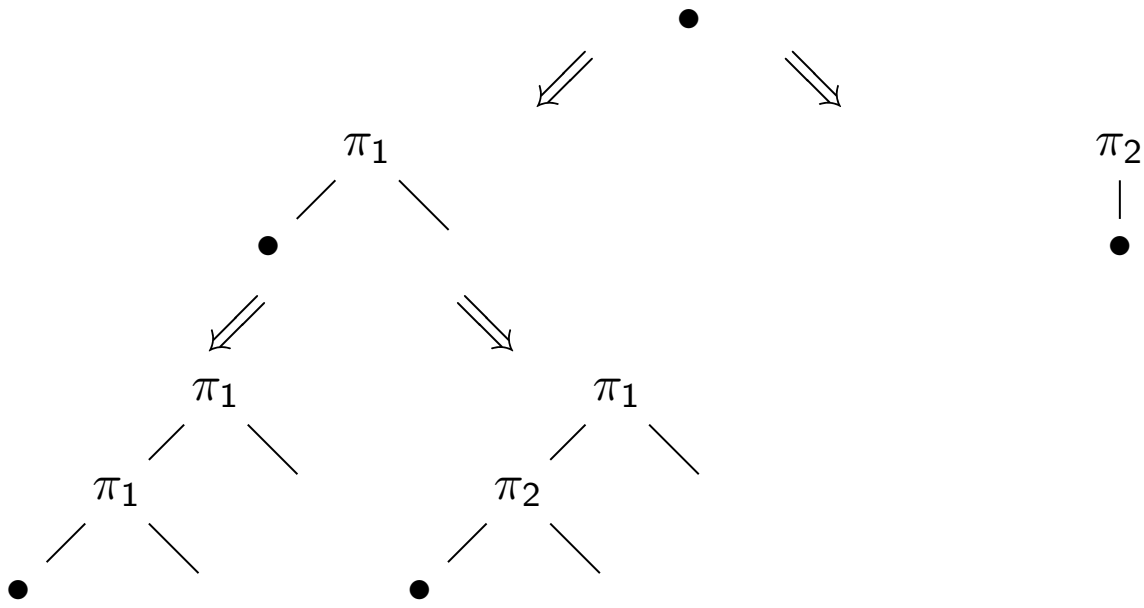
Inducing a Model

Decision Tree



Inducing a Model

Decision Tree



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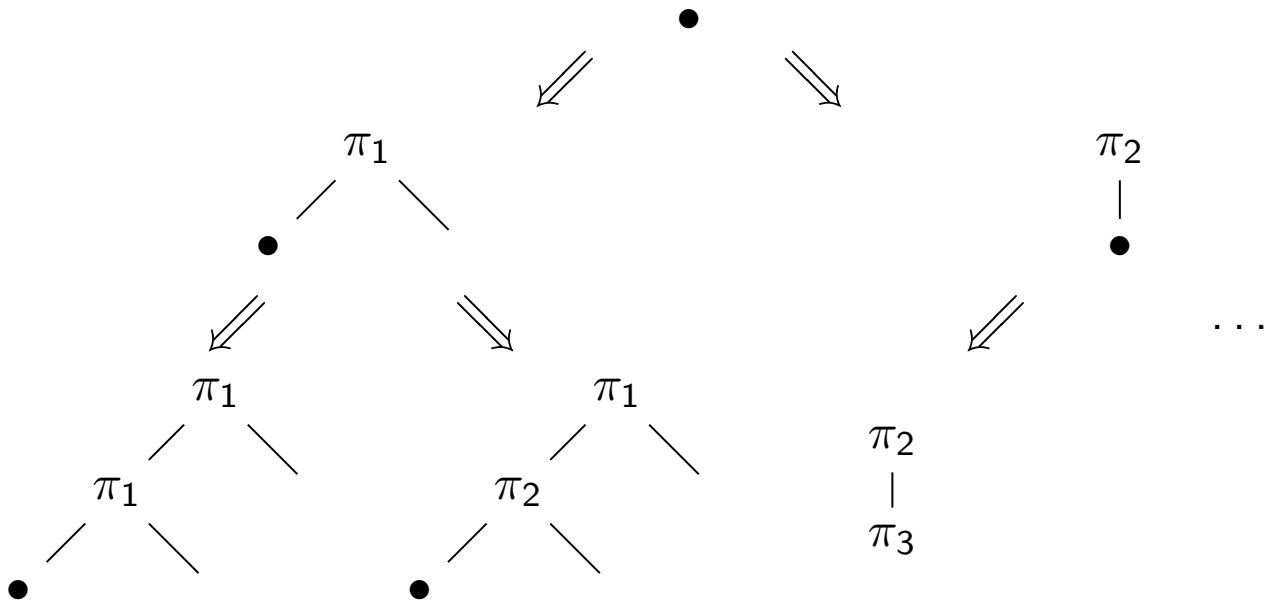

Inducing a Model

Assignments

$$\left.\begin{array}{r}\text{grammar } G \\ \text{assignment } p'\end{array}\right\} \text{“distribution”} \;\; p'(e, d, f)$$

$$\text{model } \mathcal{M}_G = \{p \in \mathcal{M}(E, F) \mid \exists p' \colon p(e, f) = \textstyle\sum_d p'(e, d, f)\}$$

Proper and Consistent Assignments

Let $q \in [0, 1]$.

$$\begin{array}{lll} \pi_1\colon & \mathsf{S} \to \langle \mathsf{S}\ \mathsf{S}, \mathsf{S}\ \mathsf{S} \rangle & \# \ q \\ \pi_2\colon & \mathsf{S} \to \langle \varepsilon, \varepsilon \rangle & \# \ 1 - q \end{array}$$

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$$Z = q \cdot Z \cdot Z + (1 - q)\ .$$

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Least solution:

$$Z = \begin{cases} 1 & \text{if } 0 \leq q \leq 0.5, \\ \frac{1}{q} - 1 & \text{otherwise.} \end{cases}$$

Training: Rule Extraction

Word-aligned Training Pair

⟨30 duonianlai de youhao hezuo, over 30 years of friendly cooperation⟩

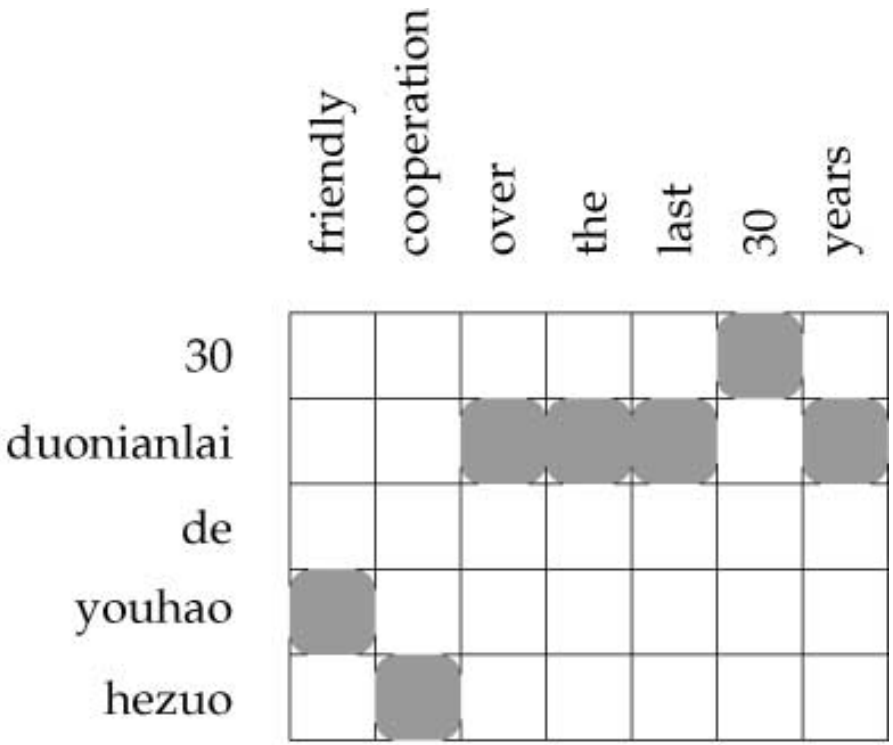


Figure 2 of (Chiang 2007)

Training: Rule Extraction

Initial Rules

X → ⟨30 duonianlai de youhao hezuo, over 30 years of friendly cooperation⟩
X → ⟨duonianlai, over the last 30 years⟩
X → ⟨youhao hezuo, friendly cooperation⟩
X → ⟨30, 30⟩
X → ⟨youhao, friendly⟩
X → ⟨hezuo, cooperation⟩

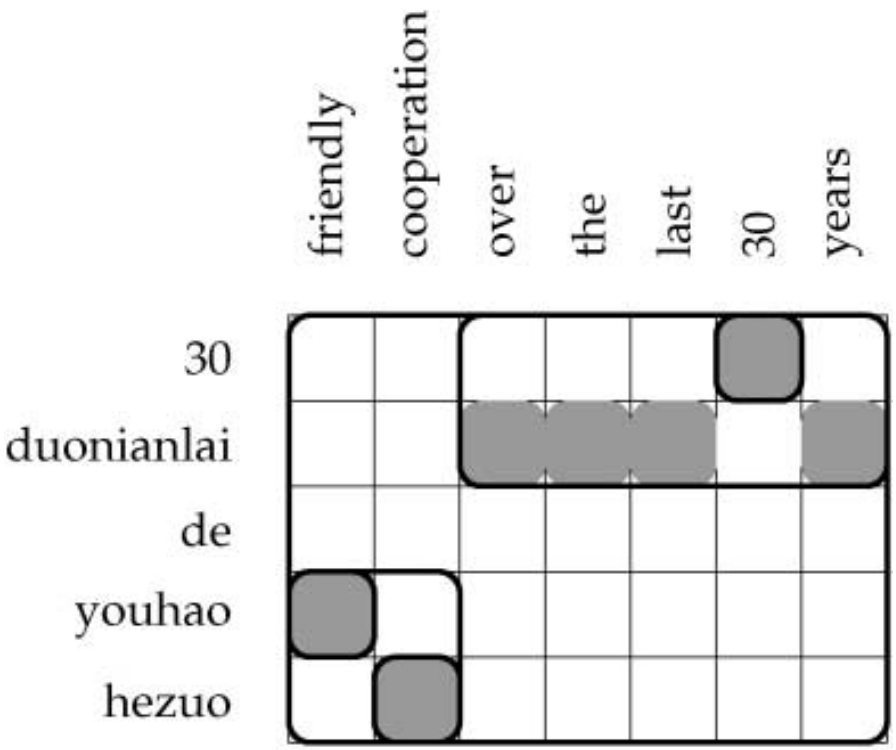


Figure 2 of (Chiang 2007)

Training: Rule Extraction

Example Rule

$X \rightarrow \langle X_{[1]}$ duonianlai de $X_{[2]}$, $X_{[2]}$ over the last years $X_{[1]} \rangle$

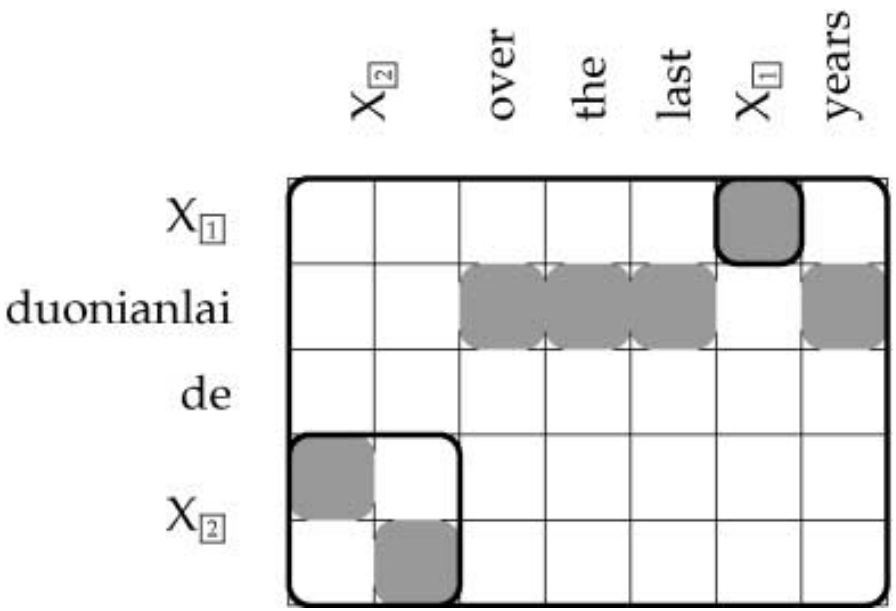


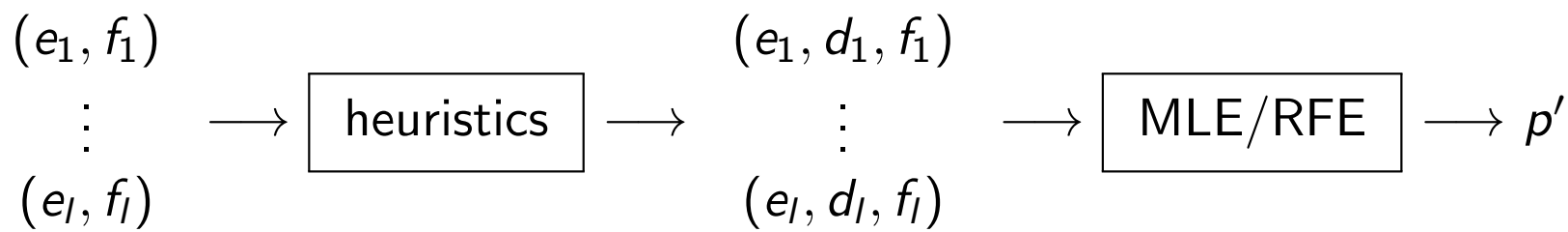
Figure 2 of (Chiang 2007)

Training: Maximum-Likelihood Estimation

- training data: sequence $\mathcal{D} = (e_1, f_1), \ldots, (e_l, f_l)$
- model: $\mathcal{M}_G = \{p \in \mathcal{M}(E, F) \mid \exists p' : p(e, f) = \sum_d p'(e, d, f)\}$
- likelihood: $P(\mathcal{D} \mid p') = \prod_j \sum_d p'(e_j, d, f_j)$
- use EM algorithm

Training: Reality

Chiang (2007), DeNeefe and Knight (2009):



Outline

Feature Selection

Hidden variable: derivation of (e, f) in G

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Decoding

$$\hat{e} = \text{argmax}_e \; p(e \mid f)$$

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NP-hard! (Casacuberta and Higuera 2000; Li, Eisner, and Khudanpur 2009)

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Cube Pruning (Chiang 2007)

Banerjee, Satanjeev and Alon Lavie (2005). "METEOR: An Automatic Metric for MT Evaluation with Improved Correlation with Human Judgments". In: Proceedings of the ACL 2005 Workshop on Intrinsic and Extrinsic Evaluation Measures for MT and/or Summarization.

Berger, Adam L., Vincent J. Della Pietra, and Stephen A. Della Pietra (1996). "A maximum entropy approach to natural language processing". In: Comp. Ling. 22 (1), pp. 39–71. ISSN: 0891-2017. URL: http://portal.acm.org/citation.cfm?id=234285.234289.

Brown, Peter F. et al. (1993). "The mathematics of statistical machine translation: parameter estimation". In: Comp. Ling. 19 (2), pp. 263–311. ISSN: 0891-2017. URL: http://portal.acm.org/citation.cfm?id=972470.972474.

Casacuberta, Francisco and Colin de la Higuera (2000). "Computational Complexity of Problems on Probabilistic Grammars and Transducers". In: LNCS.

Casella, George and Roger L. Berger (2002). Statistical Inference. Duxbury Advances Series. Duxbury.

Chiang, David (2007). "Hierarchical Phrase-Based Translation". In: Comp. Ling. 33.2, pp. 201–228. ISSN: 0891-2017. DOI: http://dx.doi.org/10.1162/coli.2007.33.2.201.

DeNeefe, Steve and Kevin Knight (2009). "Synchronous tree adjoining machine translation". In: EMNLP '09: Proceedings of the 2009

Conference on Empirical Methods in Natural Language Processing. Singapore: Association for Computational Linguistics, pp. 727–736. ISBN: 978-1-932432-62-6.

Li, Zhifei, Jason Eisner, and Sanjeev Khudanpur (2009). "Variational decoding for statistical machine translation". In: ACL-IJCNLP '09: Proceedings of the Joint Conference of the 47th Annual Meeting of the ACL and the 4th International Joint Conference on Natural Language Processing of the AFNLP: Volume 2. Suntec, Singapore: Association for Computational Linguistics, pp. 593–601. ISBN: 978-1-932432-46-6.

Och, Franz Josef (2003). "Minimum Error Rate Training in Statistical Machine Translation". In: ACL, pp. 160–167.

Och, Franz Josef and Hermann Ney (2002). "Discriminative Training and Maximum Entropy Models for Statistical Machine Translation". In: ACL, pp. 295–302.

Papineni, Kishore et al. (2002). "BLEU: a method for automatic evaluation of machine translation". In: ACL '02: Proceedings of the 40th Annual Meeting on Association for Computational Linguistics. Philadelphia, Pennsylvania: Association for Computational Linguistics, pp. 311–318. DOI: http://dx.doi.org/10.3115/1073083.1073135.

Smith, David A. and Jason Eisner (2006). "Minimum-Risk Annealing for Training Log-Linear Models". In: Proceedings of the International Conference on Computational Linguistics and the Association for

Computational Linguistics (COLING-ACL), Companion Volume. Sydney, pp. 787–794. URL: http://cs.jhu.edu/~jason/papers/#acl06-risk.

Snover, Matthew et al. (2006). "A Study of Translation Edit Rate with Targeted Human Annotation". In: Proceedings of Association for Machine Translation in the Americas.