Deciding the Twins Property for Weighted Tree Automata over Extremal Semifields

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TECHNISCHE UNIVERSITÄT DRESDEN

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Outline

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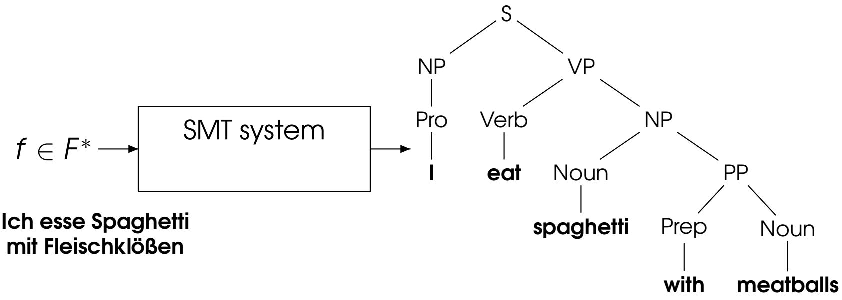
Twins Property in SMT

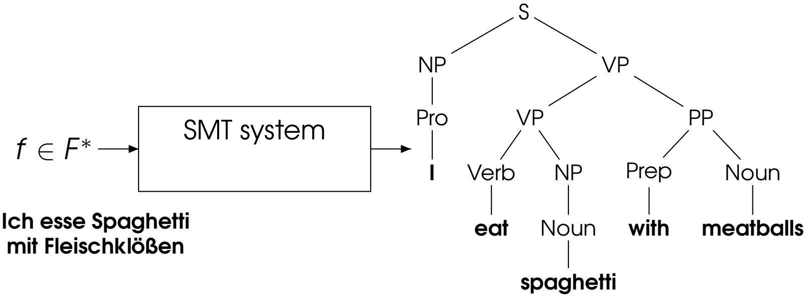
Deciding the Twins Property

Conclusion

Ambiguity in SMT
(May and Knight 2006)



Ambiguity in SMT
(May and Knight 2006)



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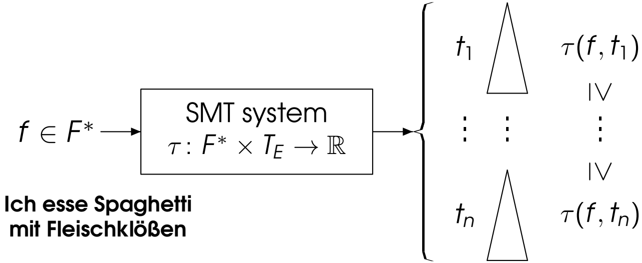

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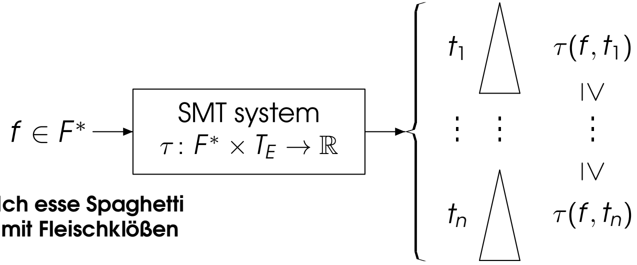

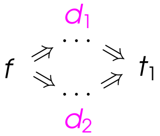

$$\tau(f, t_1) = \sum_{d\colon \pi(d)=t_1} \mathrm{wt}(d)$$

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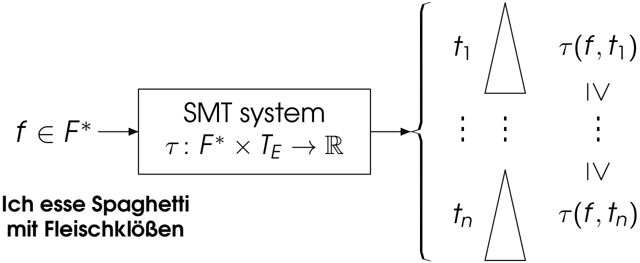

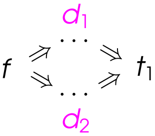

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spurious ambiguity

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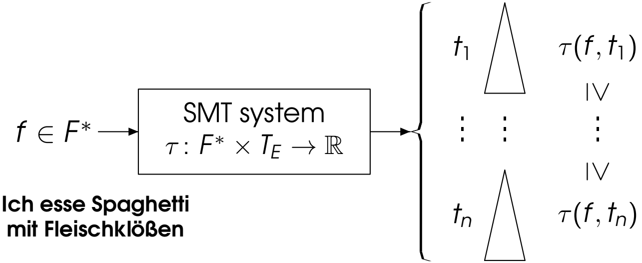

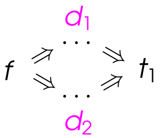

$$\tau(f, t_1) = \sum_{d\colon\ \pi(d)=t_1} \mathrm{wt}(d)$$

spurious ambiguity

translation: NP hard

(Casacuberta and de la Higuera 2000)

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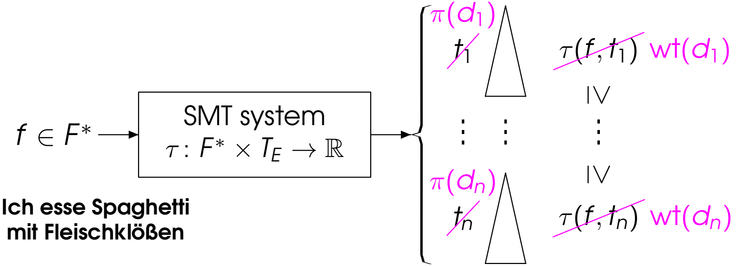

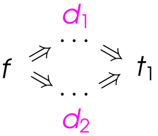

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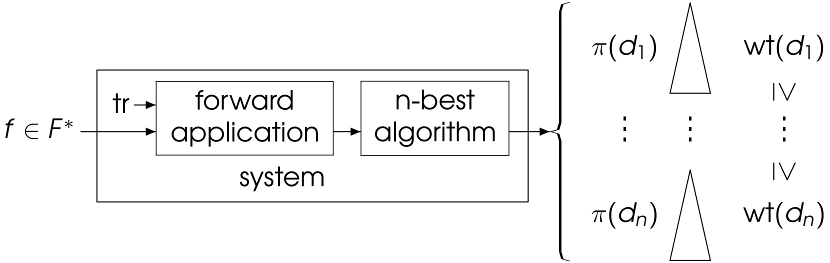
Weighted Tree Automata in SMT



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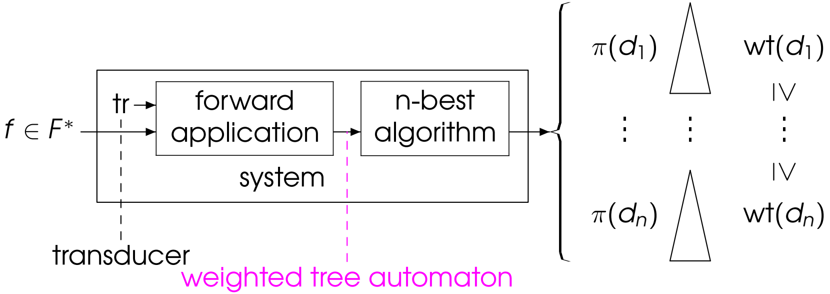

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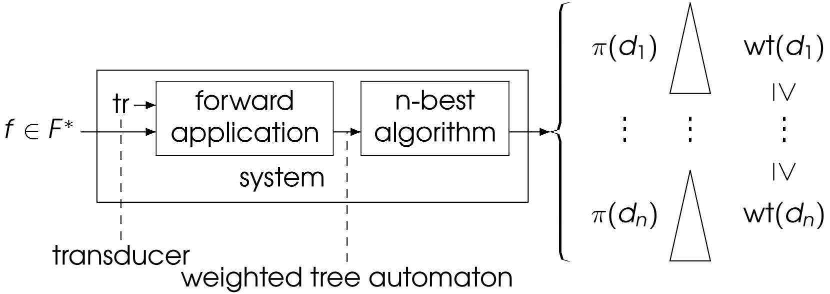

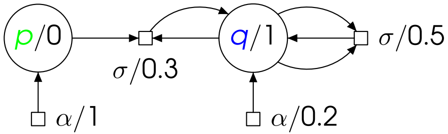

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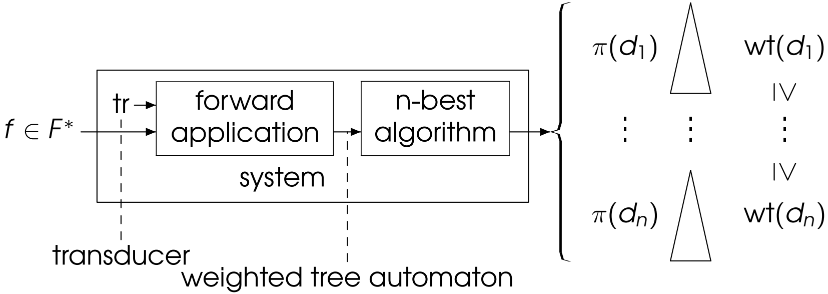

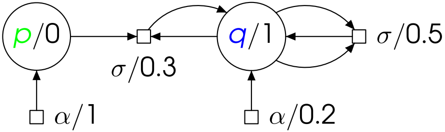

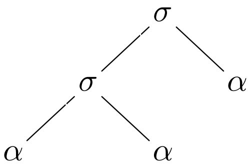

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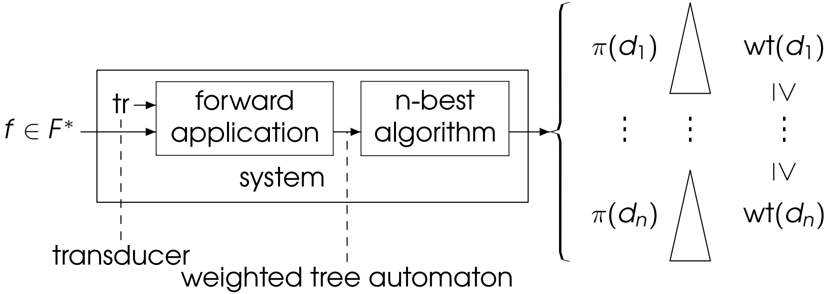
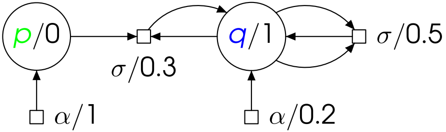
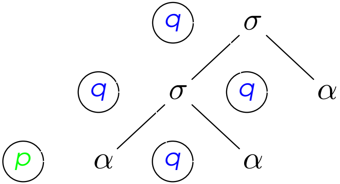
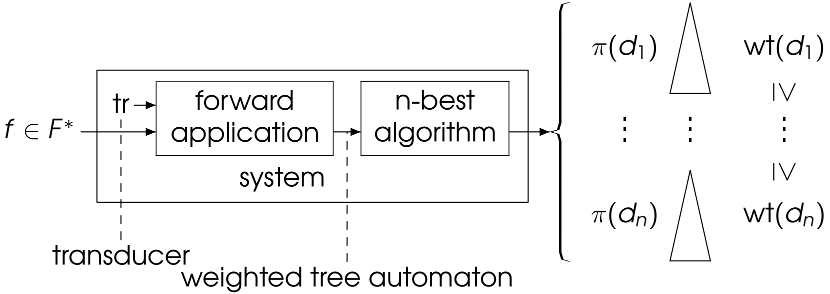


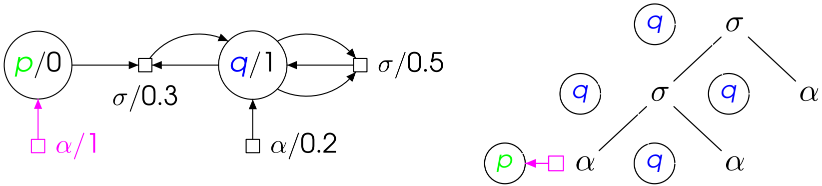




run

Weighted Tree Automata in SMT





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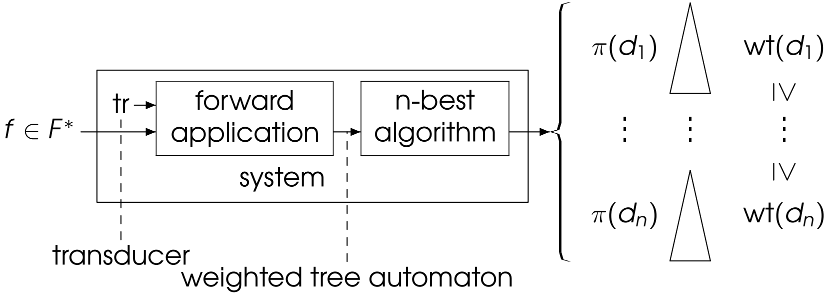
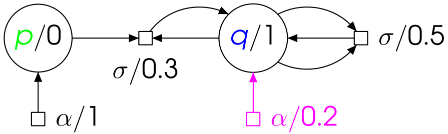
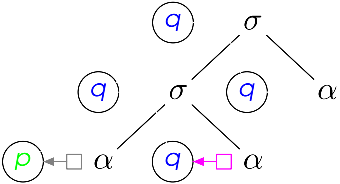
Weighted Tree Automata in SMT

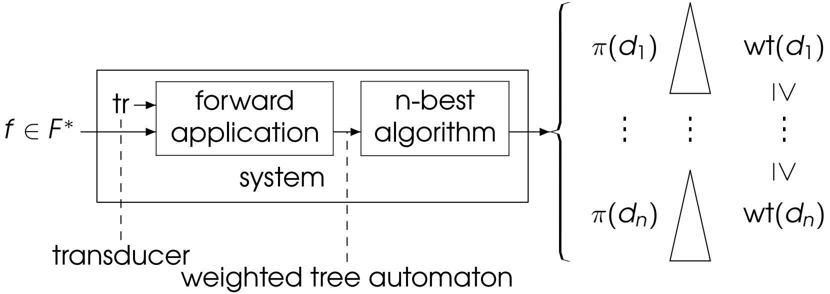


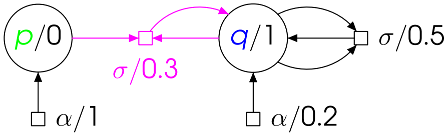


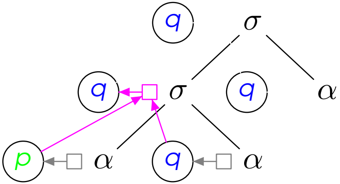


$1 \cdot 0.2$

Weighted Tree Automata in SMT

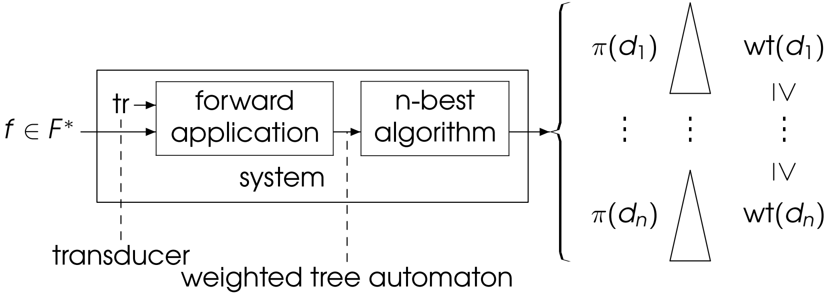


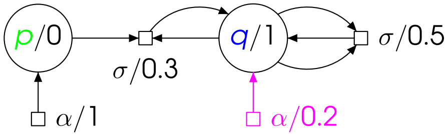


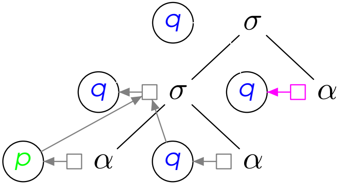


$1 \cdot 0.2 \cdot 0.3$

Weighted Tree Automata in SMT

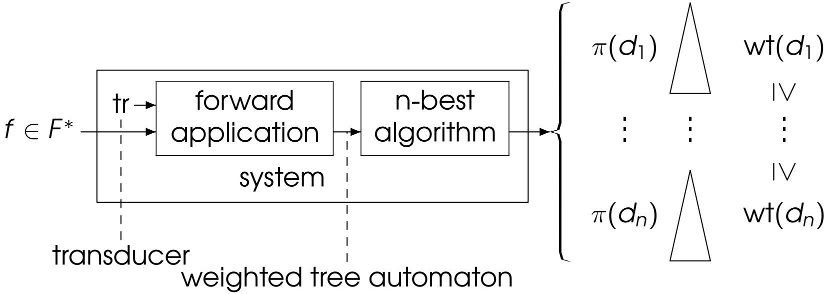


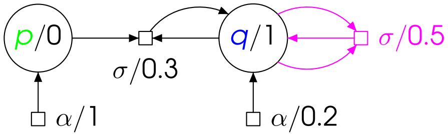


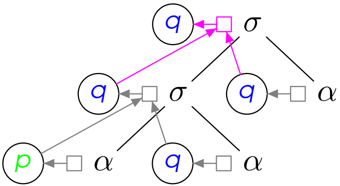


$1 \cdot 0.2 \cdot 0.3 \cdot 0.2$

Weighted Tree Automata in SMT

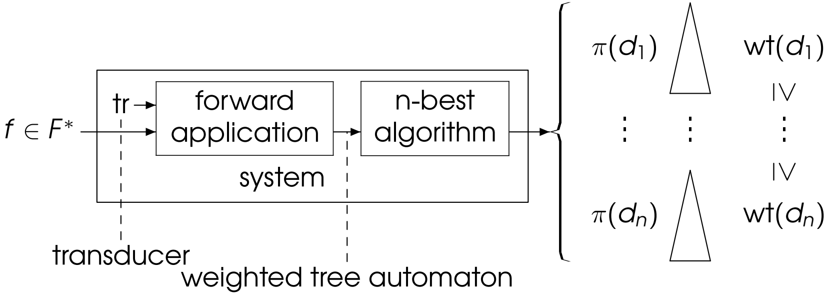


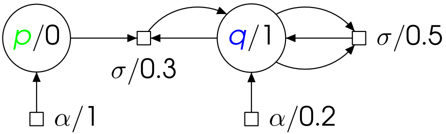


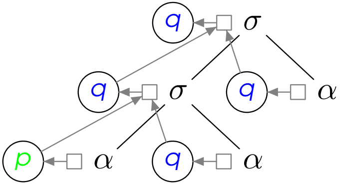


$1 \cdot 0.2 \cdot 0.3 \cdot 0.2 \cdot 0.5$

Weighted Tree Automata in SMT

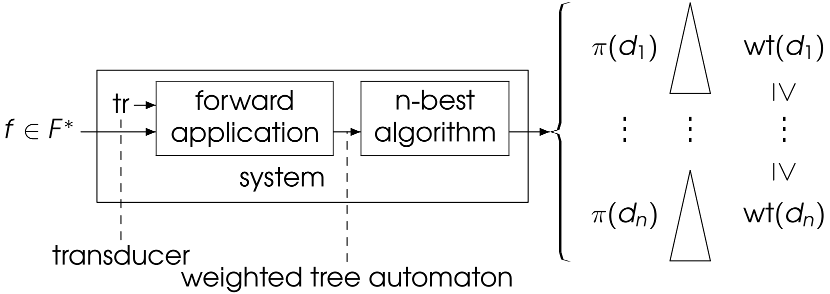


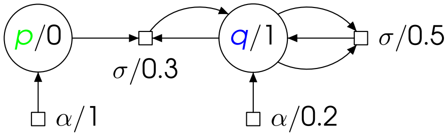


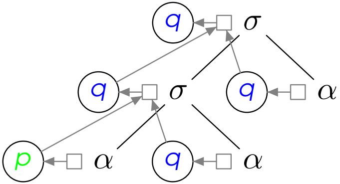


$$1 \cdot 0.2 \cdot 0.3 \cdot 0.2 \cdot 0.5$$

Weighted Tree Automata in SMT



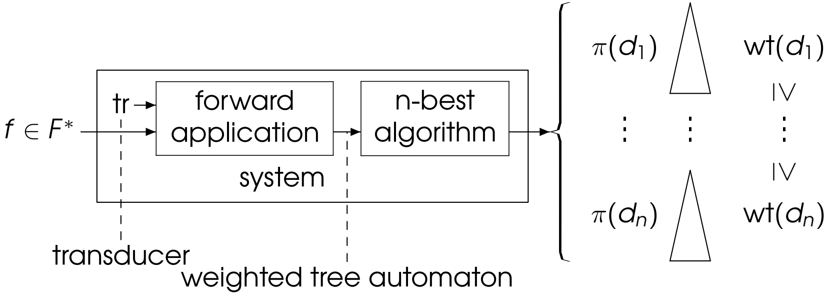


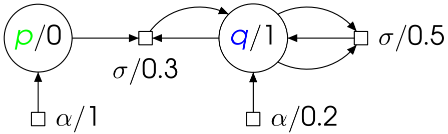


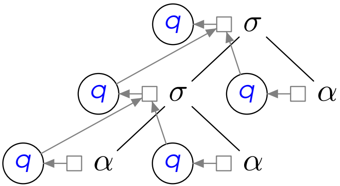
nondeterministic

$1 \cdot 0.2 \cdot 0.3 \cdot 0.2 \cdot 0.5$

Weighted Tree Automata in SMT



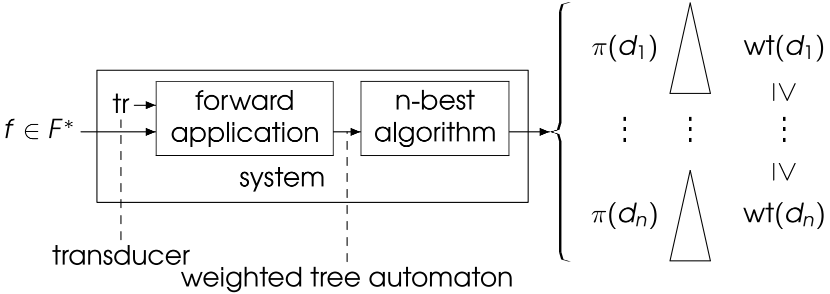


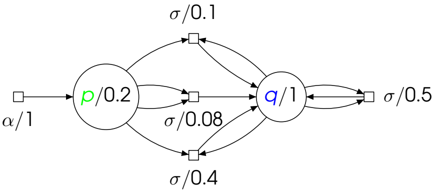


nondeterministic

$0.2 \cdot 0.2 \cdot 0.5 \cdot 0.2 \cdot 0.5$

Weighted Tree Automata in SMT





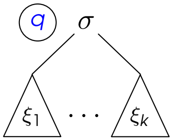
deterministic

$$\mathrm{wt}(d) = \tau(f, \pi(d))$$

Excursion: Semantics

$(S, +, \cdot, 0, 1)$

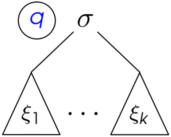
$[\![\xi]\!]_q \in S$ sum over runs



Excursion: Semantics

$(S, +, \cdot, 0, 1)$

$[\![\xi]\!]_q \in S$	sum over runs
$[\![\xi]\!] \in S^Q$	weight vector



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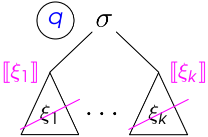

$$[\![\xi]\!] = [\![\sigma([\![\xi_1]\!], \ldots, [\![\xi_k]\!])]\!]$$

Excursion: Semantics

$(S, +, \cdot, 0, 1)$

$[\![\xi]\!]_q \in S$	sum over runs
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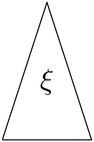

$[\![\xi]\!]$

Excursion: Semantics

$(S, +, \cdot, 0, 1)$

$[\![\xi]\!]_q \in S$	sum over runs
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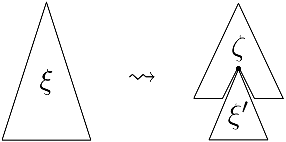


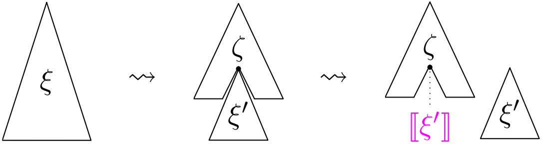
$$[\![\xi]\!] = [\![\xi' \cdot \zeta]\!]$$

Excursion: Semantics

$(S, +, \cdot, 0, 1)$

$[\![\xi]\!]_q \in S$ sum over runs

$[\![\xi]\!] \in S^Q$ weight vector



$$[\![\xi]\!] = [\![\xi' \cdot \zeta]\!] = [\![[\![\xi']\!] \cdot \zeta]\!]$$

Determinizing Weighted Tree Automata
(Büchse, May, and Vogler 2010)

Sufficient conditions: (pick one)

Determinizing Weighted Tree Automata
(Büchse, May, and Vogler 2010)

Sufficient conditions:

- acyclic

(May and Knight 2006)

Determinizing Weighted Tree Automata
(Büchse, May, and Vogler 2010)

Sufficient conditions:

- deterministic

Determinizing Weighted Tree Automata

(Büchse, May, and Vogler 2010)

Sufficient conditions:

III semiring locally finite

(Borchardt 2004)

- Boolean semiring $(\{\bot, \top\}, \vee, \wedge, \bot, \top)$
- capacity semiring $(\mathbb{R} \cup \{-\infty, \infty\}, \max, \min, -\infty, \infty)$
- powerset lattice $(\mathcal{P}(A), \cup, \cap, \emptyset, A)$

Determinizing Weighted Tree Automata
(Büchse, May, and Vogler 2010)

Sufficient conditions:

IV semiring extremal ($a + b \in \{a, b\}$)
and ...
and ...

- Viterbi semiring ($[0, 1], \max, \cdot, 0, 1$)
- tropical semiring ($\mathbb{R}^{\geq 0} \cup \{\infty\}, \min, +, \infty, 0$)
- Boolean semiring
- capacity semiring
- powerset lattice

(Mahr 1984)

Determinizing Weighted Tree Automata
(Büchse, May, and Vogler 2010)

Sufficient conditions:

IV semiring extremal ($a + b \in \{a, b\}$)
and maximal factorization (f, g)
and ...

$$\begin{pmatrix}4\\6\\2\end{pmatrix} \overset{g}{\mapsto} 2, \qquad \begin{pmatrix}4\\6\\2\end{pmatrix} \overset{f}{\mapsto} \begin{pmatrix}2\\3\\1\end{pmatrix}$$

$$2 \cdot \begin{pmatrix}2\\3\\1\end{pmatrix} = \begin{pmatrix}4\\6\\2\end{pmatrix}$$

(Kirsten and Mäurer 2005)

Determinizing Weighted Tree Automata
(Büchse, May, and Vogler 2010)

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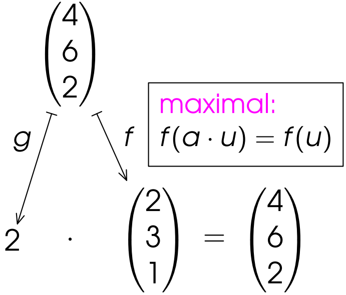

(Kirsten and Mäurer 2005)

Determinizing Weighted Tree Automata
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Sufficient conditions:

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and maximal factorization (f, g)
and twins property ($\textsc{Siblings}(\mathcal{A}) \subseteq \textsc{Twins}(\mathcal{A})$)

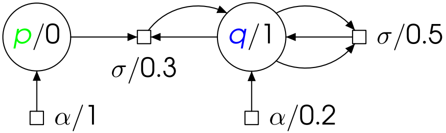
Determinizing Weighted Tree Automata
(Büchse, May, and Vogler 2010)

Sufficient conditions:

IV semiring extremal ($a + b \in \{a, b\}$)
and maximal factorization (f, g)
and twins property ($\text{SIBLINGS}(\mathcal{A}) \subseteq \text{TWINS}(\mathcal{A})$)

$(p, q) \in \text{SIBLINGS}(\mathcal{A})$

iff $\exists \xi \colon [\![\xi]\!]_p \neq 0, [\![\xi]\!]_q \neq 0$



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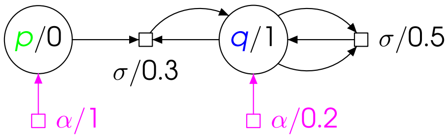
$(p, q) \in \textsc{Siblings}(\mathcal{A})$

iff $\exists \xi : [\![\xi]\!]_p \neq 0, [\![\xi]\!]_q \neq 0$



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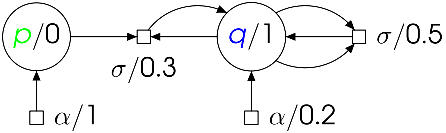
$(p, q) \in \textsc{Siblings}(\mathcal{A})$

iff $\exists \xi : [\![\xi]\!]_p \neq 0, [\![\xi]\!]_q \neq 0$



$(p, q) \in \textsc{Twins}(\mathcal{A})$

iff $\forall \zeta : [\![e_p \cdot \zeta]\!]_p \neq 0, [\![e_q \cdot \zeta]\!]_q \neq 0$
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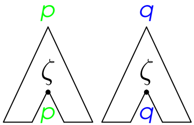
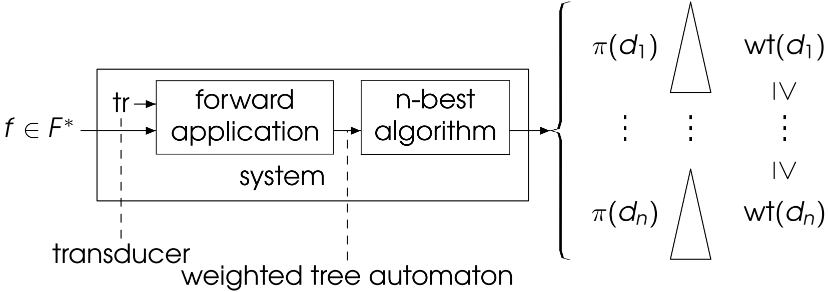
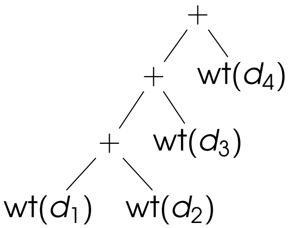


Extremal Semiring – Victorious Runs

$a + b \in \{a, b\}$



$$\tau(f, t) = \sum_{d\colon \pi(d) = t} \mathrm{wt}(d)$$



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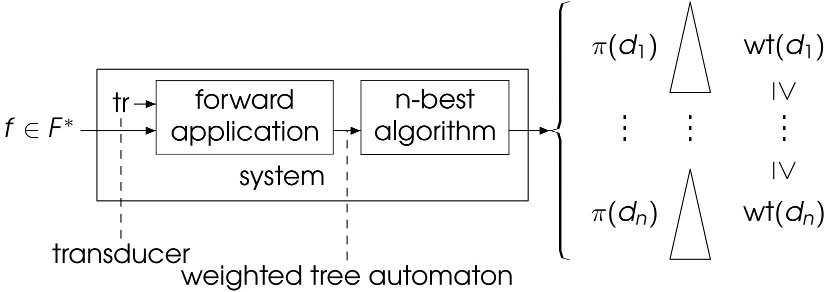

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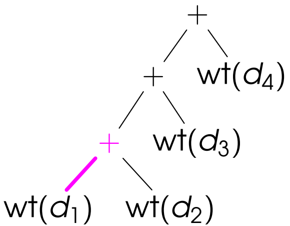

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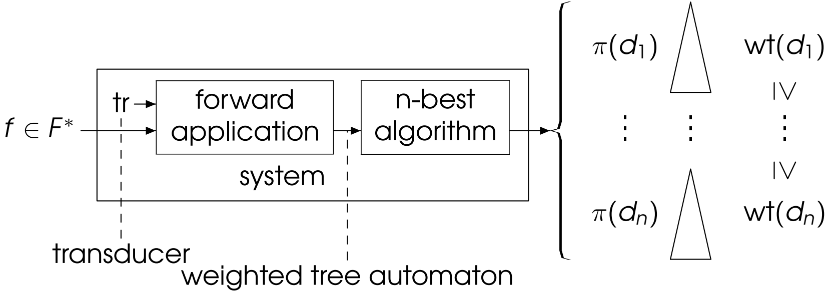

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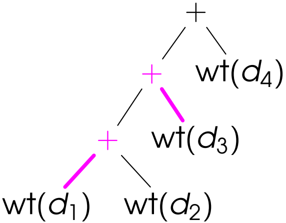

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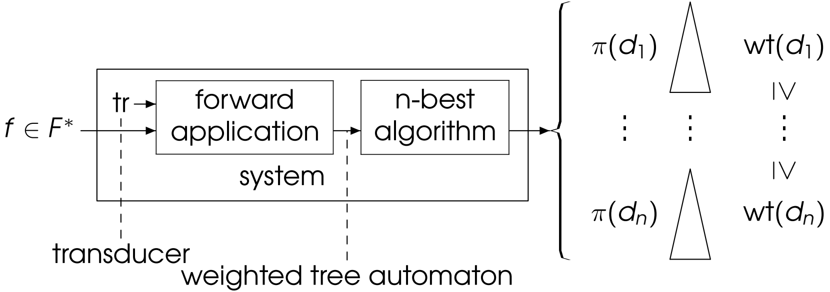

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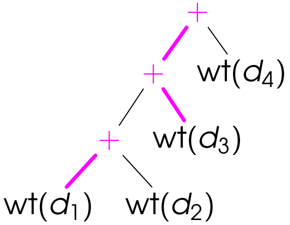

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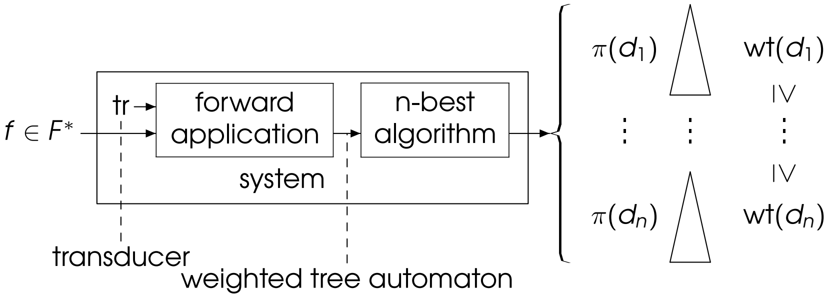
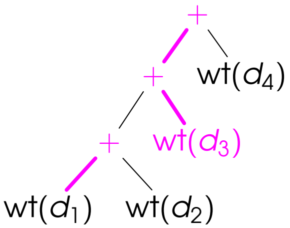


$$\tau(f,t) = \sum_{d\colon \pi(d)=t} \mathrm{wt}(d)$$
$$= \mathrm{wt}(d') \text{ for some } d'$$



Outline

Search Space (Kirsten 2012)

Recall: TP	iff	$\textsc{Siblings}(\mathcal{A}) \subseteq \textsc{Twins}(\mathcal{A})$
$(p, q) \in \textsc{Siblings}(\mathcal{A})$	iff	$\exists \xi : [\![\xi]\!]_p \neq 0, [\![\xi]\!]_q \neq 0$
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$\neg$TP

	(p, q)	$\cdots$	(p', q')
ζ_1	✓	$\cdots$	✓
ζ_2	✓	$\cdots$	✗
$\vdots$	$\vdots$		$\vdots$

counterexample will be found

TP

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nontermination!

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counterexample will be found $\rightsquigarrow$

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✓	$\cdots$	✗
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nontermination! but: regularity $\rightsquigarrow$

(p,q)	$\cdots$	(p',q')
✓	$\cdots$	✓
$\vdots$	$\ddots$	$\vdots$
✓	$\cdots$	✓

finite search space

Roadmap

step	description	conditions

Roadmap

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a	reveal regularity (Lemma 3.3)	commutative extremal semiring

Roadmap

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Revealing Regularity

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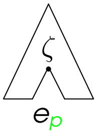
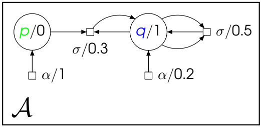
$[\![e_p \cdot \zeta]\!]_p$





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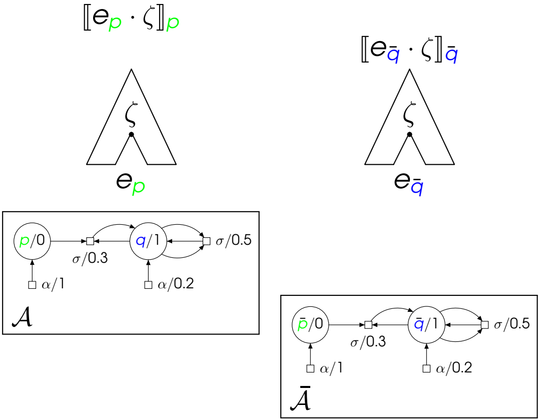

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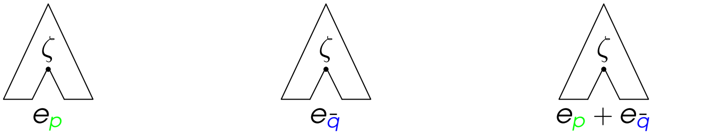

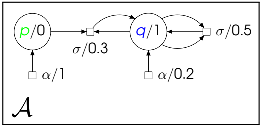

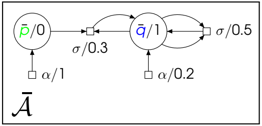

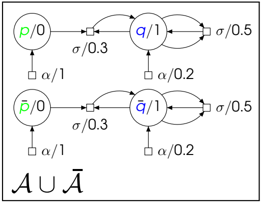

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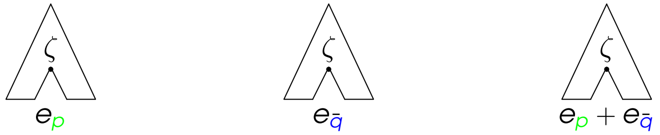

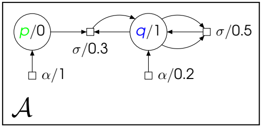

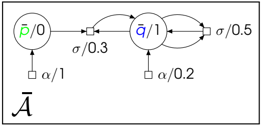

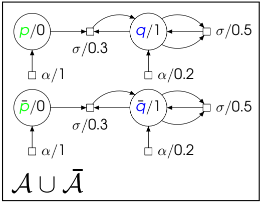

$$T_{p,q} = \{[\![(e_p + e_{\bar{q}}) \cdot \zeta]\!] \mid \zeta \in C_\Sigma\}$$

$u \in T_{p,q}$ counterexample: $u_p \neq 0, u_{\bar{q}} \neq 0, u_p \neq u_{\bar{q}}$

The Regularity Property

Recall: $T_{p,q} = \{[\![(e_p + e_{\bar{q}}) \cdot \zeta]\!] \mid \zeta \in C_\Sigma\}$ $\quad T_{p,q} \subseteq S \cdot S'$
counterexample: $u_p \neq 0, u_{\bar{q}} \neq 0, u_p \neq u_{\bar{q}}$

Let $\mathcal{A}$ have the twins property and $(p, q) \in \textsc{Siblings}(\mathcal{A})$.

Then there is a finite set $S' \subseteq S^{Q \cup \bar{Q}}$ with $T_{p,q} \subseteq S \cdot S'$.

(Lemma 3.3)

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(Lemma 3.3)

Then $f(T_{p,q} \setminus \{\tilde{0}\}) \subseteq f(S \cdot S' \setminus \{\tilde{0}\}) \subseteq f(S' \setminus \{\tilde{0}\})$.

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A vector u is a counterexample iff so is $f(u)$.

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(Lemma 3.4)

A vector u is a counterexample iff so is $f(u)$.

Roughly: $S' \approx \{[\![(e_p + e_{\bar{q}}) \cdot \zeta]\!] \mid \text{height}(\zeta) \leq 2|Q|^{2|Q|}\}$

Outline

Twins Property in SMT

Deciding the Twins Property

Conclusion

Conclusion

- twins property decidable
- complexity: PSPACE-hard (Kirsten 2012)
- search on compressed search space
- beyond extremal, beyond twins property?

Literature

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Proof – Victorious Runs

$$[\![\xi]\!] = \begin{pmatrix} [\![\xi]\!]_{q_1} \\ \vdots \\ [\![\xi]\!]_{q_k} \end{pmatrix}$$

weight
vector

Proof – Victorious Runs

$$[\![\xi]\!] = \begin{pmatrix} [\![\xi]\!]_{q_1} \\ \vdots \\ [\![\xi]\!]_{q_k} \end{pmatrix} \qquad \begin{pmatrix} \kappa_{q_1} \; \triangle \\ \vdots \\ \kappa_{q_k} \; \triangle \end{pmatrix}$$

weight vector

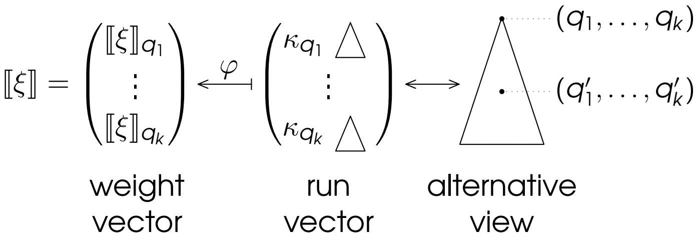
run vector

Proof – Victorious Runs

$$[\![\xi]\!] = \begin{pmatrix} [\![\xi]\!]_{q_1} \\ \vdots \\ [\![\xi]\!]_{q_k} \end{pmatrix} \overset{\varphi}{\longleftarrow\!\shortmid} \begin{pmatrix} \kappa_{q_1} \ \triangle \\ \vdots \\ \kappa_{q_k} \ \triangle \end{pmatrix}$$

weight vector | run vector

Proof – Victorious Runs



Proof – Smallest Counterexample

Recall: $T_{p,q} = \{[\![(e_p + e_{\bar{q}}) \cdot \zeta]\!] \mid \zeta \in C_\Sigma\}$ $\quad T_{p,q} \subseteq S \cdot S'$

$S' \approx \{[\![(e_p + e_{\bar{q}}) \cdot \zeta]\!] \mid \text{height}(\zeta) \leq 2|Q|^{2|Q|}\}$