Deciding the Twins Property for Weighted Tree Automata over Extremal Semifields

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Outline

Determinization of Weighted Tree Automata

Deciding the Twins Property

Conclusion

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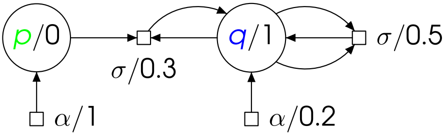
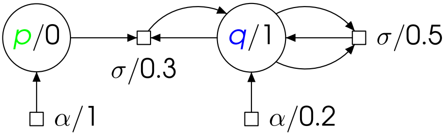
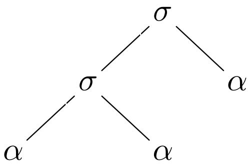
Weighted Tree Automata

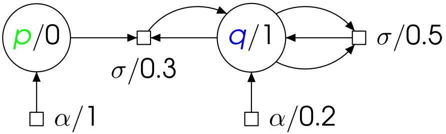


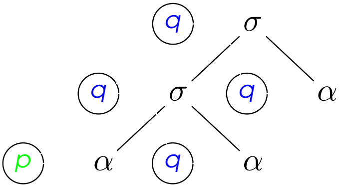
Weighted Tree Automata





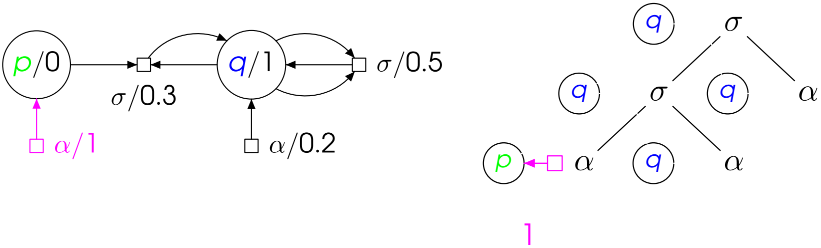
Weighted Tree Automata





run

Weighted Tree Automata



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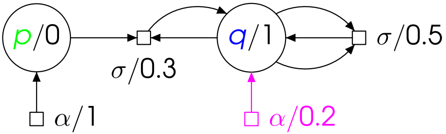
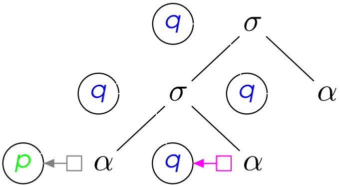
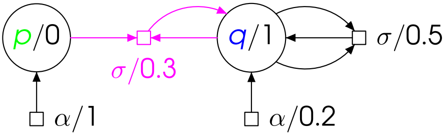
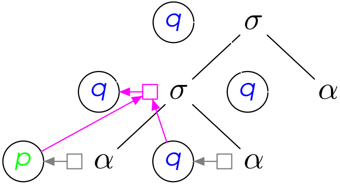




$1 \cdot 0.2$

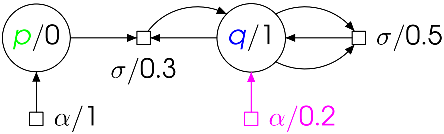
Weighted Tree Automata

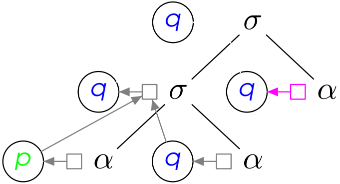




$1 \cdot 0.2 \cdot 0.3$

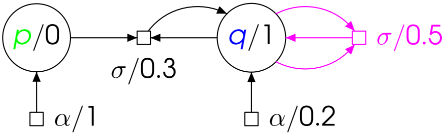
Weighted Tree Automata

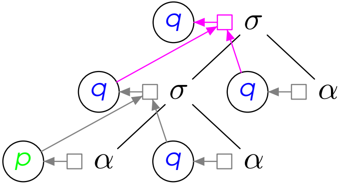




$1 \cdot 0.2 \cdot 0.3 \cdot 0.2$

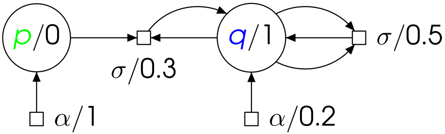
Weighted Tree Automata

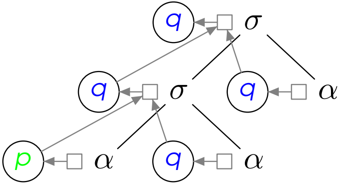




$1 \cdot 0.2 \cdot 0.3 \cdot 0.2 \cdot 0.5$

Weighted Tree Automata





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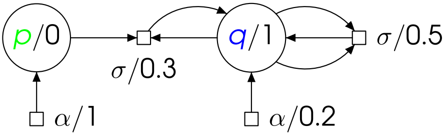
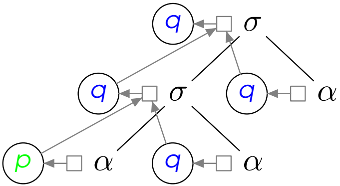
Weighted Tree Automata

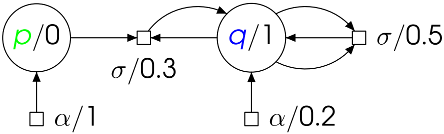


nondeterministic



$1 \cdot 0.2 \cdot 0.3 \cdot 0.2 \cdot 0.5$

Weighted Tree Automata



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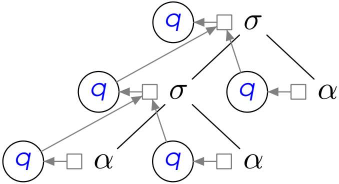
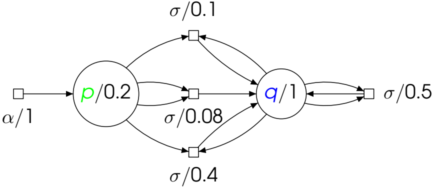


$0.2 \cdot 0.2 \cdot 0.5 \cdot 0.2 \cdot 0.5$

Weighted Tree Automata

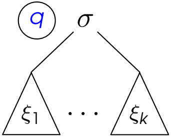


deterministic

Excursion: Semantics

$(S, +, \cdot, 0, 1)$

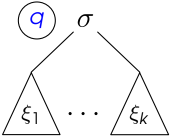
$[\![\xi]\!]_q \in S$ sum over runs



Excursion: Semantics

$(S, +, \cdot, 0, 1)$

$[\![\xi]\!]_q \in S$	sum over runs
$[\![\xi]\!] \in S^Q$	weight vector



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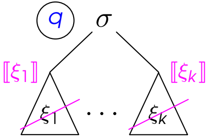

$$[\![\xi]\!] = [\![\sigma([\![\xi_1]\!], \ldots, [\![\xi_k]\!])]\!]$$

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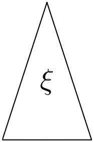

$[\![\xi]\!]$

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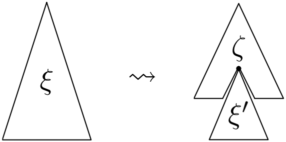

$[\![\xi]\!] = [\![\xi' \cdot \zeta]\!]$

Excursion: Semantics

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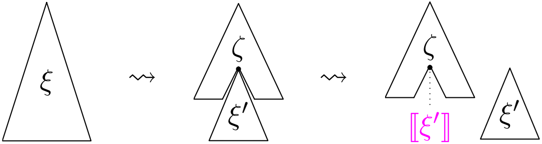

$$[\![\xi]\!] = [\![\xi' \cdot \zeta]\!] = [\![[\![\xi']\!] \cdot \zeta]\!]$$

Determinizing Weighted Tree Automata
(Büchse, May, and Vogler 2010)

Sufficient conditions: (pick one)

Determinizing Weighted Tree Automata
(Büchse, May, and Vogler 2010)

Sufficient conditions:

- acyclic

(May and Knight 2006)

Determinizing Weighted Tree Automata
(Büchse, May, and Vogler 2010)

Sufficient conditions:

- deterministic

Determinizing Weighted Tree Automata
(Büchse, May, and Vogler 2010)

Sufficient conditions:

III semiring locally finite

(Borchardt 2004)

- Boolean semiring $(\{\bot, \top\}, \vee, \wedge, \bot, \top)$
- capacity semiring $(\mathbb{R} \cup \{-\infty, \infty\}, \max, \min, -\infty, \infty)$
- powerset lattice $(\mathcal{P}(A), \cup, \cap, \emptyset, A)$
- every bdl

Determinizing Weighted Tree Automata
(Büchse, May, and Vogler 2010)

Sufficient conditions:

IV semiring extremal ($a + b \in \{a, b\}$)
and …
and …

- Viterbi semiring ([0, 1], max, ·, 0, 1)
- tropical semiring ($\mathbb{R}^{\geq 0} \cup \{\infty\}$, min, +, ∞, 0)
- Boolean semiring
- capacity semiring
- powerset lattice

(Mahr 1984)

Determinizing Weighted Tree Automata
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Sufficient conditions:

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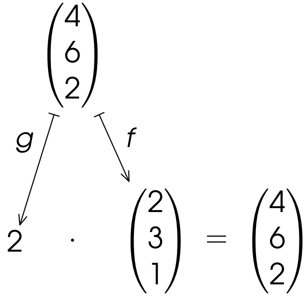

(Kirsten and Mäurer 2005)

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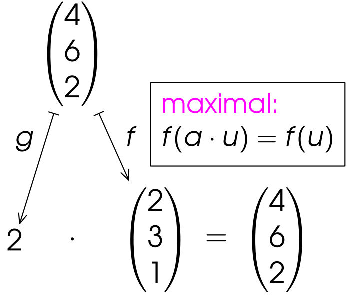

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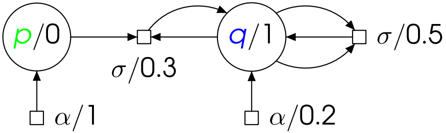

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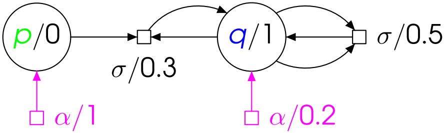

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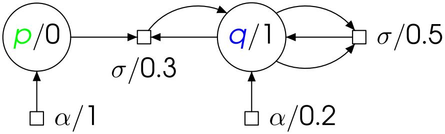
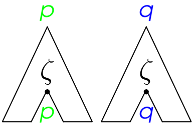


$(p, q) \in \textsc{Twins}(\mathcal{A})$

iff $\forall \zeta : [\![e_p \cdot \zeta]\!]_p \neq 0, [\![e_q \cdot \zeta]\!]_q \neq 0$
$\implies [\![e_p \cdot \zeta]\!]_p = [\![e_q \cdot \zeta]\!]_q$



Outline

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Conclusion

Search Space (Kirsten 2012)

$$\begin{array}{lll}
\text{Recall:} \qquad \text{TP} & \text{iff} & \textsc{Siblings}(\mathcal{A}) \subseteq \textsc{Twins}(\mathcal{A}) \\
(p, q) \in \textsc{Siblings}(\mathcal{A}) & \text{iff} & \exists \xi\colon [\![\xi]\!]_p \neq 0, [\![\xi]\!]_q \neq 0 \\
(p, q) \in \textsc{Twins}(\mathcal{A}) & \text{iff} & \forall \zeta\colon [\![e_p \cdot \zeta]\!]_p \neq 0, [\![e_q \cdot \zeta]\!]_q \neq 0 \\
 & & \qquad \Rightarrow [\![e_p \cdot \zeta]\!]_p = [\![e_q \cdot \zeta]\!]_q
\end{array}$$

Search Space (Kirsten 2012)

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$(p, q) \in \textsc{Twins}(\mathcal{A})$		iff	$\forall \zeta : [\![e_p \cdot \zeta]\!]_p \neq 0, [\![e_q \cdot \zeta]\!]_q \neq 0 \Rightarrow [\![e_p \cdot \zeta]\!]_p = [\![e_q \cdot \zeta]\!]_q$

¬TP

	(p, q)	$\cdots$	(p', q')
ζ_1	✓	$\cdots$	✓
ζ_2	✓	$\cdots$	✗
$\vdots$	$\vdots$		$\vdots$

counterexample will be found

TP

	(p, q)	$\cdots$	(p', q')
ζ_1	✓	$\cdots$	✓
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nontermination!

Search Space (Kirsten 2012)

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$\neg$TP

	(p, q)	$\cdots$	(p', q')
ζ_1	✓	$\cdots$	✓
ζ_2	✓	$\cdots$	✗
$\vdots$	$\vdots$		$\vdots$

counterexample will be found

TP

	(p, q)	$\cdots$	(p', q')
ζ_1	✓	$\cdots$	✓
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nontermination! but: regularity

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¬TP

	(p, q)	$\cdots$	(p', q')
ζ_1	✓	$\cdots$	✓
ζ_2	✓	$\cdots$	✗
$\vdots$	$\vdots$		$\vdots$

counterexample will be found ⇝ counterexample will be found

(p, q)	$\cdots$	(p', q')
✓	$\cdots$	✓
✓	$\cdots$	✗
$\vdots$		$\vdots$

TP

	(p, q)	$\cdots$	(p', q')
ζ_1	✓	$\cdots$	✓
ζ_2	✓	$\cdots$	✓
$\vdots$	$\vdots$		$\vdots$

nontermination! but: regularity ⇝ finite search space

(p, q)	$\cdots$	(p', q')
✓	$\cdots$	✓
$\vdots$	$\ddots$	$\vdots$
✓	$\cdots$	✓

Roadmap

step	description	conditions

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a	reveal regularity (Lemma 3.3)	commutative extremal semiring

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Roadmap

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a+b+c	Algorithm 2	extremal semifield

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Revealing Regularity

Recall: $(p, q) \in \textsc{Twins}(\mathcal{A})$ iff $\forall \zeta : [\![e_p \cdot \zeta]\!]_p \neq 0, [\![e_q \cdot \zeta]\!]_q \neq 0$
$\Rightarrow [\![e_p \cdot \zeta]\!]_p = [\![e_q \cdot \zeta]\!]_q$

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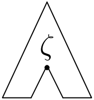
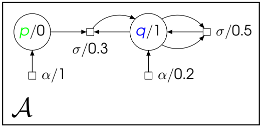
$[\![e_p \cdot \zeta]\!]_p$



e_p



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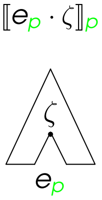
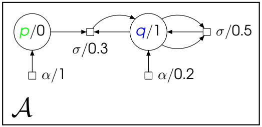
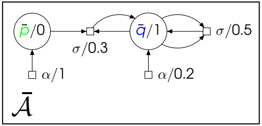




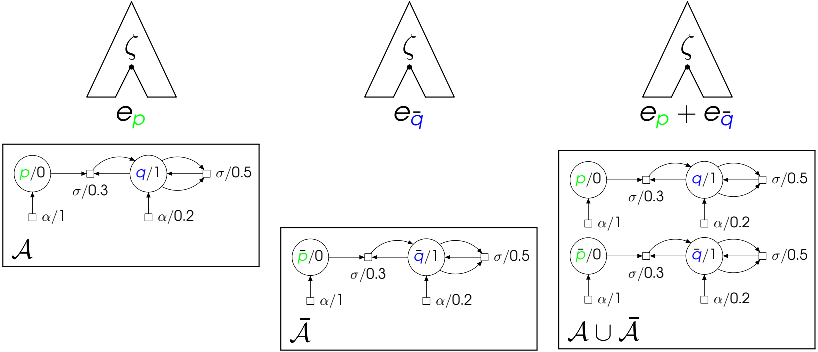


Revealing Regularity

Recall: $(p, q) \in \textsc{Twins}(\mathcal{A})$ iff $\forall \zeta \colon [\![e_p \cdot \zeta]\!]_p \neq 0, [\![e_q \cdot \zeta]\!]_q \neq 0 \Rightarrow [\![e_p \cdot \zeta]\!]_p = [\![e_q \cdot \zeta]\!]_q$

$$[\![e_p \cdot \zeta]\!]_p = \cdots\cdots\cdots\cdots\cdots = [\![(e_p + e_{\bar{q}}) \cdot \zeta]\!]_p$$

$$[\![e_{\bar{q}} \cdot \zeta]\!]_{\bar{q}} = [\![(e_p + e_{\bar{q}}) \cdot \zeta]\!]_{\bar{q}}$$



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$$[\![e_p \cdot \zeta]\!]_p \quad = \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad = \quad [\![(e_p + e_{\bar{q}}) \cdot \zeta]\!]_p$$

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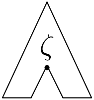
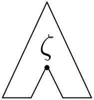
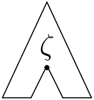


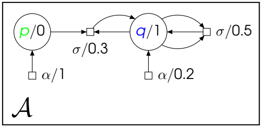
e_p

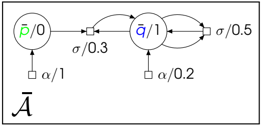


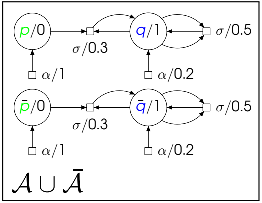
$e_{\bar{q}}$



$e_p + e_{\bar{q}}$







$$T_{p,q} = \{[\![(e_p + e_{\bar{q}}) \cdot \zeta]\!] \mid \zeta \in C_\Sigma\}$$

$u \in T_{p,q}$ counterexample: $u_p \neq 0, u_{\bar{q}} \neq 0, u_p \neq u_{\bar{q}}$

The Regularity Property

Recall: $T_{p,q} = \{[\![(e_p + e_{\bar{q}}) \cdot \zeta]\!] \mid \zeta \in C_\Sigma\}$
counterexample: $u_p \neq 0, u_{\bar{q}} \neq 0, u_p \neq u_{\bar{q}}$

Let $\mathcal{A}$ have the twins property and $(p, q) \in \textsc{Siblings}(\mathcal{A})$.

Then there is a finite set $S' \subseteq S^{Q \cup \bar{Q}}$ with $T_{p,q} \subseteq S \cdot S'$.

(Lemma 3.3)

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(Lemma 3.3)

Then $f(T_{p,q} \setminus \{\tilde{0}\}) \subseteq f(S \cdot S' \setminus \{\tilde{0}\}) \subseteq f(S' \setminus \{\tilde{0}\})$.

(Lemma 3.4)

The Regularity Property

Recall:	$T_{p,q} = \{[\![(e_p + e_{\bar{q}}) \cdot \zeta]\!] \mid \zeta \in C_\Sigma\}$ counterexample: $u_p \neq 0, u_{\bar{q}} \neq 0, u_p \neq u_{\bar{q}}$

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A vector u is a counterexample iff so is $f(u)$.

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(Lemma 3.4)

A vector u is a counterexample iff so is $f(u)$.

Roughly: $S' \approx \{[\![(e_p + e_{\bar{q}}) \cdot \zeta]\!] \mid \text{height}(\zeta) \leq 2|Q|^{2|Q|}\}$

Proof – Victorious Runs

$$[\![\xi]\!] = \begin{pmatrix} [\![\xi]\!]_{q_1} \\ \vdots \\ [\![\xi]\!]_{q_k} \end{pmatrix}$$

weight
vector

Proof – Victorious Runs

$$[\![\xi]\!] = \begin{pmatrix} [\![\xi]\!]_{q_1} \\ \vdots \\ [\![\xi]\!]_{q_k} \end{pmatrix} \qquad \begin{pmatrix} \kappa_{q_1} \; \triangle \\ \vdots \\ \kappa_{q_k} \; \triangle \end{pmatrix}$$

weight vector

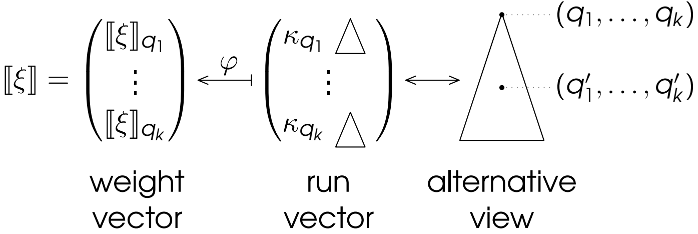
run vector

Proof – Victorious Runs

$$[\![\xi]\!] = \begin{pmatrix} [\![\xi]\!]_{q_1} \\ \vdots \\ [\![\xi]\!]_{q_k} \end{pmatrix} \xleftarrow{\varphi} \begin{pmatrix} \kappa_{q_1} \; \triangle \\ \vdots \\ \kappa_{q_k} \; \triangle \end{pmatrix}$$

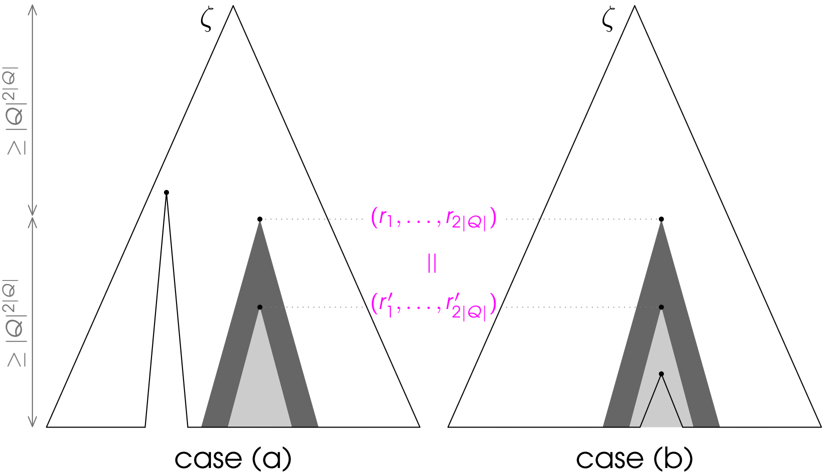
weight vector — run vector

Proof – Victorious Runs



Proof – Smallest Counterexample

Recall: $T_{p,q} = \{[\![(e_p + e_{\bar{q}}) \cdot \zeta]\!] \mid \zeta \in C_\Sigma\}$ $T_{p,q} \subseteq S \cdot S'$
$S' \approx \{[\![(e_p + e_{\bar{q}}) \cdot \zeta]\!] \mid \text{height}(\zeta) \leq 2|Q|^{2|Q|}\}$



Outline

Determinization of Weighted Tree Automata

Deciding the Twins Property

Conclusion

Conclusion

- twins property decidable
- complexity: PSPACE-hard (Kirsten 2012)
- search on compressed search space
- beyond extremal, beyond twins property?

Literature

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