

# Unranked Attributed Tree Transducers

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# Outline

## Context-sensitive Tree Transformations

- XML Application

- Attributes (Ranked Trees)

## Unranked Attributed Tree Transducers

- Attributes (Unranked Trees)

- Generators

- Unranked Attributed Tree Transducers

## Results and Open Questions

# XML Scenario: Bibliography

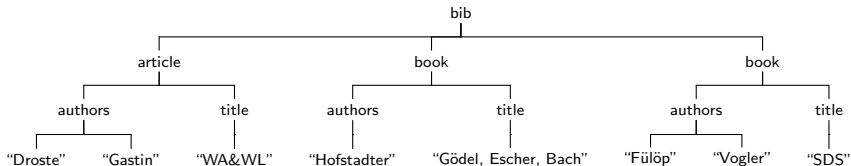


Figure: An XML database.

# XML Scenario: Bibliography

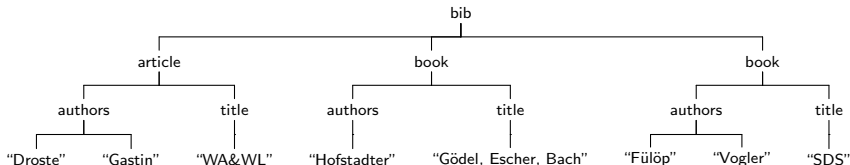


Figure: An XML database.

## Query

Get titles of entries with more than one author.



## XML Scenario: Bibliography

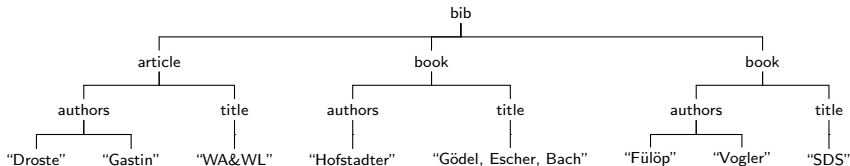
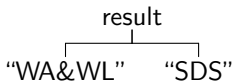


Figure: An XML database.

Answer





## XML Scenario: Bibliography

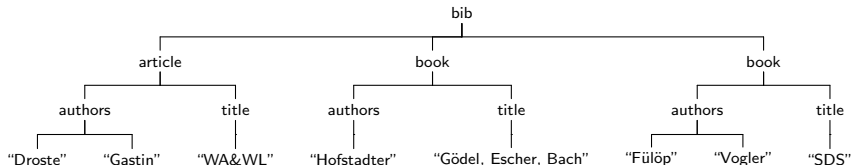


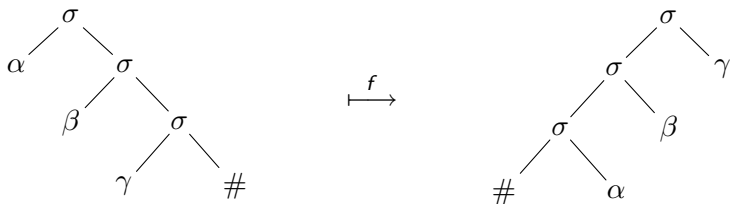
Figure: An XML database.

### Observation

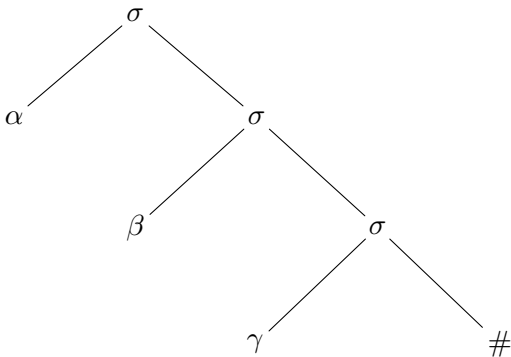
XML queries are unranked tree transformations!

# Attributes

## Motivation

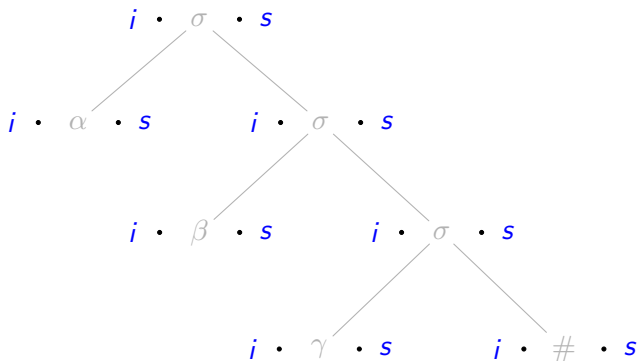


# Attributes and Attribute Instances



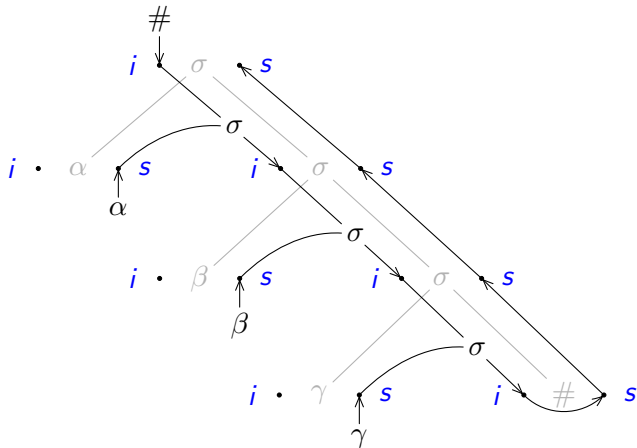
# Attributes

## and Attribute Instances



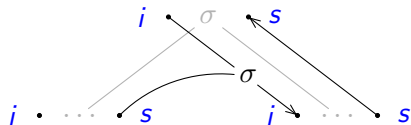


# Attributes and Attribute Instances



# Attributes

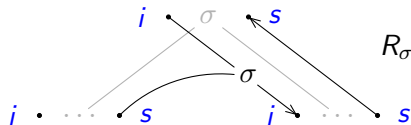
## and Attribute Occurrences





# Attributes

## and Attribute Occurrences



$$R_\sigma: (s, \pi) \rightarrow (s, \pi_2)$$

$$(i, \pi_2) \rightarrow \sigma[(i, \pi) (s, \pi_1)]$$



$$R_\alpha: (s, \pi) \rightarrow \alpha$$

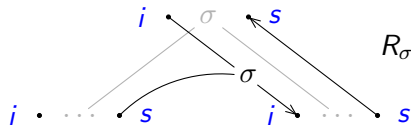


$$R_\#: (s, \pi) \rightarrow (i, \pi)$$



# Attributes

## and Attribute Occurrences



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$$R_\#: (s, \pi) \rightarrow (i, \pi)$$

$$(i, \varepsilon)$$

$$E(i) = \#()$$

# Attributes

- **Attribute Grammars [Knu68]**
- Attributed Tree Transducers [Fül81, FV98]
- Macro Attributed Tree Transducers [KV94, FV98]

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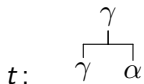
## Basic Notions and Notations

$\Sigma$	alphabet	$\Sigma = \{\alpha, \beta, \gamma\}$
$U_\Sigma$	unranked trees	$t = \gamma[\gamma[] \alpha[]] \in U_\Sigma$
$H_\Sigma$	hedges	$h = [\gamma[\gamma[] \alpha[]] \alpha[\beta[]]] \in H_\Sigma$
$\text{pos}(t)$	tree positions	$\text{pos}(t) = \{\varepsilon, 1, 2\}$
$\text{hpos}(h)$	hedge positions	$\text{hpos}(h) = \{1, 11, 12, 2, 21\}$

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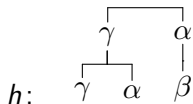
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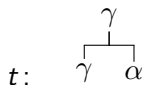
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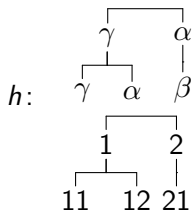
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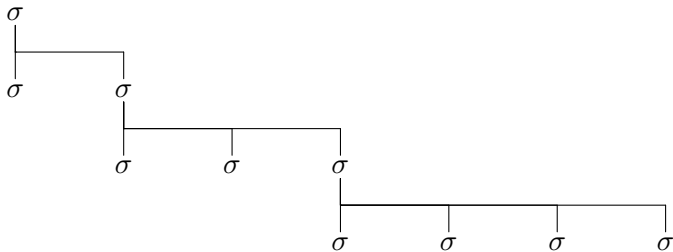
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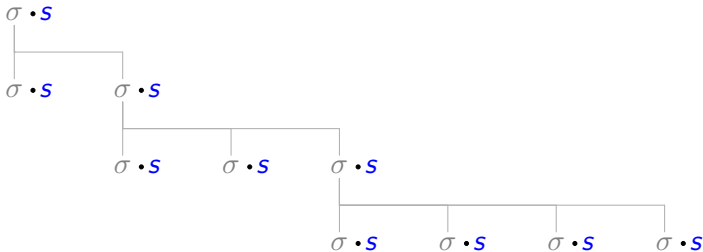
# Attributes and Unranked Trees

Example: Computing Identity



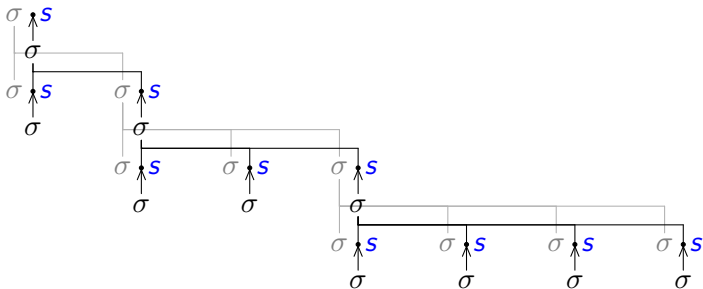
# Attributes and Unranked Trees

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# Attributes and Unranked Trees

## Example: Computing Identity



### Goal

$$(s, \pi) \rightarrow f_{s,\sigma}(k) \in "R_{\sigma,k}",$$

$$\text{where } f_{s,\sigma}(k) = \sigma[(s, \pi 1) \dots (s, \pi k)]$$

## Generators

- finite description of mapping  $f_{s,\sigma} : \mathbb{N} \rightarrow \text{RHS}$
- two-way string-to-hedge device
- encode rank  $k$ :  $\triangleright \square^k \triangleleft$

### Example: Specifying $f_{s,\sigma}$ as Generator

$\triangleright \square \square \square \triangleleft$   
 $q_0$

$(q_0, \triangleright) \mapsto [\sigma[(q_0, r)]]$   
 $(q_0, \square) \mapsto [s(q_0, r)]$   
 $(q_0, \triangleleft) \mapsto []$

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$\triangleright \square \square \square \triangleleft$

$q_0$

$[(q_0, 0)]$

$(q_0, \triangleright) \mapsto [\sigma[(q_0, r)]]$

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$[\sigma[(q_0, 1)]]$

$(q_0, \triangleright) \mapsto [\sigma[(q_0, r)]]$

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$[\sigma[(s, \pi 1) (q_0, 2)]]$

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### Example: Specifying $f_{s,\sigma}$ as Generator

$\triangleright \square \square \square \triangleleft$   
 $q_0$

$[\sigma[(s, \pi_1)(s, \pi_2)(q_0, 3)]]$

$(q_0, \triangleright) \mapsto [\sigma[(q_0, r)]]$

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$\triangleright \square \square \square \triangleleft$   
 $q_0$

$[\sigma[(s, \pi_1)(s, \pi_2)(s, \pi_3)(q_0, 4)]]$

$(q_0, \triangleright) \mapsto [\sigma[(q_0, r)]]$

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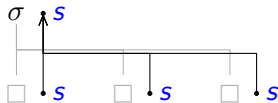
### Example: Specifying $f_{s,\sigma}$ as Generator

$\triangleright \square \square \square \triangleleft$

$[\sigma[(s, \pi_1) (s, \pi_2) (s, \pi_3)]]$

# Generators

## Synthesized Attributes

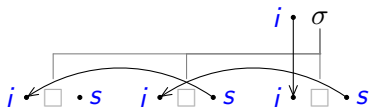


$$(s, \pi) \rightarrow \sigma[(s, \pi 1) \dots (s, \pi k)]$$



# Generators

## Inherited Attributes

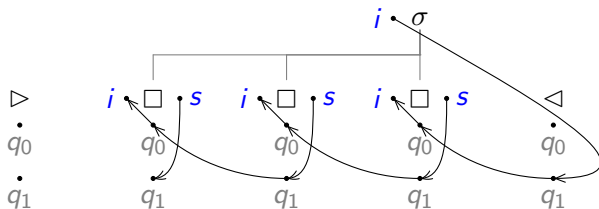


for  $1 \leq j < k$ :  $(i, \pi_j) \rightarrow (s, \pi(j+1))$

if  $k > 0$ :  $(i, \pi_k) \rightarrow (i, \pi)$

# Generators

## Inherited Attributes



$$(q_0, \square) \mapsto [(q_1, r)]$$

$$(q_1, \square) \mapsto [s]$$

$$(q_1, \triangleleft) \mapsto [i]$$

# Generators

## Definition Sketch

$G = (Q, q_0, \rho)$  generator over  $A$  and  $\Delta$  iff

- $Q$  finite set (states),
- $q_0 \in Q$  (initial state),
- $\rho : Q \times \{\triangleright, \square, \triangleleft\} \rightarrow H_\Delta(Q \times \{l, r\} \cup A)$  (transition function);

$$\rho(Q \times \{\triangleright\}) \subseteq H_\Delta(Q \times \{r\} \cup A_{\text{inh}}) \text{ and}$$

$$\rho(Q \times \{\triangleleft\}) \subseteq H_\Delta(Q \times \{l\} \cup A_{\text{inh}}).$$

semantics  $\llbracket G \rrbracket : \mathbb{N} \times \mathbb{N} \rightarrow H_\Delta([A, \_])$  partial

$$\llbracket G \rrbracket(k, i) = \xi$$

iff  $i \leq k$ ,  $\xi \in H_\Delta([A, k])$ , and  $(q_0, i) \Rightarrow_{G,k}^* \xi$ .

## Unranked Attributed Tree Transducer

$(A_{\text{syn}}, A_{\text{inh}}, \Sigma, \Delta, a_0, R, E)$  **uatt** iff

- $A_{\text{syn}}$  finite set (**synthesized attributes**),
- $A_{\text{inh}}$  finite set (**inherited attributes**),
- $\Sigma$  finite set (**input alphabet**),
- $\Delta$  finite set (**output alphabet**),
- $a_0 \in A_{\text{syn}}$  (**initial attribute**),
- $R = (R_{a,\sigma})_{a \in A, \sigma \in \Sigma}$  family of generators over  $A$  and  $\Delta$ ,
- $E : A_{\text{inh}} \rightarrow H_{\Delta}$  (**environment**).

$$(A = A_{\text{syn}} \cup A_{\text{inh}})$$

## Result (I)

$$\text{ATT}(\Sigma, \Delta) \hookrightarrow \text{uATT}(\Sigma, \Delta)$$

For every  $\tau_1 \in \text{ATT}(\Sigma, \Delta)$ , there is  $\tau_2 \in \text{uATT}(\Sigma, \Delta)$  such that  $\tau_2(t) = [\tau_1(t)]$  for every  $t \in \mathbb{T}_\Sigma$ .



## References



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On attributed tree transducers.

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