EXERCISE 6 (DETERMINISTIC BU-TA I)
Let $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}\}$. For each of the following tree languages give a deterministic bu-ta which accepts exactly that language.

1. $P = \{\xi \in T_\Sigma \mid \xi \text{ contains at least one } \alpha \text{ and one } \beta\}$
2. $P = \{\xi \in T_\Sigma \mid \xi \text{ contains an even number of } \alpha \text{ symbols}\}$

EXERCISE 7 (DETERMINISTIC BU-TA II)
Let $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}\}$. For every $k \in \mathbb{N}$, we define the tree $\xi_k \in T_\Sigma$ by $\xi_0 = \beta$ and $\xi_{k+1} = \sigma(\alpha, \xi_k)$ for every $k \in \mathbb{N}$. In addition, we define the property $P = \{\xi \in T_\Sigma \mid \exists k \in \mathbb{N}: \xi = \xi_k\}$.

Moreover, we define the deterministic bu-ta $A = (Q, \Sigma, \delta, F)$ where $Q = \{l, r, \bot\}$, $F = \{r\}$, $\delta(\alpha()) = l$, $\delta(\beta()) = r$, and for every $(q_1, q_2) \in Q \times Q$

$$
\delta_\sigma(q_1, q_2) = \begin{cases} r & \text{if } (q_1, q_2) = (l, r), \\ \bot & \text{otherwise.} \end{cases}
$$

Show the following statements.

1. $P \subseteq L_A$.
2. $L_A \subseteq P$.

EXERCISE 8 (NONDETERMINISTIC BU-TA)
Let $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}, \gamma^{(0)}\}$. Give a nondeterministic bu-ta which accepts exactly the language of all $t \in T_\Sigma$ containing a $\beta$-leaf somewhere between an $\alpha$- and a $\gamma$-leaf, reading leaves left-to-right or right-to-left. Try to use as few states and transitions as possible.

EXERCISE 9 (NONDETERMINISM AND DETERMINIZATION)
Let $\Sigma = \{\alpha^{(0)}, \gamma^{(1)}, \sigma^{(2)}\}$. Recall from the lecture the language

$$
L = \{\xi \in T_\Sigma \mid \forall w \in \text{pos}(\xi): \xi(w) = \gamma \Rightarrow \xi|_w \in T_{\Sigma \setminus \Sigma^{(2)}}\}
$$
of all trees where for every path from the root to some leaf, the rank of the nodes along the path is monotonic decreasing.

1. Construct a bu-ta $A$ accepting $L$ using the strategy of guessing and checking. Your automaton should guess whether it has already seen every gamma on the path to the root and choose the successor state accordingly. Later on, it should verify that the guess was correct. Naturally, if the guess was “no”, then a $\sigma$ should lead to failure.
2. Use the powerset construction to obtain a deterministic version of $A$. 

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Exercise 10 (Nondeterministic td-ta)
For each of the following tree languages, give a td-ta which accepts exactly that language. Which of these languages can be accepted by some deterministic td-ta?

1. $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}\}$ and
$L = \{\xi \in T_{\Sigma} \mid \xi \text{ contains at least one } \alpha \text{ and one } \beta\}.$

2. $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}\}$ and
$L = \{\xi \in T_{\Sigma} \mid \xi \text{ contains an even number of } \alpha \text{ symbols}\}.$

3. $\Sigma = \{\alpha^{(1)}, \beta^{(1)}, \gamma^{(1)}, \epsilon^{(0)}\}$ and
$L = \{\xi \in T_{\Sigma} \mid \xi \text{ contains an } \alpha \text{ somewhere above a } \beta \text{ or a } \beta \text{ somewhere above a } \gamma\}.$