3rd Exercise Sheet (April 25, 2013)

Weighted Tree Automata

**Exercise 10 (Run semantics)**
1. Consider the automaton $A$ from Ex. 9(1) and a tree $\xi = \sigma(\gamma(\alpha), \alpha)$. Determine the set of runs $R_A(\xi)$ of $A$ on $\xi$, as well as the set of valid runs $R_v^A(\xi)$.

2. Show for every r.a. $\Sigma$, bu-det FTA $A$ over $\Sigma$ and tree $\xi \in T_\Sigma$, that if $\xi \in L(A)$, then $|R_v^A(\xi)| = 1$.

**Exercise 11 (Regular tree grammars)**
1. Let $\Sigma = \{\sigma(2), \gamma(1), \alpha(0)\}$. Give regular tree grammars $G_1, G_2$ which generate the languages $L_1 = \{\xi \in T_\Sigma \mid \xi \text{ contains exactly one } \sigma\}$, respectively $L_2 = \{\xi \in T_\Sigma \mid \xi \text{ contains the pattern } \sigma(\cdot, \gamma(\cdot)) \text{ at least twice}\}$.

2. Let $G = (N, \Sigma, Z, P)$ be a regular tree grammar with $N = \{Z, A, B, C\}$, $\Sigma = \{\sigma(2), \alpha(0), \beta(0)\}$, and the set of productions $P$ given by

   $Z \to \sigma(\sigma(A, B), C), \quad Z \to B, \quad A \to \alpha, \quad A \to B,$

   $B \to \beta, \quad B \to A, \quad B \to C, \quad C \to C.$

   Use the construction from the lecture to give a regular tree grammar in normal form equivalent to $G$.

**Exercise 12 (Final-state normal form)**
Consider the language $L$ from Ex. 9.

1. Prove or refute: There is a bottom-up deterministic FTA $A = (Q, \Sigma, \delta, F)$ with $|F| = 1$ such that $L(A) = L$.

2. Construct an FTA $B$ in final-state normal form that is equivalent to the solution of Ex. 9.

**Exercise 13 (Relatedness)**
1. Give an FTA $A_1$ that is related to the regular tree grammar $G_1$ from Exercise 11(1).

2. Give a regular tree grammar which is related to the FTA $B$ from Exercise 12(2).