Weighted Tree Automata

***Exercise 14* (yield(Rec) ⊆ CF)**

Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}, \beta^{(0)}, \lambda^{(0)}\}$ be a r.a., and consider the rtg $G = (N, \Sigma, Z, P)$ with $N = \{Z, A, B, C, D, E\}$ and the productions in P given by

$$\begin{array}{llll} Z \to \sigma(A,B) & A \to \gamma(C) & B \to \sigma(E,E) & E \to \beta \\ Z \to \lambda & C \to \sigma(D,Z) & D \to \alpha\ . & \end{array}$$

1. Of which form are the trees in $L(G)$? Give the (string) languages $\text{yield}_\lambda(L(G))$ and $\text{yield}_\alpha(L(G))$.
2. Construct a CFG G' that is λ-related to G.

***Exercise 15* (CF ⊆ yield(Rec))**

Let $\Sigma = \{[,], \langle, \rangle\}$ and assume the context-free grammar $G = (N, \Sigma, Z, P)$ with $N = \{Z\}$ and the productions $Z \to ZZ$, $Z \to [Z]$, $Z \to \langle Z \rangle$, and $Z \to \varepsilon$ in P.

1. Construct an equivalent CFG G' in normal form.
2. Find a regular tree grammar H over some r.a. Δ such that $\text{yield}_e(L(H)) = L(G')$ for an $e \in \Delta$.

By the way: yield_e is a homomorphism. Which is its target algebra?