Weighted Tree Automata

Exercise 20 (Closure properties of Rec)
We consider the r.a. \( \Sigma = \{ \sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)} \} \) and the two bu-deterministic FTA \( A_1, A_2 \) where

- \( A_1 = (Q_1, \Sigma, \delta, F_1) \) with \( Q_1 = \{ e, o \} \), \( F_1 = \{ e \} \), and

  \[
  \delta_a(q_1, q_2) = \begin{cases} 
  o & \text{if } q_1q_2 \in \{ oo, ee \} \\
  e & \text{otherwise}
  \end{cases}
  \]

  for every \( q, q_2, q_2 \in Q_1 \), and

- \( A_2 = (Q_2, \Sigma, \mu, F_2) \) with \( Q_2 = \{ 0, 1, 2 \} \), \( F_2 = \{ 2 \} \) and

  \[
  \mu_a() = 0, \quad \mu_q(q) = \begin{cases} 
  2 & \text{if } q = 2 \\
  q + 1 & \text{otherwise}
  \end{cases}
  \]

  \[
  \mu_{\sigma}(q_1, q_2) = \max\{q_1, q_2\}
  \]

  for every \( q, q_2, q_2 \in Q_2 \).

Apply the construction from the lecture to obtain bu-det FTA which accept

1. \( L(A_1) \cup L(A_2) \),
2. \( L(A_1) \cap L(A_2) \), and
3. \( L(A_1) \setminus L(A_2) \).

Exercise 21 (Tree concatenation)
Let \( \Sigma = \{ \sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}, \gamma^{(0)} \} \) and \( \xi = \sigma(\alpha, \alpha, \beta) \in T_\Sigma \). Compute each of the following languages:

1. \( \xi\{ \alpha \leftarrow \gamma \}, \beta \leftarrow \{ \sigma(\alpha, \alpha) \} \),
2. \( \xi\{ \alpha \leftarrow \gamma \}, \beta \leftarrow \emptyset \),
3. \( \xi\{ \gamma \leftarrow \emptyset \} \),
4. \( \xi\{ \alpha \leftarrow \beta, \gamma \}, \beta \leftarrow \{ \gamma \} \),
5. \( \xi \cdot_a \{ \xi \} \),
6. \( \{ \alpha, \sigma(\alpha, \alpha) \} \cdot_a \{ \alpha, \sigma(\alpha, \alpha) \} \).

Recall that \( \xi \cdot_a L \) is an abbreviation for \( \xi\{ \alpha \leftarrow L \} \).

Exercise 22 (Closure of Rec under Kleene star)
1. Why is it not possible to use the closure of Rec under tree concatenation to establish the corresponding closure under Kleene star?
2. Why is it not possible to (further) simplify the construction from the lecture by mapping \( G = (N, \Sigma, Z, R) \) to \( G' = (N, \Sigma, Z, R') \) with \( R' = R \cup \{ A \rightarrow Z \mid A \rightarrow \alpha \text{ in } R \} \cup \{ Z \rightarrow \alpha \} \)?

Exercise 23 (Normal form of context-free grammars)
In the lecture, an algorithm was presented which, given as input a CFG \( G \), computes an equivalent CFG \( G' \) which is in normal form. Find a closed-form solution for \( G' \).