Weighted Tree Automata

Exercise 31 (Theorem of Kleene)

1. Let $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}\}$. Using the constructions from the lecture, show that $\{\sigma(\alpha,\beta)\}^*_\beta \cdot_\beta \{\alpha\}$ is in Rec.

2. Let $\Sigma = \{\sigma_1^{(2)}, \sigma_2^{(2)}, \alpha_1^{(0)}, \alpha_2^{(0)}\}$ and $G = (\{A_1,A_2\}, \Sigma, A_1, R)$ be a regular tree grammar with

$$R = \{A_1 \to \alpha_1,\, A_1 \to \sigma_1(A_1,A_2),\, A_2 \to \alpha_2,\, A_2 \to \sigma_2(A_2,A_1)\}\,.$$

   Using the technique from the lecture, show that $L(G)$ is in Rat.

Exercise 32 (Myhill-Nerode theorem)

1. Let $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}\}$ and $L \subseteq T_\Sigma$ be the language consisting of all trees with exactly as many α as β symbols. Use the Myhill-Nerode theorem to show that L is not recognizable.

2. But yield(L) is a context-free language! Shouldn't L be recognizable by Thatcher's theorem yield(Rec) = CF?

Exercise 33 (Monadic second-order logic)

1. Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}, \beta^{(0)}\}$ and $\mathcal{V} = \{x, y, X\}$. Construct MSO-formulas φ_1, φ_2, φ_3 s.t. for every $\xi \in T_\Sigma$ and $\mathcal{V}$-assignment ρ for ξ,

   - $(\xi,\rho) \models \varphi_1$ iff there is a downward path from the node $\rho(x)$ to the node $\rho(y)$ in ξ, i.e. there is a $w \in \mathbb{N}^*$ such that $\rho(y) = \rho(x)w$,
   - $(\xi,\rho) \models \varphi_2$ iff $\rho(X)$ is the set of all positions w in ξ such that $\xi|_w = \sigma(\alpha,\beta)$,
   - $(\xi,\rho) \models \varphi_3$ iff for every node in ξ labeled by σ, none of its child nodes is labeled by γ,

   and Free(φ_i) $\subseteq \mathcal{V}$ for $i \in \{1,2,3\}$.

2. Consider the bu-deterministic fta $\mathcal{A} = (Q, \Sigma, \delta, F)$ with $Q = \{0,1\}$, $F = \{1\}$, $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$, and $\delta_\sigma(q_1,q_2) = \max\{q_1,q_2\}$ for every q_1, $q_2 \in Q$, $\delta_\gamma(q) = 1$ for every $q \in Q$, and $\delta_\alpha() = 0$.

   Use the construction from the lecture to show that $L(\mathcal{A})$ is MSO-definable.

3. Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$ and $\varphi = \exists x.\text{label}_\gamma(x)$. Use the construction from the lecture to show that $L(\varphi) \in \text{Rec}(\Sigma)$.