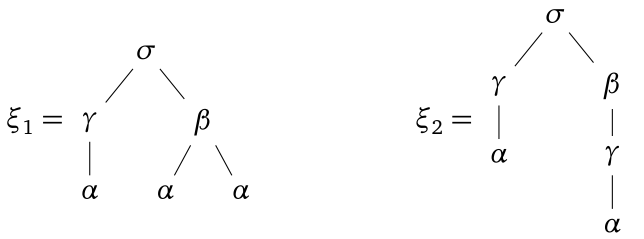
Advanced Topics on Weighted Tree Automata

Exercise 1 (Ranked alphabets and trees)

Consider the following trees:



(a) Give $\mathrm{height}(\xi_i)$, $\mathrm{size}(\xi_i)$, $\mathrm{pos}(\xi_i)$, $\mathrm{sub}(\xi_i)$ for $i \in \{1, 2\}$.

(b) Define minimal ranked alphabets Δ_1 and Δ_2 such that $\xi_1 \in T_{\Delta_1}$ and $\xi_2 \in T_{\Delta_2}$.

(c) Prove or refute: There is a ranked alphabet Σ' such that $\xi_1, \xi_2 \in T_{\Sigma'}$.

Exercise 2 (Definition by structural induction)

Let $\xi \in T_\Sigma$, $w \in \mathrm{pos}(\xi)$, $\zeta \in T_\Sigma(X_k)$, and $\zeta'_1, \ldots, \zeta'_k \in T_\Sigma(A)$. Define the following characteristics of ξ and ζ by structural induction:

(a) $\xi(w)$, the label of ξ at position w,

(b) $\xi|_w$, the subtree of ξ at position w,

(c) $\xi[\zeta]_w$, the tree obtained by substituting the subtree of ξ at position w with ζ,

(d) $\mathrm{yield}(\xi)$, the sequence of leaves of ξ from left to right,

(e) $\zeta[\zeta'_1, \ldots, \zeta'_k]$, the tree obtained from ζ by substituting, for every $i \in \{1, \ldots, k\}$, x_i by ζ'_i.

Exercise 3 (Proof by structural induction)

Let $\xi, \zeta \in T_\Sigma(A)$ and $w \in \mathrm{pos}(\xi)$. Prove or refute the following statements:

(a) $\xi(w) = \xi|_w(\varepsilon)$.

(b) $\big(\xi[\zeta]_w\big)|_w = \zeta$.

(c) $|\mathrm{pos}(\xi)| = |\mathrm{sub}(\xi)|$.

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