Exercise 1 (Ranked alphabets and trees)
Consider the following trees:

\[
\begin{align*}
\xi_1 &= \sigma \quad \gamma \quad \beta \\
&= \alpha \quad \alpha \quad \alpha \\
\xi_2 &= \gamma \quad \beta \\
&= \alpha \quad \gamma \quad \alpha
\end{align*}
\]

(a) Give height(\(\xi_i\)), size(\(\xi_i\)), pos(\(\xi_i\)), sub(\(\xi_i\)) for \(i \in \{1, 2\}\).
(b) Define minimal ranked alphabets \(\Delta_1\) and \(\Delta_2\) such that \(\xi_1 \in T_{\Delta_1}\) and \(\xi_2 \in T_{\Delta_2}\).
(c) Prove or refute: There is a ranked alphabet \(\Sigma'\) such that \(\xi_1, \xi_2 \in T_{\Sigma'}\).

Exercise 2 (Definition by structural induction)
Let \(\xi \in T_{\Sigma}, w \in \text{pos}(\xi), \zeta \in T_{\Sigma}(X_k),\) and \(\zeta'_1, \ldots, \zeta'_k \in T_{\Sigma}(A)\). Define the following characteristics of \(\xi\) and \(\zeta\) by structural induction:

(a) \(\xi(w)\), the label of \(\xi\) at position \(w\),
(b) \(\xi|_w\), the subtree of \(\xi\) at position \(w\),
(c) \(\xi[\zeta]|_w\), the tree obtained by substituting the subtree of \(\xi\) at position \(w\) with \(\zeta\),
(d) yield(\(\xi\)), the sequence of leaves of \(\xi\) from left to right,
(e) \(\zeta[\zeta'_1, \ldots, \zeta'_k]\), the tree obtained from \(\zeta\) by substituting, for every \(i \in \{1, \ldots, k\}\), \(x_i\) by \(\zeta'_i\).

Exercise 3 (Proof by structural induction)
Let \(\xi, \zeta \in T_{\Sigma}(A)\) and \(w \in \text{pos}(\xi)\). Prove or refute the following statements:

(a) \(\xi(w) = \xi|_w(\varepsilon)\).
(b) \((\xi[\zeta]|_w)|_w = \zeta\).
(c) \(|\text{pos}(\xi)| = |\text{sub}(\xi)|\).