Advanced Topics on Weighted Tree Automata

Exercise 4 (Composition)

Consider the bu-tts $M_{\text{re}} = (\{*\}, \Sigma, \Delta, \{*\}, R_{\text{re}})$ and $M_{\text{re}} = (\{*, n, f\}, \Delta, \Delta, \{*, f\}, R_{\text{ch}})$ where

$$R_{\text{re}} = \{\ \alpha \to *(\alpha),\ \gamma(*(x_1)) \to *(\gamma_f(x_1)),\ \gamma(*(x_1)) \to *(\gamma(x_1)),\ \sigma(*(x_1), *(x_2)) \to *(\sigma(x_1, x_2))\ \}$$

$$\begin{aligned} R_{\text{ch}} = \{\ & \alpha \to *(\alpha),\ \gamma(*(x_1)) \to n(\gamma(x_1)),\ \gamma(n(x_1)) \to n(\gamma(x_1)), \\ & \gamma_f(*(x_1)) \to f(\gamma_f(x_1)),\ \gamma_f(n(x_1)) \to f(\gamma_f(x_1)), \\ & \sigma(*(x_1), *(x_2)) \to *(\sigma(x_1, x_2)),\ \sigma(*(x_1), n(x_2)) \to n(\sigma(x_1, x_2)), \\ & \sigma(*(x_1), f(x_2)) \to f(\sigma(x_1, x_2)),\ \sigma(n(x_1), *(x_2)) \to n(\sigma(x_1, x_2)), \\ & \sigma(n(x_1), n(x_2)) \to n(\sigma(x_1, x_2)),\ \sigma(f(x_1), *(x_2)) \to f(\sigma(x_1, x_2)), \\ & \sigma(f(x_1), n(x_2)) \to f(\sigma(x_1, x_2))\ \} \end{aligned}$$

(a) Describe the transformations induced by M_{re} and M_{ch}.

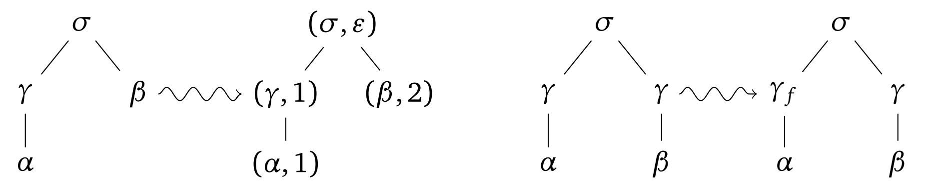
(b) Describe the transformation $\tau(M_{\text{re}}) \circ \tau(M_{\text{ch}})$.

(c) Give a bu-tt M such that $\tau(M) = \tau(M_{\text{re}}) \circ \tau(M_{\text{ch}})$.

Exercise 5 (Relabeling and checking)

(a) Give a bu-tt M_1 that, for every tree $\xi \in T_\Sigma$, enhances for every position $w \in \text{pos}(\xi)$ the label at w with the last digit of w.

(b) Let $\gamma \in \Sigma$. Give a bu-tt M_2 that, for every tree $\xi \in T_\Sigma$, replaces the first occurrence (according to depth-first order) of γ in ξ by γ_f without changing the rest of ξ.



(a) transformation $\tau(M_1)$ (b) transformation $\tau(M_2)$

Exercise 6 (Generalized sequential machines and bu-tt)

Let $G = (Q, \Sigma, \Delta, q_0, F, R)$ be a gsm. Give bu-tts that simulate the run of G

(a) on the nodes of monadic trees from front to root.

(b) on the front of trees from left to right.

Solution for Exercise 6:

(b) Use the usual Bar-Hillel Perles Shamir construction: Since G can replace one input symbol by several output symbols, we introduce a new symbol $@_l$ for every fan-out l of the rules in G. Also the constructed bu-tt has to guess the previous state at the leaves, since it can only see one input symbol. At each symbol there is a tuple of states, the left projection represents the source state and the right projection represents the target state. At each node (except the leaves), the neighboring states (right state of left child and left state of right child) of the children are then checked to be equal.

We define

$M_b = (Q \times Q, \Sigma', \Delta' \cup \{@_l \mid l \in [\max\{|w| \mid (p\alpha \to wq) \in R\}]\}, \{(q_0, q_f) \mid q_f \in F\}, R_b)$, where

$$\begin{aligned}R_b = \{&\alpha \to (p,q)(@_l(w_1,\dots,w_l)) \mid p\alpha \to w_1 \cdots w_k q \in R, w_1,\dots,w_k \in \Sigma\}\\ &\cup \{\sigma((p_1,q_1)(x_1),\dots,(p_k,q_k)(x_k)) \to (p_1,q_k)(\sigma(x_1,\dots,x_k))\\ &\quad \mid \sigma \in \Gamma \setminus \Sigma, q_1 = p_2, q_2 = p_3, \dots, q_{k-1} = p_k\}\end{aligned}$$

http://www.orchid.inf.tu-dresden.de Tobias.Denkinger@tu-dresden.de