Exercise 7 (Composition and decomposition)
Let $M = (Q, \Sigma, \Delta, F, R)$ be a bu-tt. Prove the following statements:
(a) If $M$ is a bottom-up state relabeling and $(s, t) \in \tau(M)$, then $\text{pos}(s) = \text{pos}(t)$.
(b) If $M$ is a bottom-up finite state tree automaton and $(s, t) \in \tau(M)$, then $s = t$.
(c) There exists a $c \in \mathbb{N}$ such that for every $(s, t) \in \tau(M)$ holds $\text{height}(t) \leq c \cdot \text{height}(s)$.

Exercise 8 (Subclasses of BOT)
Describe the relations of the classes of relations BOT, d-BOT, t-BOT, HOM, QREL, REL, l-BOT, n-BOT, and FTA by means of a set diagram (also known as Venn diagram).

Exercise 9 (Nondeterminism and determinism)
Let $\Sigma = \{\top^{(0)}, \bot^{(0)}, \neg^{(1)}, \wedge^{(2)}\}$ be a ranked alphabet. Note that the trees over $\Sigma$ represent a subset of the formulae of propositional logic.
(a) Give a bu-tt $M$ that eliminates double negations, i.e., for every $\xi \in T_\Sigma$, $M$ reduces every subtree of the form $\neg(\neg(\xi))$ to $\xi$.
(b) Give a deterministic bu-tt $M_{det}$, such that $\tau(M_{det}) = \tau(M)$.
(c) For any given bu-tt $M'$, is there a deterministic bu-tt $M'_{det}$ such that $\tau(M'_{det}) = \tau(M')$?
(d) What restriction is required from $M'$ in order for $M'_{det}$ to exist?