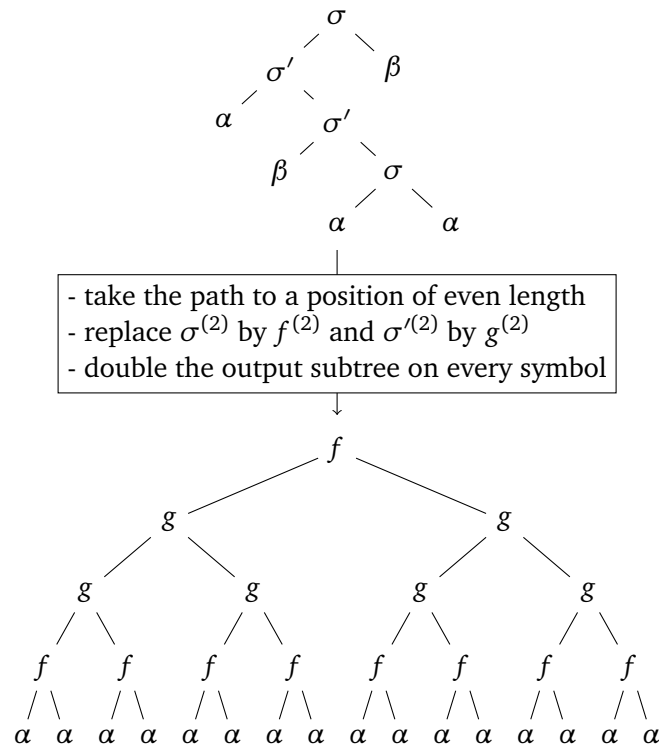


# Advanced Topics on Weighted Tree Automata

*Exercise 10 (BOT  $\subseteq$  REL  $\circ$  FTA  $\circ$  HOM)*

Let  $\Sigma = \{\sigma^{(2)}, \sigma'^{(2)}, \alpha^{(0)}, \beta^{(0)}\}$  and  $\Delta = \{f^{(2)}, g^{(2)}, \alpha^{(0)}, \beta^{(0)}\}$ . We consider the tree transformation  $\tau \subseteq T_\Sigma \times T_\Delta$  such that  $(s, t) \in \tau$  if there is a position  $w = w_1 \dots w_n \in \text{pos}(s)$  with even length (viewed as string over  $\mathbb{N}$ ) such that  $s(w) \in \Sigma^{(0)}$  and  $t$  is obtained from  $s$  by walking up the path starting from  $w$  towards the root of  $s$  and at  $w$  outputting  $s(w)$  and at each position  $w_1 \dots w_i$  with  $1 \leq i < n$  (a) replacing  $\sigma$  by  $f$  and  $\sigma'$  by  $g$  and (b) copying the output subtree computed at  $w_1 \dots w_{i+1}$ .



- (a) Give a bu-tt  $B$  such that  $\tau = \tau(B)$ .
- (b) Give a relabeling  $B_1$ , a bottom-up finite state tree automaton  $B_2$ , and a tree homomorphism  $B_3$  such that  $\tau(B) = \tau(B_1) \circ \tau(B_2) \circ \tau(B_3)$ .
- (c) Give a derivation of  $\sigma(\sigma(\sigma(\alpha, \alpha), \alpha), \alpha)$  in  $B$  and the related derivations in  $B_1$ ,  $B_2$ , and  $B_3$ .