Advanced Topics on Weighted Tree Automata

Exercise 11 (Powerset Construction)

Let $\Sigma = \{\alpha^{(0)}, \sigma^{(2)}\}$ be a ranked alphabet. Consider the bottom-up finite state tree automaton $N = (\{q_0, q_1\}, \Sigma, \Sigma, \{q_0\}, R)$ where

$$R = \left\{\alpha \to q_0(\alpha)\right\} \cup \left\{\sigma(q_i(x_1), q_j(x_2)) \to q_{1-I(k)}(\sigma(x_1, x_2)) \;\middle|\; i, j \in \{0,1\}, k \in [2], I = \begin{pmatrix} i & j \end{pmatrix}^T \right\}$$

Use the powerset construction to give a deterministic bottom-up finite state tree automaton N_{det} such that $\tau(N) = \tau(N_{\text{det}})$.

Exercise 12 (BOT ∘ FTA ⊆ BOT and BOT ∘ HOM ⊆ BOT)

Let $B = (Q, \Sigma, \Delta, F, R)$ be a bu-tt, $N = (P, \Delta, \Delta, F_N, R_N)$ a deterministic bottom-up finite state tree automaton, and $H = (\{*\}, \Delta, \Omega, \{*\}, R_H)$ a tree homomorphism. Also let $X_{\max} = \{x_i \mid i \in [\max \operatorname{rk}(\Sigma)]\}$.

(a) Consider the bottom-up finite state tree automaton $N' = (P, \Delta, \Delta \cup X_{\max}, F_N, R_N)$ and the bu-tt $\bar{B} = (Q \times P, \Sigma, \Delta, F \times F_N, \bar{R})$ where for every $k \in \mathbb{N}$ and $p, p_1, \ldots, p_k \in P$ we have

$$\begin{aligned} &\sigma(q_1(x_1), \ldots, q_k(x_k)) \to q(t') \in R \wedge t'[p_1(x_1), \ldots, p_k(x_k)] \Rightarrow^*_{N'} p(t') \\ \Longleftrightarrow\; &\sigma((q_1, p_1)(x_1), \ldots, (q_k, p_k)(x_k)) \to (q, p)(t') \in \bar{R}\,. \end{aligned}$$

Show that for every $s \in T_\Sigma$, $q \in Q$, $q \in P$, and $t \in T_\Delta$ the following equivalence holds:

$$s \Rightarrow^*_{\bar{B}} (q, p)(t) \iff s \Rightarrow^*_B q(t) \wedge t \Rightarrow^*_N p(t)\,.$$

(b) Identify the problems that arise in the construction of $\bar{B}$ if N is not deterministic.

(c) Consider the tree homomorphism $H' = (\{*\}, \Delta, \Omega \cup X_{\max}, \{*\}, R_H)$ and the bu-tt $\hat{B} = (Q, \Sigma, \Omega, F, \hat{R})$ where

$$\begin{aligned} &\sigma(q_1(x_1), \ldots, q_k(x_k)) \to q(u') \in R \wedge u'[*(x_1), \ldots, *(x_k)] \Rightarrow^*_{H'} *(t') \\ \Longleftrightarrow\; &\sigma(q_1(x_1), \ldots, q_k(x_k)) \to q(t') \in \hat{R}\,. \end{aligned}$$

Show that for every $s \in T_\Sigma$, $q \in Q$, and $t \in T_\Delta$ the following equivalence holds:

$$s \Rightarrow^*_{\hat{B}} q(t) \iff \exists u \in T_\Delta : s \Rightarrow^*_B q(u) \wedge u \Rightarrow^*_H *(t)\,.$$

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