

## Advanced Topics on Weighted Tree Automata

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*Exercise 13 (Regular tree grammars)*

Consider the ranked alphabet  $\Sigma = \{\alpha^{(0)}, \sigma^{(2)}\}$ , the tree  $\xi = \sigma(\sigma(\alpha, \alpha), \sigma(\alpha, \alpha)) \in T_\Sigma$ , and the regular tree grammar  $G = (\{S, A\}, \Sigma, S, \{S \rightarrow A, S \rightarrow \sigma(S, S), A \rightarrow \alpha, A \rightarrow \sigma(\alpha, S)\})$ .

- (a) Give a derivation and the corresponding derivation tree of  $\xi$  in  $G$ .  
How many derivation trees of  $\xi$  do exist in  $G$ ?
- (b) Give an RTG  $H$  and a tree  $\zeta$  such that  $\zeta$  has infinitely many derivations in  $H$ .
- (c) Give an RTG  $G'$  such that  $G'$  is in normal form and  $L(G') = L(G)$ .  
Give a derivation tree of  $\xi$  in  $G'$ .
- (d) Prove by construction that for every RTG  $G$  there exists an RTG  $G'$  in normal form such that  $L(G') = L(G)$ .