Advanced Topics on Weighted Tree Automata

Exercise 17 (Deletion followed by partialness)

Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{0}\}$ and $\Delta = \{\sigma^{(2)}, \gamma^{(1)}, \gamma'^{(1)}, \alpha^{0}\}$ be ranked alphabets. Consider the tree $\xi = \sigma\big(\gamma(\alpha), \sigma(\alpha, \alpha)\big)$.

(a) The tree transformation τ decides at every σ whether to delete one of the subtrees (replacing σ with γ') or none. If a subtree is deleted it then ensures that the remaining subtree contains a γ. Give a td-tt T such that $\tau = \tau(T)$.

(b) Give all derivations of T for ξ.

Exercise 18 (Decomposition of TOP)

Let $\Sigma = \{\gamma^{(1)}, \alpha^{(0)}\}$ and $\Delta = \{\sigma^{(2)}, O^{(1)}, E^{(1)}, \alpha^{(0)}\}$ be ranked alphabets. Furthermore let $\xi = \gamma\big(\gamma(\alpha)\big)$.

(a) Give a td-tt T such that $\tau(T)$ transforms every tree in T_Σ into a tree in T_Δ such that each γ is replaced by σ where the subtree of γ is copied and, starting with O at the top, alternately O and E are inserted before each symbol.
Give a derivation of T for ξ.

(b) Give a top-down tree homomorphism H and a linear top-down tree transducer T' such that $\tau(T) = \tau(H) \circ \tau(T')$.
Give derivations of H and T' for ξ.

Exercise 19 (ln-BOT = ln-TOP)

Let $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}\}$, $\Delta = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$ be ranked alphabets, and $\xi = \sigma\big(\sigma(\alpha, \alpha), \alpha\big) \in T_\Sigma$. Consider the linear non-deleting bu-tt $B = \big(\{q_0, q_1\}, \Sigma, \Delta, \{q_0\}, R_B\big)$ and the linear non-deleting td-tt $T = \big(\{q_0, q_1\}, \Sigma, \Delta, \{q_0\}, R_T\big)$ where

$$\begin{aligned} R_B = \{\ & \alpha \to q_0(\alpha), \quad \sigma(q_0(x_1), q_0(x_2)) \to q_1(\sigma(x_1, x_2)), \\ & \alpha \to q_1(\alpha), \quad \sigma(q_1(x_1), q_1(x_2)) \to q_0(\gamma(\sigma(x_2, x_1)))\ \} \quad \text{and} \end{aligned}$$

$$\begin{aligned} R_T = \{\ & q_0(\alpha) \to \alpha, \quad q_0(\sigma(x_1, x_2)) \to \sigma(q_1(x_2), q_1(x_1)), \\ & q_1(\alpha) \to \alpha, \quad q_1(\sigma(x_1, x_2)) \to \sigma(\gamma(q_0(x_1)), \gamma(q_0(x_2)))\ \} \end{aligned}$$

(a) Give a linear non-deleting td-tt T' such that $\tau(B) = \tau(T')$.
Give derivations of B and T' on ξ.

(b) Give a linear non-deleting bu-tt B' such that $\tau(T) = \tau(B')$.
Give derivations of T and B' on ξ.