9th Exercise Sheet (December 11, 2013)

Advanced Topics on Weighted Tree Automata

Exercise 20 (h-TOP = HOM and r-TOP = REL)

(a) Prove by construction that h-TOP = HOM.

(b) Prove by construction that r-TOP = REL.

Hint: Define relatedness for a top-down tree homomorphism (relabeling) and a bottom-up tree homomorphism (relabeling). Show that the respective transducers induce the same tree transformation if they are related (Lemma). Use the Lemma to obtain the equivalence of the respective classes.

Exercise 21 (l-TOP ⊂ l-BOT)

Consider the linear td-tt $T = (\{q_0, q_1\}, \Sigma, \Delta, \{q_0\}, R)$ where

$$\begin{aligned} R = \{\ & q_0(\sigma(x_1,x_2)) \to \gamma'(q_1(x_1)), \quad q_0(\sigma(x_1,x_2)) \to \sigma(q_0(x_1), q_0(x_2)), \\ & q_0(\sigma(x_1,x_2)) \to \gamma'(q_1(x_2)), \quad q_1(\sigma(x_1,x_2)) \to \sigma(q_1(x_1), q_1(x_2)), \\ & q_1(\sigma(x_1,x_2)) \to \gamma'(q_1(x_1)), \quad q_0(\gamma(x_1)) \to \gamma(q_0(x_1)), \\ & q_1(\sigma(x_1,x_2)) \to \gamma'(q_1(x_2)), \quad q_1(\gamma(x_1)) \to \gamma(q_0(x_1)), \quad q_0(\alpha) \to \alpha\ \} \end{aligned}$$

Give a linear bu-tt B such that $\tau(T) = \tau(B)$.

Exercise 22 (GSM)

GSM is the class of string transformations $\tau \subseteq \Sigma^* \times \Delta^*$ that are be induced by some gsm.

(a) Give formal definitions for the derivation relation of a gsm and the string transformation induced by a gsm.

(b) Prove by construction that GSM is closed under composition.
Hint: Use a product construction where the right hand side of a rule of the first gsm is processed by the second gsm (pipelining).

(c) Let G be a gsm. Give a gsm G^R such that $\tau(G^R) = \{(w_l^R, w_r^R) \mid (w_l, w_r) \in \tau(G)\}$ where for some alphabet Σ and every $w \in \Sigma$: w^R denotes the reverse of w.

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