Advanced Topics on Weighted Tree Automata

Exercise 23 (Bimorphism characterization of BOT)

Recall the decomposition result $\mathrm{BOT} \subseteq \mathrm{REL} \circ \mathrm{FTA} \circ \mathrm{HOM}$. For every bottom-up tree transducer B we can construct a bottom-up tree relabeling R, a bottom-up finite state tree automaton A, and a bottom-up tree homomorphism H such that $\tau(B) = \tau(R) \circ \tau(A) \circ \tau(H)$.

(a) Show that $\big(\tau(R)\big)^{-1} \in \mathrm{HOM}$.

(b) Devise a short-hand notation for bottom-up finite state tree automata and tree transductions in HOM.

(c) Give a bimorphism characterization of BOT, i.e., give a formal definition of (the syntax of) bimorphisms, and its induced tree transformation. Show that the class of tree transformations induced by bimorphisms subsumes BOT.

(d) Consider the bu-tt $B = \big(\{*, q, q_f\}, \Sigma, \Sigma, \{q_f\}, R\big)$ where $\Sigma = \big\{\alpha^{(0)}, \beta^{(0)}, \gamma^{(1)}, \sigma^{(2)}\big\}$ and

$$R = \Big\{ \ \alpha \to q(\alpha), \quad \alpha \to *(\alpha), \quad \beta \to *(\beta), \quad \gamma(q(x_1)) \to q(\gamma(x_1)), \quad \gamma(*(x_1)) \to *(\gamma(x_1)), \\ \sigma(*(x_1), q(x_2)) \to q_f(x_1), \quad \sigma(*(x_1), *(x_2)) \to *(\sigma(x_1, x_2)) \ \Big\}$$

Give a bimorphism $\mathscr{B}$ such that $\tau(B) = \tau(\mathscr{B})$.
Give a derivation of $\xi = \sigma\big(\gamma(\beta), \gamma(\alpha)\big)$ in B.
Give a tree $\zeta \in T_\Omega$ such that $\zeta \in L(A)$ and $\varphi(\zeta) = \xi$.

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