

Advanced Topics on Weighted Tree Automata

Exercise 24 (Simulation of BOT and TOP)

Let $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}, \beta^{(0)}\}$ be a ranked alphabet. Consider the bu-tt $B = (Q_B, \Sigma, \Sigma, F, R_B)$ and the td-tt $T = (Q_T, \Sigma, \Sigma, I, R_T)$ where $Q_B = \{*, q, q_f\}$, $F = \{q_f\}$, $Q_T = \{*, q\}$, $I = \{*\}$, and

$$\begin{array}{ll}
 R_B = \{ \sigma(*(x_1), *(x_2)) \rightarrow *(\sigma(x_1, x_2)), & R_T = \{ q(\sigma(x_1, x_2)) \rightarrow \sigma(q(x_1), q(x_2)), \\
 \sigma(*(x_1), q(x_2)) \rightarrow q_f(x_1), & *(\sigma(x_1, x_2)) \rightarrow \sigma(q(x_1), *(x_1)), \\
 \gamma(*(x_1)) \rightarrow *(\gamma(x_1)), & *(\sigma(x_1, x_2)) \rightarrow \sigma(*(x_1), q(x_1)), \\
 \gamma(q(x_1)) \rightarrow q(\gamma(x_1)), & *(\gamma(x_1)) \rightarrow \gamma(*(x_1)), \\
 \gamma(q_f(x_1)) \rightarrow q_f(\gamma(x_1)), & q(\gamma(x_1)) \rightarrow \gamma(q(x_1)), \\
 \alpha \rightarrow *(\alpha), \alpha \rightarrow q(\alpha), \beta \rightarrow q(\beta) \} & *(\alpha) \rightarrow \alpha, q(\alpha) \rightarrow \alpha, *(\beta) \rightarrow \beta \}
 \end{array}$$

- (a) Identify the bottom-up and top-down specific properties of the tree transformations induced by B and T respectively.
- (b) Give td-tt T_1 and T_2 and bu-tt B_1 and B_2 such that $\tau(B) = \tau(T_1) \circ \tau(T_2)$ and $\tau(T) = \tau(B_1) \circ \tau(B_2)$.