Advanced Topics on Weighted Tree Automata

Exercise 25 (Baker's Theorem for BOT)

Baker's Theorem for BOT states a sufficient criterion for the composition of two bottom-up tree transformations to remain in BOT:

> **Theorem**. Let B_1 and B_2 be bu-tt. Then $\tau(B_1) \circ \tau(B_2) \in \mathrm{BOT}$ if the following two conditions hold:
>
> 1. B_1 is linear or B_2 is deterministic;
> 2. B_1 is nondeleting or B_2 is total.

(a) Give two bu-tt B_1' and B_2' that fulfill Condition 1 but not Condition 2. For each bu-tt use the minimum number of rules necessary.

(b) Construct the instance B' (for B_1' and B_2') of the bu-tt B defined in the proof of Theorem 6.3.

(c) Give a tree transformation (s,t) such that $\neg\big((s,t) \in \tau(B_1') \circ \tau(B_2') \iff (s,t) \in \tau(B')\big)$.

Exercise 26 (Baker's Theorem for TOP)

Baker's Theorem for TOP states a sufficient criterion for the composition of two top-down tree transformations to remain in TOP:

> **Theorem**. Let T_1 and T_2 be td-tt. Then $\tau(T_1) \circ \tau(T_2) \in \mathrm{TOP}$ if the following two conditions hold:
>
> 1. T_1 is deterministic or T_2 is linear;
> 2. T_1 is total or T_2 is nondeleting.

Let $T_1 = \big(Q, \Sigma, \Delta, I_1, R_1\big)$ and $T_2 = \big(P, \Delta, \Omega, I_2, R_2\big)$ be td-tt.

(a) Construct a td-tt T such that $\tau(T_1) \circ \tau(T_2) = \tau(T)$ if the above conditions hold.

(b) Prove that $\tau(T_1) \circ \tau(T_2) = \tau(T)$.

(c) Give two td-tt T_1' and T_2' that fulfill Condition 1 but not Condition 2. For each td-tt use the minimum number of rules necessary.

(d) Construct the instance T' (for T_1' and T_2') of the td-tt T defined in Exercise 26(a).

(e) Give a tree transformation (s,t) such that $\neg\big((s,t) \in \tau(T_1') \circ \tau(T_2') \iff (s,t) \in \tau(T')\big)$.

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