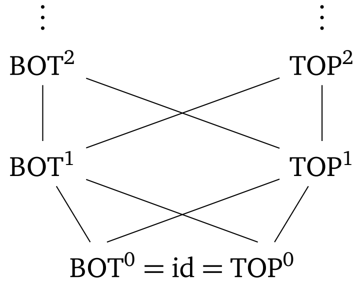
Advanced Topics on Weighted Tree Automata

Exercise 27 ($BOT^n \subseteq TOP^{n+1}$ and $TOP^n \subseteq BOT^{n+1}$)

Using the fact that $\forall n \geq 1\colon \mathrm{BOT}^n \bigcirc\!\!\!\!\bigcirc \mathrm{TOP}^n$ [Eng82], prove the correctness of the following Hasse-diagram:



Exercise 28 (Top-down tree transducer with regular look-ahead)

Let $T = \big(Q, \Sigma, \Delta, I, R\big)$ be a top-down tree transducer with regular look-ahead.

(a) Give a formal definition of the derivation relation of T.

(b) Give a formal definition of the tree transformation induced by T.

Exercise 29 ($TOP^R \subseteq$ d-QREL $\circ$ TOP)

Let $L_1, \ldots, L_n \in \mathrm{REC}(\Sigma)$ be recognizable tree languages, $\mathscr{U} = \{0,1\}^n$, and Σ and Ω be ranked alphabets where $\Omega = \big\{ \langle \sigma, (u_1, \ldots, u_k) \rangle^{(k)} \mid \sigma \in \Sigma,\ \mathrm{rk}(\sigma) = k,\ u_1, \ldots, u_k \in \mathscr{U} \big\}$. Note that if $k = 0$ we write σ rather than $\langle \sigma, () \rangle$. Consider the recursively defined non-deterministic total function $B_{L_1,\ldots,L_n}\colon T_\Sigma \to \mathscr{P}(T_\Omega)$ where for every $\sigma \in \Sigma$ and $t_1, \ldots, t_k \in T_\Sigma$:

$$B_{L_1,\ldots,L_n}(\sigma(t_1, \ldots, t_k)) = \big\{ \langle \sigma, (u_1, \ldots, u_k) \rangle (t'_1, \ldots, t'_k) \mid t'_1 \in B_{L_1,\ldots,L_n}(t_1),\ \ldots,\ t'_k \in B_{L_1,\ldots,L_n}(t_k),$$
$$\forall i \in [k],\ \forall j \in [n]\colon\ u_i \in \mathscr{U}, u_i(j) = \text{if } t_i \in L_j \text{ then } 1 \text{ else } 0 \big\}$$

(a) Give a deterministic state-relabeling bu-tt B that computes $B_{L_1,\ldots,L_n}$.

(b) Let $\Sigma = \big\{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}, \beta^{(0)}\big\}$ be a ranked alphabet, $n = 2$, and $L_1 = \{\gamma^k(\beta) \mid k \in \mathbb{N}\}$ and $L_2 = \{\gamma^k(\alpha) \mid k \in \mathbb{N}\}$ be recognizable tree languages. Construct a deterministic state-relabeling bu-tt B' that computes B_{L_1,L_2}.

Exercise 30 (l-BOT = l-TOP^R)

Consider the linear bu-tt $B = \big(Q, \Sigma, \Delta, F, R\big)$ where $\Sigma = \Delta = \big\{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}, \beta^{(0)}\big\}$, $Q = \{*, q, q_f\}$, $F = \{q_f\}$, and

$$R = \big\{\ \alpha \to q(\alpha), \quad \alpha \to *(\alpha), \quad \beta \to *(\beta),$$
$$\gamma(q(x_1)) \to q(\gamma(x_1)), \quad \gamma(*(x_1)) \to *(\gamma(x_1)),$$
$$\sigma(*(x_1), q(x_2)) \to q_f(x_1), \quad \sigma(*(x_1), *(x_2)) \to *(\sigma(x_1, x_2))\ \big\}$$

(a) Give a linear td-tt with regular look-ahead T such that $\tau(T) = \tau(B)$.

(b) Construct a deterministic state-relabeling bu-tt B' and a linear td-tt T' such that $\tau(T) = \tau(B') \circ \tau(T')$.

References

[Eng82] J. Engelfriet. Three hierarchies of transducers. *Math. Systems Theory*, 15(2):95–125, 1982.

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