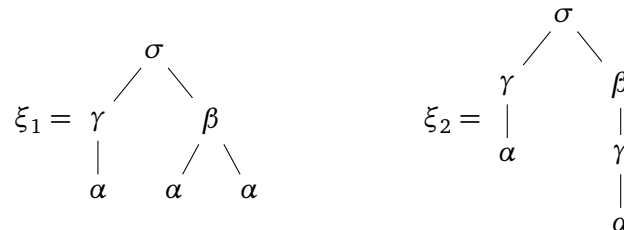


Formale Übersetzungsmodelle

Exercise 1 (Ranked alphabets and trees)

Consider the following trees:



- Give $\text{height}(\xi_i)$, $\text{size}(\xi_i)$, $\text{pos}(\xi_i)$, $\text{sub}(\xi_i)$ for $i \in \{1, 2\}$.
- Define minimal ranked alphabets Δ_1 and Δ_2 such that $\xi_1 \in T_{\Delta_1}$ and $\xi_2 \in T_{\Delta_2}$.
- Extend the intersection, union, and subset relation to ranked alphabets.
- Prove or refute: There is a ranked alphabet Σ' such that $\xi_1, \xi_2 \in T_{\Sigma'}$.

Exercise 2 (Definition by structural induction)

Let $\xi \in T_{\Sigma}$, $w \in \text{pos}(\xi)$, $\zeta \in T_{\Sigma}(X_k)$, and $\zeta'_1, \dots, \zeta'_k \in T_{\Sigma}(A)$. Define the following characteristics of ξ and ζ by structural induction:

- $\xi(w)$, the label of ξ at position w ,
- $\xi|_w$, the subtree of ξ at position w ,
- $\xi[\zeta]_w$, the tree obtained by substituting the subtree of ξ at position w with ζ ,
- $\text{yield}(\xi)$, the sequence of leaves of ξ from left to right,
- $\zeta[\zeta'_1, \dots, \zeta'_k]$, the tree obtained from ζ by substituting, for every $i \in \{1, \dots, k\}$, x_i by ζ'_i .

Exercise 3 (Proof by structural induction)

Let $\xi, \zeta \in T_{\Sigma}(A)$ and $w \in \text{pos}(\xi)$. Prove or refute the following statements:

- $\xi(w) = \xi|_w(\epsilon)$.
- $(\xi[\zeta]_w)|_w = \zeta$.
- $|\text{pos}(\xi)| = |\text{sub}(\xi)|$.

Exercise 4 (Correctness of structural induction)

Extra task: Use your knowledge from other parts of mathematics (e.g. universal algebra, order theory, etc.) to show that the principles of (a) definition and of (b) proof by structural induction on trees are correct. There are several ways to prove this.