

## Formale Übersetzungsmodelle

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### Exercise 13 (Decomposition of a bu-tt)

Let  $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$ . Consider the bu-tt  $M = (\{q_0, q_1, p\}, \Sigma, \Sigma, \{q_0\}, R)$ , where  $R$  is given by

$$\begin{aligned} \alpha \rightarrow q_0(\alpha), \quad \alpha \rightarrow p(\alpha), \quad \gamma(p(x_1)) \rightarrow p(\gamma(x_1)), \\ \sigma(q_0(x_1), p(x_2)) \rightarrow q_1(\sigma(x_2, x_1)), \quad \sigma(q_1(x_1), p(x_2)) \rightarrow q_0(\sigma(x_1, x_1)). \end{aligned}$$

- Describe  $\tau(M)$ .
- Construct, according to the decomposition result from the lecture, a relabeling  $M_1$ , an fta  $M_2$ , and a homomorphism  $M_3$  such that  $\tau(M) = \tau(M_1) \circ \tau(M_2) \circ \tau(M_3)$ .

### Exercise 14 (Decomposition of bu-tt, but with qrels)

- Sketch the construction of the qrel in the proof of  $\text{BOT} \subseteq \text{QREL} \circ \text{HOM}$ .
- Apply this construction to exercise 13, i.e., give a qrel  $M'_1$  and a homomorphism  $M'_2$  such that  $\tau(M) = \tau(M'_1) \circ \tau(M'_2)$ .

### Exercise 15 (Bimorphisms)

Prove the following (slightly modified) decomposition result for bu-tt:

$$[1]\text{-BOT} \subseteq \text{d-REL}^{-1} \circ \text{FTA} \circ [1]\text{-HOM}.$$

### Exercise 16 (Finishing the proof)

Complete the proof of the decomposition result from the lecture by proving the following statement: For every  $s \in T_\Sigma$ ,  $q \in Q$ , and  $t \in T_\Delta$ , if there is a  $u \in T_\Omega$  such that  $s \Rightarrow_{B_1}^* *(u)$ ,  $u \Rightarrow_{B_2}^* q(u)$ , and  $u \Rightarrow_{B_3}^* *(t)$ , then  $s \Rightarrow_B^* q(t)$ .