Formale Übersetzungsmodelle

Exercise 33 (Baker's Theorem for TOP, I)

Let $\Sigma = \{\gamma^{(1)}, \delta^{(1)}, \alpha^{(0)}\}$, $\Delta = \{\tau^{(2)}, \sigma^{(2)}, \alpha^{(0)}\}$, and $\Gamma = \{\vartheta^{(3)}, \tau^{(2)}, \alpha^{(0)}\}$ be ranked alphabets. Consider the td-tt $T_1 = (Q, \Sigma, \Delta, I_1, R_1)$ and $T_2 = (P, \Delta, \Gamma, I_2, R_2)$, where

- $Q = \{q_1, q_2\}$, $I_1 = \{q_1\}$, $P = I_2 = \{p\}$,
- R_1 contains the rules

$$q_1(\gamma(x_1)) \to \sigma\big(\alpha, \tau(q_2(x_1), \alpha)\big) \qquad q_2(\delta(x_1)) \to q_1(x_1) \qquad q_1(\alpha) \to \alpha\,,$$

- R_2 contains the rules

$$p(\sigma(x_1, x_2)) \to \vartheta(p(x_1), p(x_2), p(x_2)) \qquad p(\tau(x_1, x_2)) \to \tau(p(x_2), p(x_1)) \qquad p(\alpha) \to \alpha\,.$$

(a) Do T_1 and T_2 satisfy the conditions demanded in Baker's theorem for TOP?

(b) Apply Baker's construction to T_1 and T_2!

(c) Denote the result of the above construction by T. Is $\tau(T) = \tau(T_1) \circ \tau(T_2)$?

Exercise 34 (Baker's Theorem for TOP, II)

Let $\Sigma = \{\gamma^{(1)}, \alpha^{(0)}\}$, $\Delta = \Sigma \cup \{\delta^{(1)}\}$, and $\Gamma = \{\sigma^{(2)}, \tau^{(2)}, \alpha^{(0)}\}$ be ranked alphabets. Consider the td-tt $T_1 = (Q, \Sigma, \Delta, I_1, R_1)$ and $T_2 = (P, \Delta, \Gamma, I_2, R_2)$, where

- $Q = I_1 = \{q\}$, $P = I_2 = \{p\}$,
- R_1 contains the rules

$$q(\gamma(x_1)) \to \gamma(q(x_1)) \qquad q(\gamma(x_1)) \to \delta(q(x_1)) \qquad q(\alpha) \to \alpha$$

- R_2 contains the rules

$$p(\gamma(x_1)) \to \sigma(p(x_1), p(x_1)) \qquad p(\delta(x_1)) \to \tau(p(x_1), p(x_1)) \qquad p(\alpha) \to \alpha$$

(a) Do T_1 and T_2 satisfy the conditions demanded in Baker's theorem for TOP?

(b) Apply Baker's construction to T_1 and T_2!

(c) Denote the result of the above construction by T. Is $\tau(T) = \tau(T_1) \circ \tau(T_2)$?

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Exercise 35 (Baker's Theorem for TOP, III)

Let $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}\}$, and $\Delta = \{\gamma^{(1)}\alpha^{(0)}\}$ be ranked alphabets. Consider the td-tt $T_1 = (Q, \Sigma, \Sigma, I_1, R_1)$ and $T_2 = (P, \Sigma, \Delta, I_2, R_2)$, where

- $Q = I_1 = \{q\}$, $P = I_2 = \{p\}$,
- R_1 contains the rules

$$q(\sigma(x_1, x_2)) \to \sigma(q(x_1), q(x_2)) \qquad q(\alpha) \to \alpha$$

- R_2 contains the rules

$$p(\sigma(x_1, x_2)) \to \gamma(p(x_1)) \qquad p(\alpha) \to \alpha$$

(a) Do T_1 and T_2 satisfy the conditions demanded in Baker's theorem for TOP?

(b) Apply Baker's construction to T_1 and T_2!

(c) Denote the result of the above construction by T. Is $\tau(T) = \tau(T_1) \circ \tau(T_2)$?

Exercise 36 (Hierarchies of Transducers)

As proved by Engelfriet,[1] the hierarchy $(\mathrm{TOP}^n)_{n\in\mathbb{N}}$ is proper, i.e.,

$$\mathrm{TOP}^0 \subsetneq \mathrm{TOP}^1 \subsetneq \mathrm{TOP}^2 \subsetneq \mathrm{TOP}^3 \subsetneq \cdots .$$

Use this fact and your knowledge from the lecture to give a Hasse diagram relating the classes TOP^n, and BOT^n, for every $n \in \mathbb{N}$.

[1] Corollary to Thm. 3.14 in J. Engelfriet, Three Hierarchies of Transducers, *Math. Syst. Theory* 15, pp. 95-125 (1982)