

Formale Baumsprachen

Task 4 (deterministic bu-ta)

Let $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}, \beta^{(0)}\}$. Give deterministic bu-ta \mathcal{A}_1 and \mathcal{A}_2 such that $L_1 = L(\mathcal{A}_1)$ and $L_2 = L(\mathcal{A}_2)$ where

- (a) $L_1 = \{\xi \in T_\Sigma \mid \xi \text{ contains at least one } \alpha \text{ and one } \beta\}$ and
- (b) $L_2 = \{\xi \in T_\Sigma \mid \xi \text{ contains an even number of } \alpha \text{ symbols}\}$.

Task 5 (finite state automata)

Recall the concept of string automata. Let Σ be an alphabet and $\# \notin \Sigma$. We define the ranked alphabet $\Sigma_\# = \Sigma_\#^{(0)} \cup \Sigma_\#^{(1)}$ where $\Sigma_\#^{(0)} = \{\#\}$ and $\Sigma_\#^{(1)} = \Sigma$. Moreover, we define the $\Sigma_\#$ -algebra (Σ^*, θ) where $\theta(\#) = \varepsilon$ and $\theta(a)(w) = wa$ for every $a \in \Sigma$ and $w \in \Sigma^*$.

- (a) Show that Σ^* is initial in the class of $\Sigma_\#$ -algebras.
- (b) We consider $\Sigma = \{a, b\}$ and the language $L = \{a^n b^m \mid n, m \in \mathbb{N}\}$. Sketch the diagram of a total deterministic finite-state automaton accepting L and model the transition table using a finite $\Sigma_\#$ -algebra Q . How can we interpret the uniquely determined homomorphism $h: \Sigma^* \rightarrow Q$?
- (c) Convince yourself that any total deterministic finite-state automaton can be modeled as a quadruple $\mathcal{A} = (Q, \Sigma, \theta, F)$ where (Q, θ) is a finite $\Sigma_\#$ -algebra and $F \subseteq Q$. Define the language accepted by \mathcal{A} using the homomorphism $h: \Sigma^* \rightarrow Q$.

Task 6 (bud-Rec(Σ) \subseteq Rec(Σ))

Let Σ be a ranked alphabet. In the lecture we have shown that $\text{Rec}(\Sigma)$ is a subset of $\text{bud-Rec}(\Sigma)$ using the powerset construction. Show that $\text{bud-Rec}(\Sigma)$ is a subset of $\text{Rec}(\Sigma)$.