Formale Baumsprachen

Task 31 (power set construction on weighted tree automata)

Consider the Viterbi-semiring $S = ([0,1], \max, \cdot, 0, 1)$ and the weighted tree automaton $\mathcal{A} = (Q, \Sigma, S, \delta, F)$ where $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}\}$, $Q = \{Z, B\}$, $F = 1.Z$, and

$$\delta_\alpha(\varepsilon, B) = 1 \qquad \delta_\alpha(\varepsilon, Z) = 0.2 \qquad \delta_\sigma = (0.5).(BZ, Z)\ .$$

(a) Define a powerset construction for weighted tree automata similar to the unweighted case.

(b) Use the definition from Task 31 (a) on $\mathcal{A}$. What problem arises?

Task 32 (closure of recognizable step functions)

Show that recognizable step functions are closed under pointwise addition $+$ and Hadamard product $\odot$.

Task 33 (unrestricted MSO and recognizability)

Consider the ranked alphabet $\Sigma = \{\gamma^{(1)}, \alpha^{(0)}\}$, the semiring $\mathbb{N}$, and let $\varphi = \forall x.\forall y.2$ and $\psi = \forall X.2$ where $\varphi, \psi \in \mathrm{MSO}(\mathbb{N}, \Sigma)$.

(a) Show that φ is not restricted.

(b) Show that $[\![\varphi]\!]$ is not recognizable.

(c) Show that $[\![\psi]\!]$ is not recognizable.

Task 34 (encoding weighted tree automata as REMSO formulae)

Let $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}\}$ be a ranked alphabet and $\Delta = \{a, b, c\}$ be an unranked alphabet. Moreover, let $\mathcal{A} = (Q, \Sigma, \mathcal{P}(\Delta^*), \delta, F)$ be a weighted tree automaton over the semiring $\mathcal{P}(\Delta^*)$ of formal languages where $Q = \{f, q, p\}$, $F(f) = \{\varepsilon\}$, $F(q) = F(p) = \emptyset$, and

$$\delta_\sigma = \{\varepsilon\}.(qq, q) + \{b\}.(qf, f) + \{c\}.(qp, f) + \{a\}.(fq, f)$$

as well as

$$\delta_\alpha = \{\varepsilon\}.(\varepsilon, q) + \{\varepsilon\}.(\varepsilon, p).$$

(a) Determine the semantics of $\mathcal{A}$ on a tree $\xi \in T_\Sigma$.

(b) Apply the construction in Theorem 5.11 to obtain a formula $\varphi \in \mathrm{REMSO}(\mathcal{P}(\Delta^*), \Sigma)$ such that $[\![\varphi]\!] = \mathrm{r}_{\mathcal{A}}$.

Task 35 (idempotent semirings are locally finite)

Show that every commutative semiring $(S, +, \cdot, 0, 1)$ with idempotent addition and multiplication (i.e., $a + a = a = a \cdot a$ for every $a \in S$) is locally finite.

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Task 36 (rings, additively idempotent, and zero-sum free semirings)

Prove or refute the following statements:

(a) Only the trivial ring (i.e., the ring whose carrier has only one element) is zero-sum free.

(b) Every additively idempotent semiring is zero-sum free.

(c) Every zero-sum free semiring is additively idempotent.

Task 37 (f.o. universal quantification of recognizable step functions is recognizable)

Let $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}\}$ be a ranked alphabet and $\varphi = \forall x.\psi$ with $\psi = (2 \wedge \mathrm{label}_\sigma(x)) \vee (3 \wedge \mathrm{label}_\alpha(x))$ where $\varphi, \psi \in \mathrm{MSO}(\mathbb{N}, \Sigma)$.

(a) Convince yourself that $[\![\psi]\!]_{\{x\}}$ is a recognizable step function

$$[\![\psi]\!]_{\{x\}} = \textstyle\sum_{i=1}^{k} n_i \cdot \mathbb{1}_{L_i},$$

where $k, n_1, ..., n_k \in \mathbb{N}$, and $L_1, ..., L_k \subseteq \mathrm{T}_{\Sigma_{\{x\}}}$ are recognizable tree languages that partition $\mathrm{T}_{\Sigma_{\{x\}}}$.

(b) Devise finite tree automata $M_1, ..., M_k$ that recognize, respectively, the tree languages $L_1, ..., L_k$.

(c) Apply the technique from the lecture to construct finite tree automata $\tilde{M}_1, ..., \tilde{M}_k$ from $M_1, ..., M_k$ such that for every $j \in [k]$:

$$L(\tilde{M}_j) = \big\{(\xi, \nu) \in \mathrm{T}_{\tilde{\Sigma}_\emptyset} \mid \forall w \in \mathrm{pos}(\xi)\colon \nu(w) = j \text{ implies } \xi[x \to w] \in L_j\big\},$$

where $\tilde{\Sigma} = \Sigma \times [k]$, utilizing the bijection between $\mathrm{T}_{\tilde{\Sigma}_\emptyset}$ and $\{(\xi, \nu) \mid \xi \in \mathrm{T}_{\tilde{\Sigma}_\emptyset}, \nu\colon \mathrm{pos}(\xi) \to [k]\}$.

(d) Based on Task 37 (c) we can assume a finite tree automaton recognizing $\tilde{L} = \bigcap_{i=1}^{k} L(\tilde{M}_i)$. Sketch how the rest of the proof of Droste and Vogler [DV06, Lemma 5.5] goes through for this example.

References

[DV06] Manfred Droste and Heiko Vogler. “Weighted tree automata and weighted logics”. In: *Theoretical Computer Science* 366.3 (2006), pp. 228–247. issn: 0304-3975. doi: `10.1016/j.tcs.2006.08.025`.

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